

AP[®] CALCULUS AB 2002 SCORING GUIDELINES

Question 2

The rate at which people enter an amusement park on a given day is modeled by the function E defined by

$$E(t) = \frac{15600}{(t^2 - 24t + 160)}.$$

The rate at which people leave the same amusement park on the same day is modeled by the function L defined by

$$L(t) = \frac{9890}{(t^2 - 38t + 370)}.$$

Both $E(t)$ and $L(t)$ are measured in people per hour and time t is measured in hours after midnight. These functions are valid for $9 \leq t \leq 23$, the hours during which the park is open. At time $t = 9$, there are no people in the park.

- (a) How many people have entered the park by 5:00 P.M. ($t = 17$)? Round answer to the nearest whole number.
- (b) The price of admission to the park is \$15 until 5:00 P.M. ($t = 17$). After 5:00 P.M., the price of admission to the park is \$11. How many dollars are collected from admissions to the park on the given day? Round your answer to the nearest whole number.
- (c) Let $H(t) = \int_9^t (E(x) - L(x)) dx$ for $9 \leq t \leq 23$. The value of $H(17)$ to the nearest whole number is 3725. Find the value of $H'(17)$ and explain the meaning of $H(17)$ and $H'(17)$ in the context of the park.
- (d) At what time t , for $9 \leq t \leq 23$, does the model predict that the number of people in the park is a maximum?

(a) $\int_9^{17} E(t) dt = 6004.270$
6004 people entered the park by 5 pm.

(b) $15 \int_9^{17} E(t) dt + 11 \int_{17}^{23} E(t) dt = 104048.165$

The amount collected was \$104,048.

or

$$\int_{17}^{23} E(t) dt = 1271.283$$

1271 people entered the park between 5 pm and

11 pm, so the amount collected was

$$\$15 \cdot (6004) + \$11 \cdot (1271) = \$104,041.$$

(c) $H'(17) = E(17) - L(17) = -380.281$

There were 3725 people in the park at $t = 17$.

The number of people in the park was decreasing at the rate of approximately 380 people/hr at time $t = 17$.

(d) $H'(t) = E(t) - L(t) = 0$

$$t = 15.794 \text{ or } 15.795$$

3 { 1 : limits
1 : integrand
1 : answer

1 : setup

3 { 1 : value of $H'(17)$
2 : meanings
1 : meaning of $H(17)$
1 : meaning of $H'(17)$
< -1 > if no reference to $t = 17$

2 { 1 : $E(t) - L(t) = 0$
1 : answer

2)

AP[®] CALCULUS AB
2004 SCORING GUIDELINES

Question 1

Traffic flow is defined as the rate at which cars pass through an intersection, measured in cars per minute. The traffic flow at a particular intersection is modeled by the function F defined by

$$F(t) = 82 + 4 \sin\left(\frac{t}{2}\right) \text{ for } 0 \leq t \leq 30,$$

where $F(t)$ is measured in cars per minute and t is measured in minutes.

- (a) To the nearest whole number, how many cars pass through the intersection over the 30-minute period?
- (b) Is the traffic flow increasing or decreasing at $t = 7$? Give a reason for your answer.
- (c) What is the average value of the traffic flow over the time interval $10 \leq t \leq 15$? Indicate units of measure.
- (d) What is the average rate of change of the traffic flow over the time interval $10 \leq t \leq 15$? Indicate units of measure.

(a) $\int_0^{30} F(t) dt = 2474$ cars

$$3 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$$

(b) $F'(7) = -1.872$ or -1.873
Since $F'(7) < 0$, the traffic flow is decreasing at $t = 7$.

1 : answer with reason

(c) $\frac{1}{5} \int_{10}^{15} F(t) dt = 81.899$ cars/min

$$3 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$$

(d) $\frac{F(15) - F(10)}{15 - 10} = 1.517$ or 1.518 cars/min²

1 : answer

Units of cars/min in (c) and cars/min² in (d)

1 : units in (c) and (d)

3)

AP[®] CALCULUS AB
2005 SCORING GUIDELINES

Question 2

The tide removes sand from Sandy Point Beach at a rate modeled by the function R , given by

$$R(t) = 2 + 5 \sin\left(\frac{4\pi t}{25}\right).$$

A pumping station adds sand to the beach at a rate modeled by the function S , given by

$$S(t) = \frac{15t}{1 + 3t}.$$

Both $R(t)$ and $S(t)$ have units of cubic yards per hour and t is measured in hours for $0 \leq t \leq 6$. At time $t = 0$, the beach contains 2500 cubic yards of sand.

- (a) How much sand will the tide remove from the beach during this 6-hour period? Indicate units of measure.
 (b) Write an expression for $Y(t)$, the total number of cubic yards of sand on the beach at time t .
 (c) Find the rate at which the total amount of sand on the beach is changing at time $t = 4$.
 (d) For $0 \leq t \leq 6$, at what time t is the amount of sand on the beach a minimum? What is the minimum value? Justify your answers.

