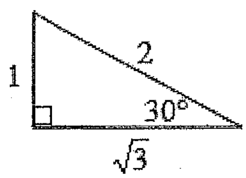
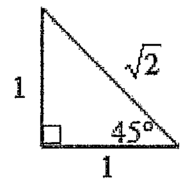
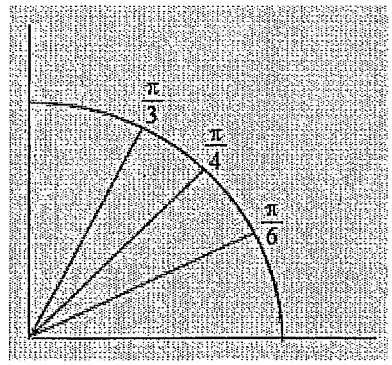
AP Calculus AB **Trig Day 1: Precalc review**

Review ratios for 30, 60, 90 and 45, 45, 90 triangles.







Ex. 1 Use these to find the value of all 6 trig functions at 60°.

Ex. 2 Use these to find the value of all 6 trig functions at 225°.

1 radian is simply the angle formed when a radius is wrapped around the outside of the circle. It takes 2π radians to get all the way around the circle, so it takes π radians to get half way around. This means that π radians equates to 180°.

Ex. 3 Find period, amplitude of $y = 2\cos\left(\frac{\pi x}{3}\right) - 6$

Ex. 4: Solve for x: $\sin 2x = \sin x$ if $0 \le x < 2\pi$

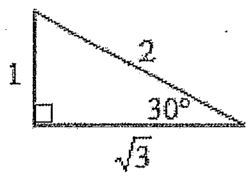
AP Calculus AB   Trig Day 1: Precalc review

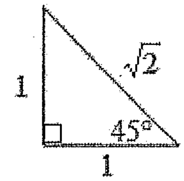
$(\cos\theta, \sin\theta)$

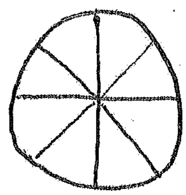
$(-,+)$	$(+,+)$
$(-,-)$	$(+,-)$

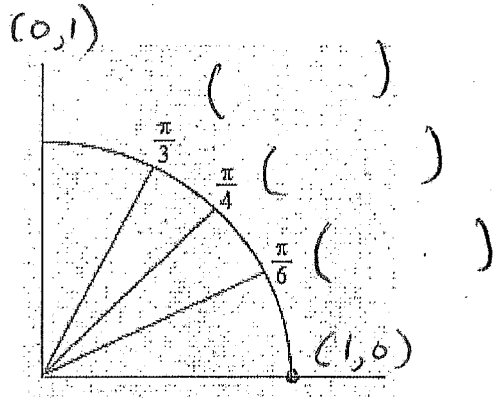
$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

Review ratios for 30, 60, 90 and 45, 45, 90 triangles.









Ex. 1 Use these to find the value of all 6 trig functions at 60°. $(\pi/3)$

$\sin(\pi/3) = \sqrt{3}/2$ $\qquad \csc(\pi/3) = \dfrac{2}{\sqrt{3}}$ or $\dfrac{2\sqrt{3}}{3}$

$\cos(\pi/3) = 1/2$ $\qquad \sec(\pi/3) = 2$

$\tan(\pi/3) = \sqrt{3}$ $\qquad \cot(\pi/3) = \dfrac{1}{\sqrt{3}}$ or $\dfrac{\sqrt{3}}{3}$

Ex. 2 Use these to find the value of all 6 trig functions at 225°.

$\sin(5\pi/4) = \dfrac{-\sqrt{3}}{2}$ $\qquad \csc(5\pi/4) = \dfrac{-2}{\sqrt{2}}$ or $-\sqrt{2}$ $\quad (5\pi/4)$

$\cos(5\pi/4) = \dfrac{-\sqrt{2}}{2}$ $\qquad \sec(5\pi/4) = -\sqrt{2}$

$\tan(5\pi/4) = 1$ $\qquad \cot(5\pi/4) = 1$

1 radian is simply the angle formed when a radius is wrapped around the outside of the circle. It takes 2π radians to get all the way around the circle, so it takes π radians to get half way around. This means that π radians equates to 180°.

