

Key

AP Topic 5a: Antiderivative Word Problem FROs

Concept Reminders: (Units of Measure)

1) If $v(t)$ is in **meters/sec**, then $v'(t)$ will be in **meters/sec²** because we are applying a derivative, which is a rate of change of the starting function (finding **slope** of graph through derivative)

Example: $\frac{6m/s - 2m/s}{2s - 1s} = \frac{4m/s}{1s} = 4m/s^2$

2) If $v(t)$ is in **meters/sec**, then $\int v(t)dt$ will change to a new unit of measure in **meters**, since the accumulation of rate (finding **area** under graph through integration) will result in a new unit of measure, giving us displacement.

Example: travel 60 mph over a 5 hr period: $\frac{60 \text{ miles}}{\text{hr}} \times 5 \text{ hr} = 300 \text{ miles} \rightarrow \int_0^5 60 dt = 60t \Big|_0^5 = 60(5) - 60(0) = 300 \text{ miles}$

1) (calculator question) (2002 AP exam #2)

The rate at which people enter an amusement park on a given day is modeled by the function E defined by

$$E(t) = \frac{15600}{(t^2 - 24t + 160)}$$

The rate at which people leave the same amusement park on the same day is modeled by the function L defined by

$$L(t) = \frac{9890}{(t^2 - 38t + 370)}$$

Both $E(t)$ and $L(t)$ are measured in **people per hour** and time t is measured in hours after midnight. These functions are valid for $9 \leq t \leq 23$, the hours during which the park is open. At time $t = 9$, there are no people in the park.

- a) How many people have entered the park by 5:00 pm ($t = 17$)? Round your answer to the nearest whole number.
- b) The price of admission to the park is \$15 until 5:00 pm ($t = 17$). After 5:00 pm, the price of admission to the park is \$11. How many dollars are collected from admissions to the park on the given day? Round your answer to the nearest whole number.
- c) Let $H(t) = \int_9^t (E(x) - L(x))dx$ for $9 \leq t \leq 23$. The value of $H(17)$ to the nearest whole number is 3725. Find the value of $H'(17)$, and explain the meaning of $H(17)$ and $H'(17)$ in the context of the amusement park.
- d) At what time t , for $9 \leq t \leq 23$, does the model predict that the number of people in the park is a maximum?

a) $\int_9^{17} E(t) dt = 6004$ people entered in the park by 5 pm.

people/hr

b) $15 \cdot \int_9^{17} E(t) dt + 11 \cdot \int_{17}^{23} E(t) dt = \$104,048$

$\$90,064.05 + \$139,84.11$

c) next page

d) next page

$$c) H(t) = \int_9^t E(x) - L(x) dx$$

$$H(17) = \int_9^{17} E(x) - L(x) dx$$

$H(17) = 3725$ people left
in the park 5pm

$\int_9^{17} E(x) dx$
of people
entered in the
park b/t
9-5pm

$\int_9^{17} L(x) dx$
of people
left the park
b/t 9-5pm

$$\rightarrow H'(t) = \frac{d}{dt} \int_9^t E(x) - L(x) dx$$

$$H'(t) = E(t) - L(t)$$

$$H'(17) = E(17) - L(17) = -380.28 \text{ people/hr}$$

of people in the park is decreasing
at rate of 380 people/hr.

people/hr
Rate at which
the people in
park changing

d) # of people at maximum



* set $H'(t) = 0$

$$0 = E(t) - L(t)$$

Graph this

x-int

$$E(t) = L(t)$$

d) $t = 15.794$ is
when maximum # of
people in park

Easier example:

$$E(t) = 100 \text{ people/hr} \rightarrow 5 \text{ hrs} \rightarrow 500 \text{ people}$$

$$\int_0^5 100 dt = 500 \text{ people}$$

#2.

2004 #1 (calculator question)

Traffic flow is defined as the rate at which cars pass through an intersection, measured in cars per minute. The traffic flow at a particular intersection is modeled by the function F defined by $F(t) = 82 + 4 \sin\left(\frac{t}{2}\right)$ for $0 \leq t \leq 30$, where $F(t)$ is measured in cars per minute and t is measured in minutes.

- To the nearest whole number, how many cars pass through the intersection over the 30-minute period?
- Is the traffic flow increasing or decreasing at $t = 7$? Give a reason for your answer.
- What is the average value of the traffic flow over the time interval $10 \leq t \leq 15$? Indicate units of measure.
- What is the average rate of change of the traffic flow over the time interval $10 \leq t \leq 15$? Indicate units of measure.

$$a) \int_0^{30} F(t) dt = 2474 \text{ cars}$$

b) Find derivative of $F(x)$, then evaluate $F'(7)$

$$F'(7) = -1.872$$

*calculator Math 8

(\rightarrow nDeriv($F(t)$, x , 7))

Since $F'(7) < 0$, traffic flow is decreasing at $t = 7$.

c) Avg. value theorem

$$\frac{1}{b-a} \int_a^b F(t) dt = \frac{1}{15-10} \int_{10}^{15} F(t) dt = 81.899 \text{ cars/min}$$

d) Avg. rate of change of traffic flow = $\frac{\text{change in traffic flow}}{\text{change in time}}$

$$= \frac{F(15) - F(10)}{15 - 10} = 1.517 \text{ cars/min}^2$$

3) 2005 #2 (calculator question)

The tide removes sand from Sandy Point Beach at a rate modeled by the function R , given by

$$R(t) = 2 + 5 \sin\left(\frac{4\pi t}{25}\right).$$

A pumping station adds sand to the beach at a rate modeled by the function S , given by $S(t) = \frac{15t}{1+3t}$.