(a) $\int_0^6 R(t) dt = 31.815$ or 31.816 yd^3

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer with units} \end{cases}$

(b) $Y(t) = 2500 + \int_0^t (S(x) - R(x)) dx$

3 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{answer} \end{cases}$

(c) $Y'(t) = S(t) - R(t)$

$Y'(4) = S(4) - R(4) = -1.908$ or $-1.909 \text{ yd}^3/\text{hr}$

1 : answer

(d) $Y'(t) = 0$ when $S(t) - R(t) = 0$.

The only value in $[0, 6]$ to satisfy $S(t) = R(t)$ is $a = 5.117865$.

3 : $\begin{cases} 1 : \text{sets } Y'(t) = 0 \\ 1 : \text{critical } t\text{-value} \\ 1 : \text{answer with justification} \end{cases}$

t	$Y(t)$
0	2500
a	2492.3694
6	2493.2766

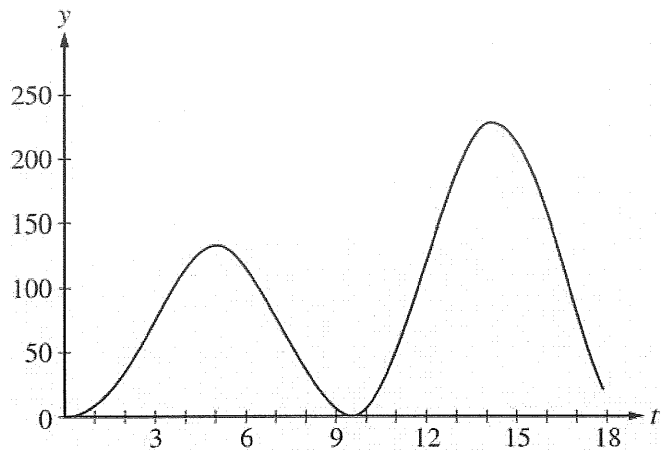
The amount of sand is a minimum when $t = 5.117$ or 5.118 hours. The minimum value is 2492.369 cubic yards.

4)

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2006 SCORING GUIDELINES

Question 2

At an intersection in Thomasville, Oregon, cars turn left at the rate $L(t) = 60\sqrt{t} \sin^2\left(\frac{t}{3}\right)$ cars per hour over the time interval $0 \leq t \leq 18$ hours. The graph of $y = L(t)$ is shown above.



- (a) To the nearest whole number, find the total number of cars turning left at the intersection over the time interval $0 \leq t \leq 18$ hours.
- (b) Traffic engineers will consider turn restrictions when $L(t) \geq 150$ cars per hour. Find all values of t for which $L(t) \geq 150$ and compute the average value of L over this time interval. Indicate units of measure.
- (c) Traffic engineers will install a signal if there is any two-hour time interval during which the product of the total number of cars turning left and the total number of oncoming cars traveling straight through the intersection is greater than 200,000. In every two-hour time interval, 500 oncoming cars travel straight through the intersection. Does this intersection require a traffic signal? Explain the reasoning that leads to your conclusion.

(a) $\int_0^{18} L(t) dt \approx 1658$ cars

(b) $L(t) = 150$ when $t = 12.42831, 16.12166$
 Let $R = 12.42831$ and $S = 16.12166$
 $L(t) \geq 150$ for t in the interval $[R, S]$
 $\frac{1}{S - R} \int_R^S L(t) dt = 199.426$ cars per hour

- (c) For the product to exceed 200,000, the number of cars turning left in a two-hour interval must be greater than 400.

$$\int_{13}^{15} L(t) dt = 431.931 > 400$$

OR

The number of cars turning left will be greater than 400 on a two-hour interval if $L(t) \geq 200$ on that interval.
 $L(t) \geq 200$ on any two-hour subinterval of $[13.25304, 15.32386]$.

Yes, a traffic signal is required.

2 : { 1 : setup
1 : answer

3 : { 1 : t -interval when $L(t) \geq 150$
1 : average value integral
1 : answer with units

4 : { 1 : considers 400 cars
1 : valid interval $[h, h + 2]$
1 : value of $\int_h^{h+2} L(t) dt$
1 : answer and explanation

OR

4 : { 1 : considers 200 cars per hour
1 : solves $L(t) \geq 200$
1 : discusses 2 hour interval
1 : answer and explanation

5)