~~**Ex. 3** Determine the value of all 6 trig functions for θ = 5π / 3.~~

$y = a\cos(bx - c) + d$

Find period, amplitude of $y = 2\cos\left(\frac{\pi x}{3}\right) - 6$

$\text{period} = \dfrac{2\pi}{b}$

$\text{period} = \dfrac{2\pi}{\pi/3} = 2 \cdot 3 = 6$

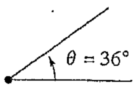
$\text{amplitude} = 2$

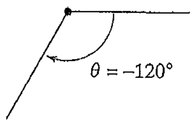
$\boxed{\text{Ex. 4}}$ $\sin 2x = \sin x \quad 0 \le x < 2\pi$

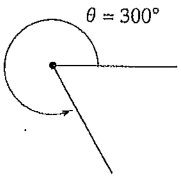
$\boxed{x = 0, \pi, \pi/3, 5\pi/3}$

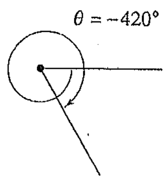
EXERCISES FOR APPENDIX A.3

In Exercises 1 and 2, determine two coterminal angles (one positive and one negative) for each given angle. Express your answers in degrees.

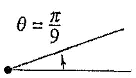
1. (a)


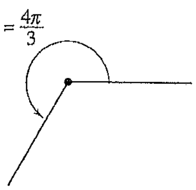
(b)


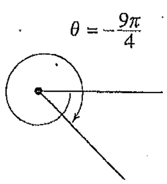
2. (a)


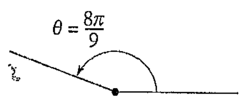
(b)


In Exercises 3 and 4, determine two coterminal angles (one positive and one negative) for each given angle. Express your answers in radians.

3. (a)


(b)


4. (a)


(b)


In Exercises 5 and 6, express the angles in radian measure as multiples of π and as decimals accurate to three decimal places.

5. (a) 30° (b) 150° (c) 315° (d) 120°
6. (a) −20° (b) −240° (c) −270° (d) 144°

In Exercises 7 and 8, express the angles in degree measure.

7. (a) $\frac{3\pi}{2}$ (b) $\frac{7\pi}{6}$ (c) $-\frac{7\pi}{12}$ (d) -2.367

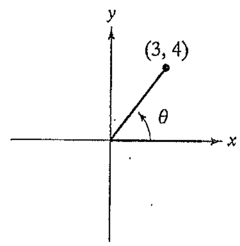
8. (a) $\frac{7\pi}{3}$ (b) $-\frac{11\pi}{30}$ (c) $\frac{11\pi}{6}$ (d) 0.438

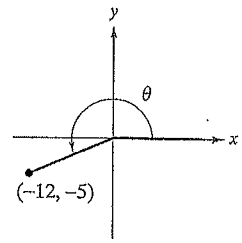
9. Let r represent the radius of a circle, θ the central angle (measured in radians), and s the length of the arc subtended by the angle. Use the relationship $s = r\theta$ to complete the table.

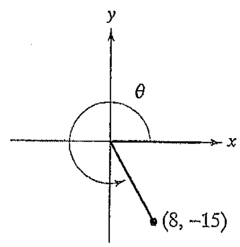
r	8 ft	15 in.	85 cm		
s	12 ft			96 in.	8642 mi
θ		1.6	$\frac{3\pi}{4}$	4	$\frac{2\pi}{3}$

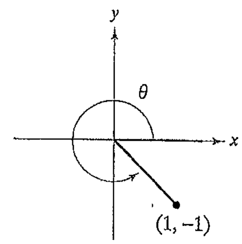
10. *Angular Speed* A car is moving at the rate of 50 miles per hour, and the diameter of its wheels is 2.5 feet.
   (a) Find the number of revolutions per minute that the wheels are rotating.
   (b) Find the angular speed of the wheels in radians per minute.

In Exercises 11 and 12, determine all six trigonometric functions for the angle θ.

11. (a)


(b)


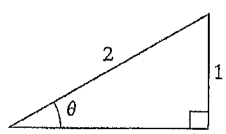
12. (a)


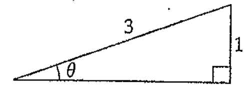
(b)


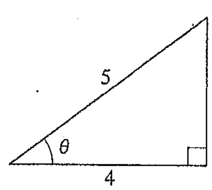
In Exercises 13 and 14, determine the quadrant in which θ lies.