Both $R(t)$ and $S(t)$ have units of cubic yards per hour and t is measured in hours for $0 \leq t \leq 6$. At time $t = 0$, the beach contains 2500 cubic yards of sand.

- How much sand will the tide remove from the beach during this 6-hour period? Indicate units of measure.
- Write an expression for $Y(t)$, the total number of cubic yards of sand on the beach at time t .
- Find the rate at which the total amount of sand on the beach is changing at time $t = 4$.
- For $0 \leq t \leq 6$, at what time t is the amount of sand on the beach a minimum? What is the minimum value? Justify your answers.

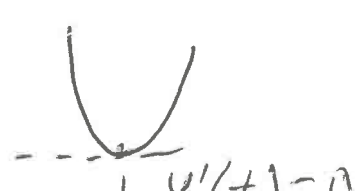
a) $\int_0^6 R(t) dt = 31.815 \text{ yds}^3$

b) * $\text{Total Amount} = \text{Initial Amount} + \text{Amount Added} - \text{Amount Removed}$

$$Y(t) = 2500 + \int_0^t S(x) dx - \int_0^t R(x) dx$$

c) $Y'(t) = 0 + S(t) - R(t)$

$$Y'(4) = S(4) - R(4) = -1.908 \text{ yds/hr}$$

d) 
set $Y'(t) = 0$

$$0 = S(t) - R(t)$$

look for
x-int

Graph
this

$$t = 5.117$$

$t = 5.117$ | Apply ① Test endpoints.
EVT ② Test critical pts.

$$Y(0) = 2500 \text{ yds.}$$

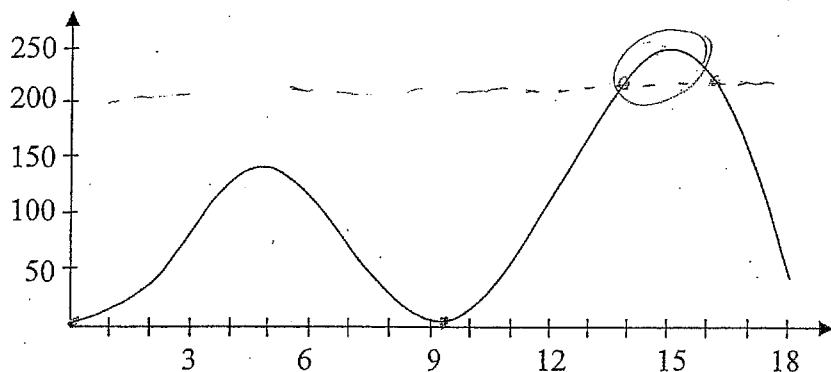
$$Y(5.117) = 2500 + \int_0^{5.117} S(x) dx - \int_0^{5.117} R(x) dx = 2492.369$$

$$Y(6) = 2500 + \int_0^6 S(x) dx - \int_0^6 R(x) dx = 2493.276$$

Abs. minimum is 2492.369 yd^3 at $t = 5.117 \text{ hrs.}$

4.

2006 #2 (calculator question)



At an intersection in Thomasville, Oregon, cars turn left at the rate $L(t) = 60\sqrt{t} \sin^2\left(\frac{t}{3}\right)$ cars per hour over the time interval $0 \leq t \leq 18$ hours. The graph of $y = L(t)$ is shown above.

- a) To the nearest whole number, find the total number of cars turning left at the intersection over the time interval $0 \leq t \leq 18$ hours.
- b) Traffic engineers will consider turn restrictions when $L(t) \geq 150$ cars per hour. Find all values of t for which $L(t) \geq 150$ and compute the average value of L over this time interval. Indicate units of measure.
- c) Traffic engineers will install a signal if there is any two-hour time interval during which the product of the total number of cars turning left and the total number of oncoming cars traveling straight through the intersection is greater than 200,000. In every two-hour time interval, 500 oncoming cars travel straight through the intersection. Does this intersection require a traffic signal? Explain the reasoning that leads to your conclusion.