AP[®] CALCULUS AB
2007 SCORING GUIDELINES

Question 2

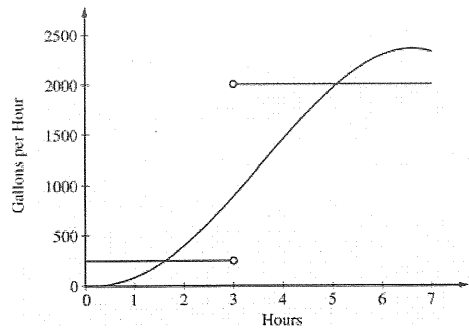
The amount of water in a storage tank, in gallons, is modeled by a continuous function on the time interval $0 \leq t \leq 7$, where t is measured in hours. In this model, rates are given as follows:

- (i) The rate at which water enters the tank is

$$f(t) = 100t^2 \sin(\sqrt{t}) \text{ gallons per hour for } 0 \leq t \leq 7.$$

- (ii) The rate at which water leaves the tank is

$$g(t) = \begin{cases} 250 & \text{for } 0 \leq t < 3 \\ 2000 & \text{for } 3 < t \leq 7 \end{cases} \text{ gallons per hour.}$$



The graphs of f and g , which intersect at $t = 1.617$ and $t = 5.076$, are shown in the figure above. At time $t = 0$, the amount of water in the tank is 5000 gallons.

- (a) How many gallons of water enter the tank during the time interval $0 \leq t \leq 7$? Round your answer to the nearest gallon.
- (b) For $0 \leq t \leq 7$, find the time intervals during which the amount of water in the tank is decreasing. Give a reason for each answer.
- (c) For $0 \leq t \leq 7$, at what time t is the amount of water in the tank greatest? To the nearest gallon, compute the amount of water at this time. Justify your answer.

(a) $\int_0^7 f(t) dt \approx 8264$ gallons

- (b) The amount of water in the tank is decreasing on the intervals $0 \leq t \leq 1.617$ and $3 \leq t \leq 5.076$ because $f(t) < g(t)$ for $0 \leq t < 1.617$ and $3 < t < 5.076$.

- (c) Since $f(t) - g(t)$ changes sign from positive to negative only at $t = 3$, the candidates for the absolute maximum are at $t = 0, 3$, and 7 .

t (hours)	gallons of water
0	5000
3	$5000 + \int_0^3 f(t) dt - 250(3) = 5126.591$
7	$5126.591 + \int_3^7 f(t) dt - 2000(4) = 4513.807$

The amount of water in the tank is greatest at 3 hours. At that time, the amount of water in the tank, rounded to the nearest gallon, is 5127 gallons.

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

2 : $\begin{cases} 1 : \text{intervals} \\ 1 : \text{reason} \end{cases}$

5 : $\begin{cases} 1 : \text{identifies } t = 3 \text{ as a candidate} \\ 1 : \text{integrand} \\ 1 : \text{amount of water at } t = 3 \\ 1 : \text{amount of water at } t = 7 \\ 1 : \text{conclusion} \end{cases}$

6)

AP[®] CALCULUS AB
2009 SCORING GUIDELINES

Question 2

The rate at which people enter an auditorium for a rock concert is modeled by the function R given by $R(t) = 1380t^2 - 675t^3$ for $0 \leq t \leq 2$ hours; $R(t)$ is measured in people per hour. No one is in the auditorium at time $t = 0$, when the doors open. The doors close and the concert begins at time $t = 2$.

- (a) How many people are in the auditorium when the concert begins?
- (b) Find the time when the rate at which people enter the auditorium is a maximum. Justify your answer.
- (c) The total wait time for all the people in the auditorium is found by adding the time each person waits, starting at the time the person enters the auditorium and ending when the concert begins. The function w models the total wait time for all the people who enter the auditorium before time t . The derivative of w is given by $w'(t) = (2 - t)R(t)$. Find $w(2) - w(1)$, the total wait time for those who enter the auditorium after time $t = 1$.
- (d) On average, how long does a person wait in the auditorium for the concert to begin? Consider all people who enter the auditorium after the doors open, and use the model for total wait time from part (c).

(a) $\int_0^2 R(t) dt = 980$ people

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

- (b) $R'(t) = 0$ when $t = 0$ and $t = 1.36296$
The maximum rate may occur at 0, $a = 1.36296$, or 2.

$$R(0) = 0$$

$$R(a) = 854.527$$

$$R(2) = 120$$

The maximum rate occurs when $t = 1.362$ or 1.363.

3 : $\begin{cases} 1 : \text{considers } R'(t) = 0 \\ 1 : \text{interior critical point} \\ 1 : \text{answer and justification} \end{cases}$

(c) $w(2) - w(1) = \int_1^2 w'(t) dt = \int_1^2 (2 - t)R(t) dt = 387.5$

The total wait time for those who enter the auditorium after time $t = 1$ is 387.5 hours.

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(d) $\frac{1}{980}w(2) = \frac{1}{980} \int_0^2 (2 - t)R(t) dt = 0.77551$

On average, a person waits 0.775 or 0.776 hour.

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$