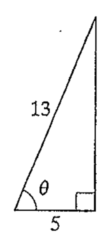
13. (a) $\sin\theta < 0$ and $\cos\theta < 0$
   (b) $\sec\theta > 0$ and $\cot\theta < 0$
14. (a) $\sin\theta > 0$ and $\cos\theta < 0$
   (b) $\csc\theta < 0$ and $\tan\theta > 0$

In Exercises 15–18, evaluate the trigonometric function.

15. $\sin\theta = \frac{1}{2}$
   $\cos\theta =$


16. $\sin\theta = \frac{1}{3}$
   $\tan\theta =$


17. $\cos\theta = \frac{4}{5}$
   $\cot\theta =$


18. $\sec\theta = \frac{13}{5}$
   $\csc\theta =$


4

In Exercises 19–22, evaluate the sine, cosine, and tangent of each angle *without* using a calculator.

19. (a) $60°$ (b) $120°$ (c) $\frac{\pi}{4}$ (d) $\frac{5\pi}{4}$

20. (a) $-30°$ (b) $150°$ (c) $-\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

21. (a) $225°$ (b) $-225°$ (c) $\frac{5\pi}{3}$ (d) $\frac{11\pi}{6}$

22. (a) $750°$ (b) $510°$ (c) $\frac{10\pi}{3}$ (d) $\frac{17\pi}{3}$

In Exercises 23–26, use a calculator to evaluate the trigonometric functions to four significant digits.

23. (a) $\sin 10°$ (b) $\csc 10°$

24. (a) $\sec 225°$ (b) $\sec 135°$

25. (a) $\tan \frac{\pi}{9}$ (b) $\tan \frac{10\pi}{9}$

26. (a) $\cot(1.35)$ (b) $\tan(1.35)$

In Exercises 27–30, find two solutions of each equation. Express the results in radians ($0 \le \theta < 2\pi$). Do not use a calculator.

27. (a) $\cos\theta = \frac{\sqrt{2}}{2}$ (b) $\cos\theta = -\frac{\sqrt{2}}{2}$

28. (a) $\sec\theta = 2$ (b) $\sec\theta = -2$

29. (a) $\tan\theta = 1$ (b) $\cot\theta = -\sqrt{3}$

30. (a) $\sin\theta = \frac{\sqrt{3}}{2}$ (b) $\sin\theta = -\frac{\sqrt{3}}{2}$

In Exercises 31–38, solve the equation for θ ($0 \le \theta < 2\pi$).

31. $2\sin^2\theta = 1$

32. $\tan^2\theta = 3$

33. $\tan^2\theta - \tan\theta = 0$

34. $2\cos^2\theta - \cos\theta = 1$

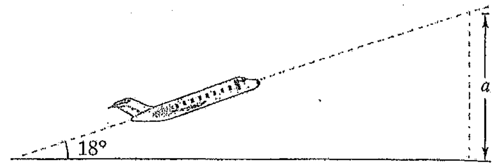
35. $\sec\theta\csc\theta = 2\csc\theta$

36. $\sin\theta = \cos\theta$

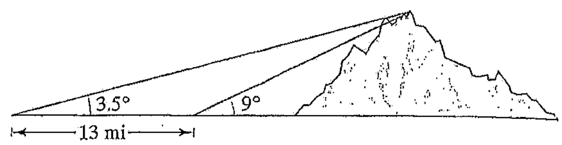
37. $\cos^2\theta + \sin\theta = 1$

38. $\cos\frac{\theta}{2} - \cos\theta = 1$

39. ***Airplane Ascent*** An airplane leaves the runway climbing at $18°$ with a speed of 275 feet per second (see figure). Find the altitude a of the plane after 1 minute.



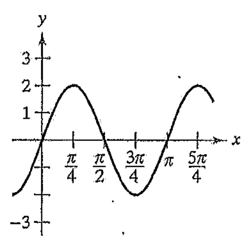
40. ***Height of a Mountain*** In traveling across flat land, you notice a mountain directly in front of you. Its angle of elevation (to the peak) is $3.5°$. After you drive 13 miles closer to the mountain, the angle of elevation is $9°$. Approximate the height of the mountain.