$$a) \int_0^{18} L(t) dt = 1658 \text{ cars}$$

$$b) \text{ set } L(t) = 150 \text{ or } L(t) - 150 = 0 \quad \frac{1}{b-a} \int_a^b L(t) dt = \frac{1}{16.122 - 12.42} \int_{12.42}^{16.122} L(t) dt$$

$$t = 12.42, 16.122$$

$$= 199.426 \text{ cars/hr.}$$

$$c) \text{ Total \# of cars turning left} \times \text{total \# of cars going straight} > 200,000?$$

$$\int_{13}^{15} L(t) dt \times 500 = 215,965.5$$

So a traffic signal is required

5)

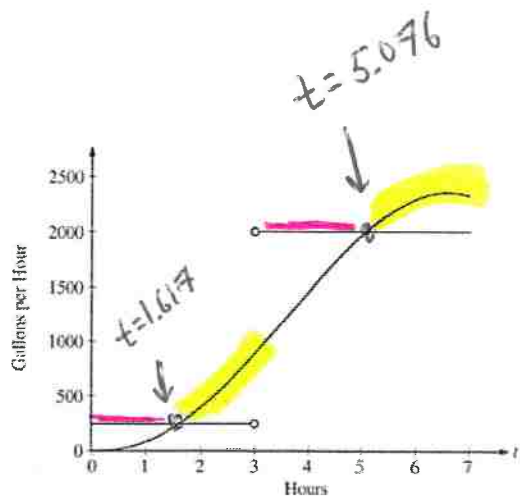
The amount of water in a storage tank, in gallons, is modeled by a continuous function on the time interval $0 \leq t \leq 7$, where t is measured in hours. In this model, rates are given as follows:

(i) The rate at which water enters the tank is

$$f(t) = 100t^2 \sin(\sqrt{t}) \text{ gallons per hour for } 0 \leq t \leq 7.$$

(ii) The rate at which water leaves the tank is

$$g(t) = \begin{cases} 250 & \text{for } 0 \leq t < 3 \\ 2000 & \text{for } 3 < t \leq 7 \end{cases} \text{ gallons per hour.}$$



The graphs of f and g , which intersect at $t = 1.617$ and $t = 5.076$, are shown in the figure above. At time $t = 0$, the amount of water in the tank is 5000 gallons.

- How many gallons of water enter the tank during the time interval $0 \leq t \leq 7$? Round your answer to the nearest gallon.
- For $0 \leq t \leq 7$, find the time intervals during which the amount of water in the tank is decreasing. Give a reason for each answer.
- For $0 \leq t \leq 7$, at what time t is the amount of water in the tank greatest? To the nearest gallon, compute the amount of water at this time. Justify your answer.

$$a) \int_0^7 f(t) dt = 8264 \text{ gallons}$$

EVT ① endpoints
② relative max

b) Amt of water in tank is decreasing $0 < t < 1.617$ and
and $3 < t < 5.076$ because $g(t) > f(t)$

c) test $t = 0, t = 7, t = 3$

Amount $A(t) \rightarrow$ Initial Amt + Amt Added - Amt Removed

$$A(0) = 5000 \text{ gallons}$$

$$A(3) = 5000 + \int_0^3 f(t) dt - \int_0^3 250 dt = 5126.591 \text{ gallons}$$

$$A(7) = 5126.591 + \int_3^7 f(t) dt - \int_3^7 2000 dt = 4513.807 \text{ gallons}$$

Abs max is 5126.591 gallons at $t = 3$ hrs.

6.

2009 #2 (calculator question)

The rate at which people enter an auditorium for a rock concert is modeled by the function R given by $R(t) = 1380t^2 - 675t^3$ for $0 \leq t \leq 2$ hours; $R(t)$ is measured in people per hour. No one is in the auditorium at time $t = 0$, when the doors open. The doors close and the concert begins at time $t = 2$.

- How many people are in the auditorium when the concert begins?
- Find the time when the rate at which people enter the auditorium is a maximum. Justify your answer.
- The total wait time for all the people in the auditorium is found by adding the time each person waits, starting at the time the person enters the auditorium and ending when the concert begins. The function w models the total wait time for all the people who enter the auditorium before time t . The derivative of w is given by $w'(t) = (2-t)R(t)$. Find $w(2) - w(1)$, the total wait time for those who enter the auditorium after time $t = 1$.
- On average, how long does a person wait in the auditorium for the concert to begin? Consider all people who enter the auditorium after the doors open, and use the model for total wait time from part (c).

$$a) \int_0^2 R(t) dt = 980 \text{ people}$$

$$b) R'(t) = 0 \quad t = 1.36296$$

test endpoints and critical pt.

$$R(0) = 0$$

$$R(1.36296) = 854.527$$

$$R(2) = 120$$

Max rate is 854.527 people/hr.
at $t = 1.362$

c) Since $w'(t) = (2-t)R(t)$, use FTC to find $w(2) - w(1)$

$$\int_1^2 w'(t) dt = w(2) - w(1)$$

$$\int_1^2 (2-t)R(t) dt = 387.5 \text{ hrs. is total wait time.}$$

$$d) \text{ Avg. wait time} = \frac{1}{980} \int_0^2 (2-t)R(t) dt = 0.775 \text{ hrs.}$$