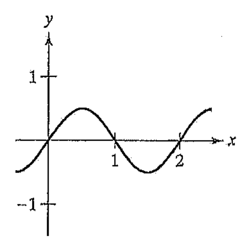


(not to scale)

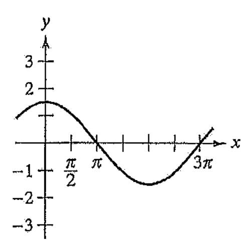
In Exercises 41–44, determine the period and amplitude of each function.

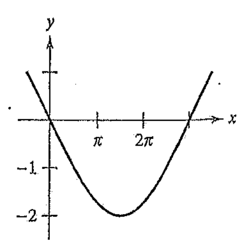
41. (a) $y = 2\sin 2x$ (b) $y = \frac{1}{2}\sin \pi x$





42. (a) $y = \frac{3}{2}\cos\frac{x}{2}$ (b) $y = -2\sin\frac{x}{3}$





43. $y = 3\sin 4\pi x$

44. $y = \frac{2}{3}\cos\frac{\pi x}{10}$

5

Section A.3 (page A25)

1. (a) $396°, -324°$ (b) $240°, -480°$

3. (a) $\frac{19\pi}{9}, -\frac{17\pi}{9}$ (b) $\frac{10\pi}{3}, -\frac{2\pi}{3}$

5. (a) $\frac{\pi}{6}$, 0.524 (b) $\frac{5\pi}{6}$, 2.618

(c) $\frac{7\pi}{4}$, 5.498 (d) $\frac{2\pi}{3}$, 2.094

7. (a) $270°$ (b) $210°$ (c) $-105°$ (d) $-135.6°$

9.

r	8 ft	15 in.	85 cm	24 in.	$\frac{12{,}963}{\pi}$ mi
s	12 ft	24 in.	63.75π cm	96 in.	8642 mi
θ	1.5	1.6	$\frac{3\pi}{4}$	4	$\frac{2\pi}{3}$

11. (a) $\sin\theta = \frac{4}{5}$ $\csc\theta = \frac{5}{4}$
$\cos\theta = \frac{3}{5}$ $\sec\theta = \frac{5}{3}$
$\tan\theta = \frac{4}{3}$ $\cot\theta = \frac{3}{4}$

(b) $\sin\theta = -\frac{5}{13}$ $\csc\theta = -\frac{13}{5}$
$\cos\theta = -\frac{12}{13}$ $\sec\theta = -\frac{13}{12}$
$\tan\theta = \frac{5}{12}$ $\cot\theta = \frac{12}{5}$

13. (a) Quadrant III (b) Quadrant IV

15. $\frac{\sqrt{3}}{2}$ 17. $\frac{4}{3}$

19. (a) $\sin 60° = \frac{\sqrt{3}}{2}$
$\cos 60° = \frac{1}{2}$
$\tan 60° = \sqrt{3}$

(b) $\sin 120° = \frac{\sqrt{3}}{2}$
$\cos 120° = -\frac{1}{2}$
$\tan 120° = -\sqrt{3}$

(c) $\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$
$\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$
$\tan\frac{\pi}{4} = 1$

(d) $\sin\frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$
$\cos\frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$
$\tan\frac{5\pi}{4} = 1$

21. (a) $\sin 225° = -\frac{\sqrt{2}}{2}$
$\cos 225° = -\frac{\sqrt{2}}{2}$
$\tan 225° = 1$

(b) $\sin(-225°) = \frac{\sqrt{2}}{2}$
$\cos(-225°) = -\frac{\sqrt{2}}{2}$
$\tan(-225°) = -1$

(c) $\sin\frac{5\pi}{3} = -\frac{\sqrt{3}}{2}$
$\cos\frac{5\pi}{3} = \frac{1}{2}$
$\tan\frac{5\pi}{3} = -\sqrt{3}$

(d) $\sin\frac{11\pi}{6} = -\frac{1}{2}$
$\cos\frac{11\pi}{6} = \frac{\sqrt{3}}{2}$
$\tan\frac{11\pi}{6} = -\frac{\sqrt{3}}{3}$

23. (a) 0.1736 (b) 5.759 25. (a) 0.3640 (b) 0.3640

27. (a) $\theta = \frac{\pi}{4}, \frac{7\pi}{4}$ (b) $\theta = \frac{3\pi}{4}, \frac{5\pi}{4}$

29. (a) $\theta = \frac{\pi}{4}, \frac{5\pi}{4}$ (b) $\theta = \frac{5\pi}{6}, \frac{11\pi}{6}$

31. $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ 33. $\theta = 0, \frac{\pi}{4}, \pi, \frac{5\pi}{4}$

35. $\theta = \frac{\pi}{3}, \frac{5\pi}{3}$ 37. $\theta = 0, \frac{\pi}{2}, \pi$ 39. 5099 feet

41. (a) Period: π
Amplitude: 2

(b) Period: 2
Amplitude: $\frac{1}{2}$

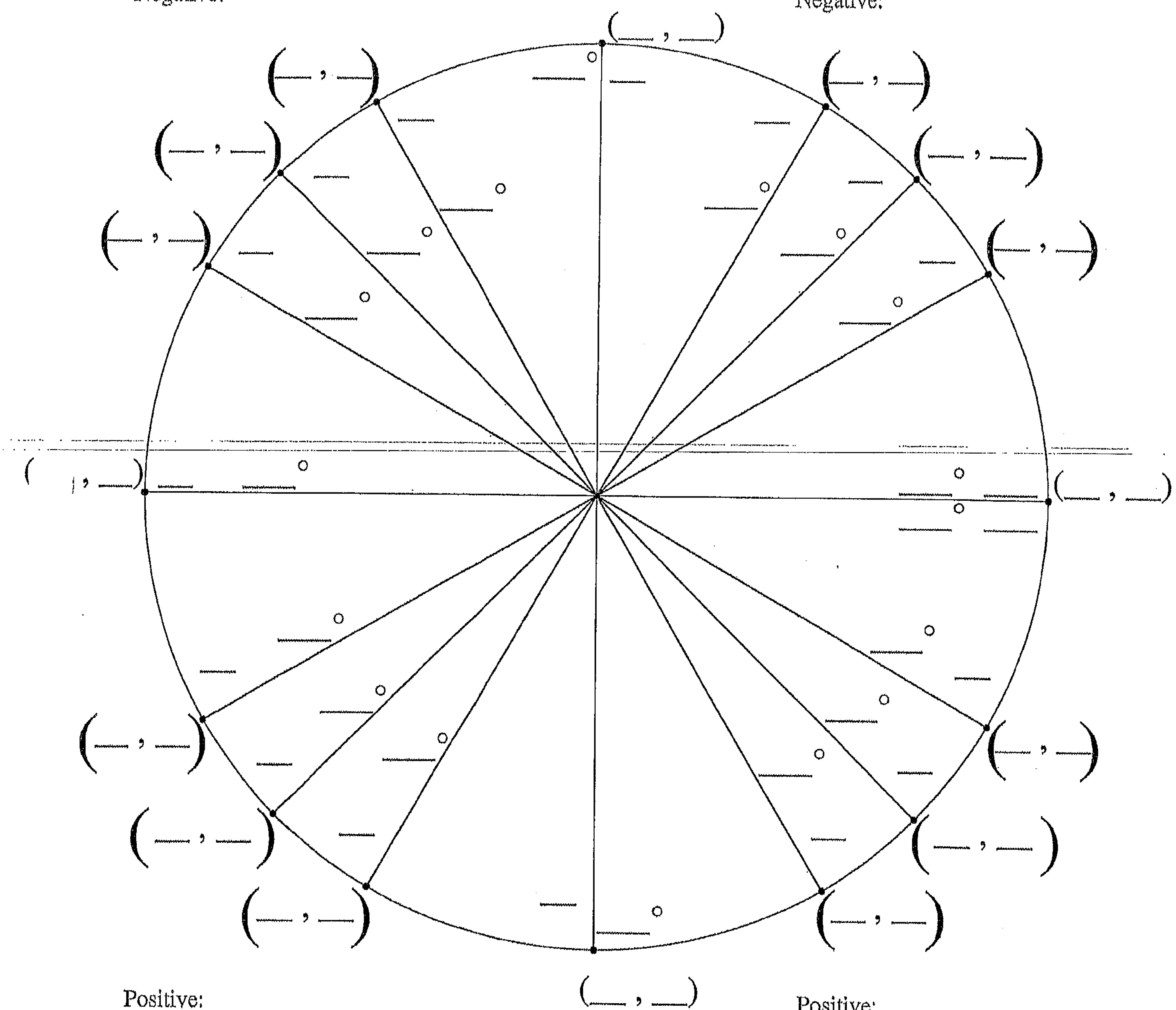
43. Period: $\frac{1}{2}$
Amplitude: 3

6a

Fill in The Unit Circle

Positive:
Negative:

Positive:
Negative:



Positive:
Negative:

Positive:
Negative:

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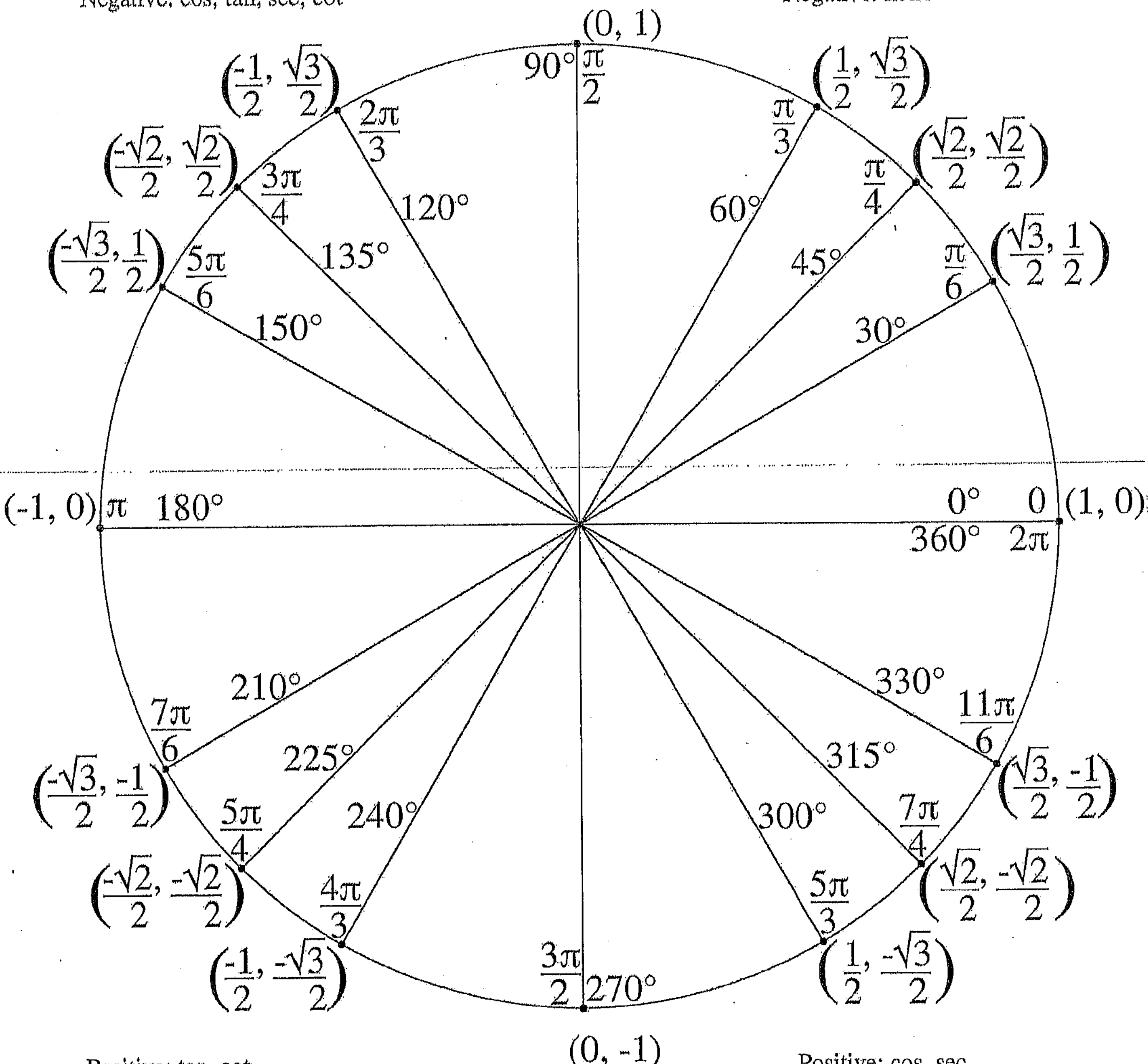
6b

The Unit Circle

Positive: sin, csc
Negative: cos, tan, sec, cot

Positive: sin, cos, tan, sec, csc, cot
Negative: none



Positive: tan, cot
Negative: sin, cos, sec, csc

Positive: cos, sec
Negative: sin, tan, csc, cot

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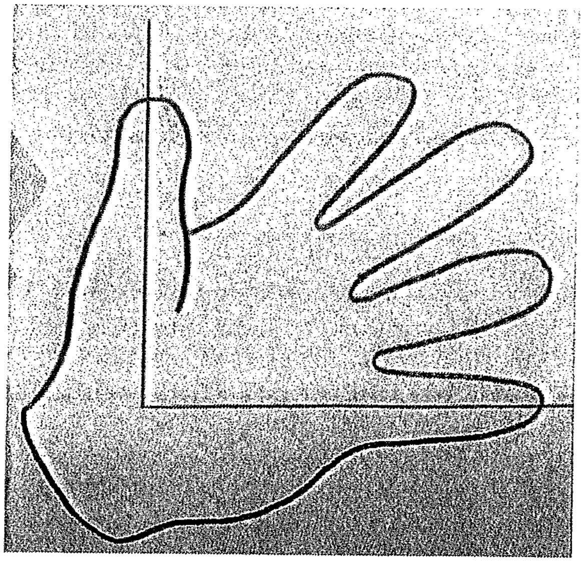
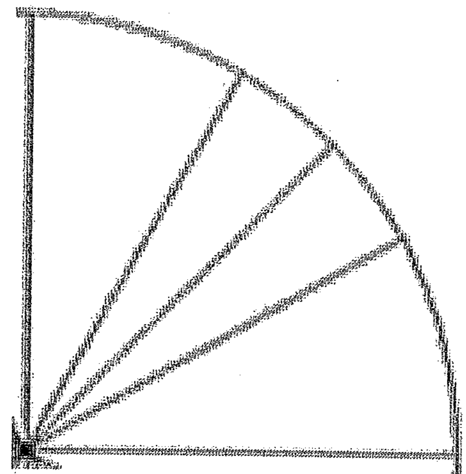
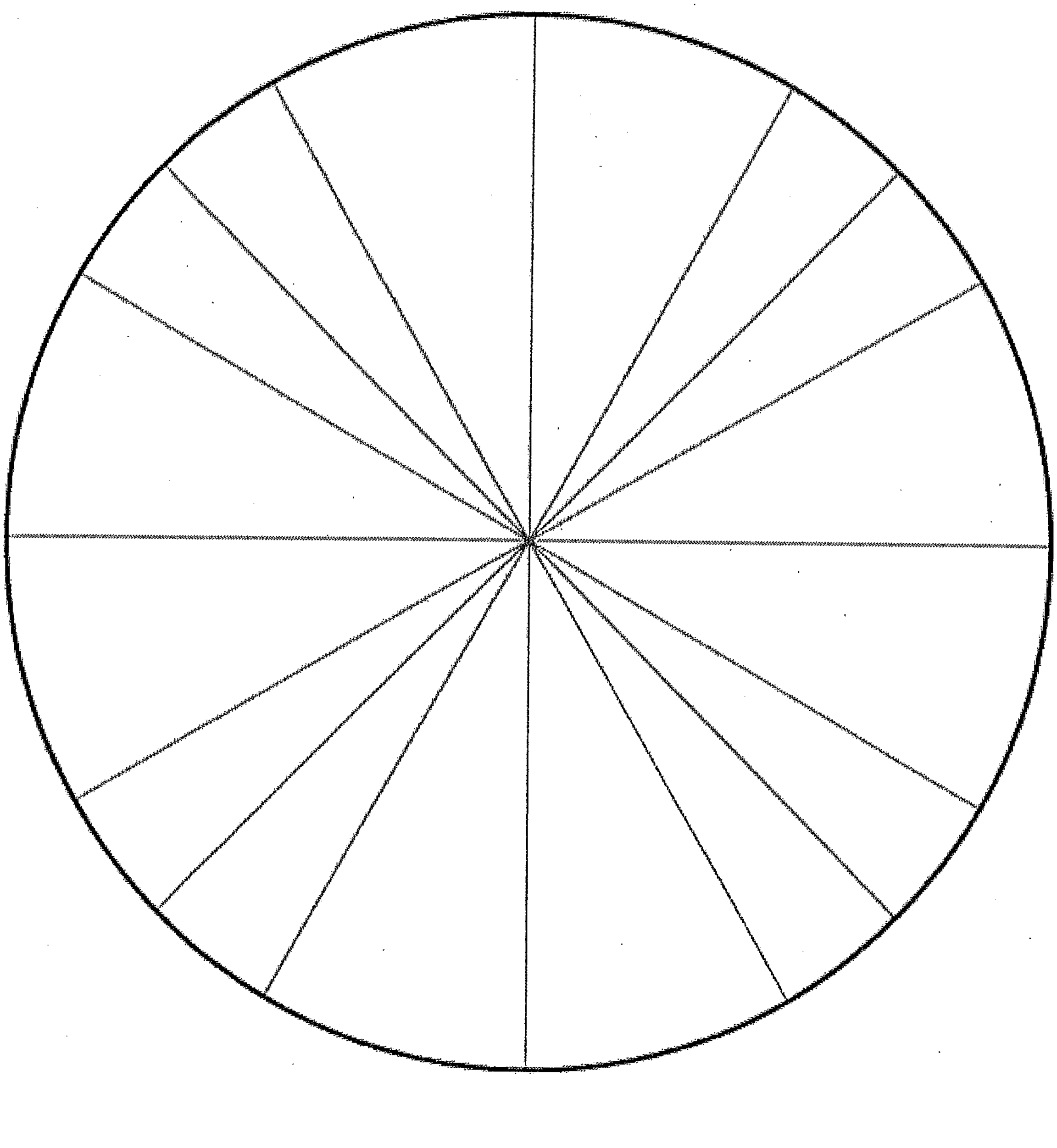
6c

Unit Circle – Left-Hand Memorization Method



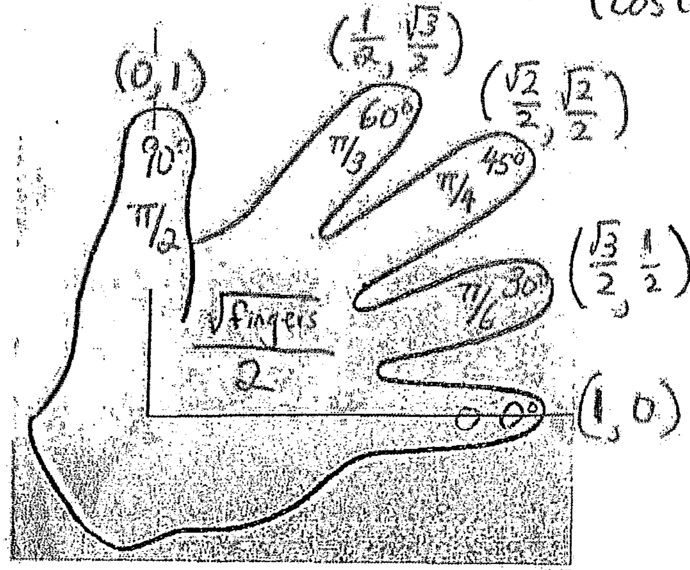


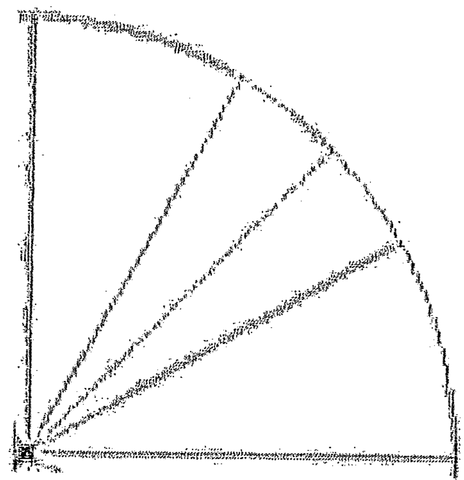


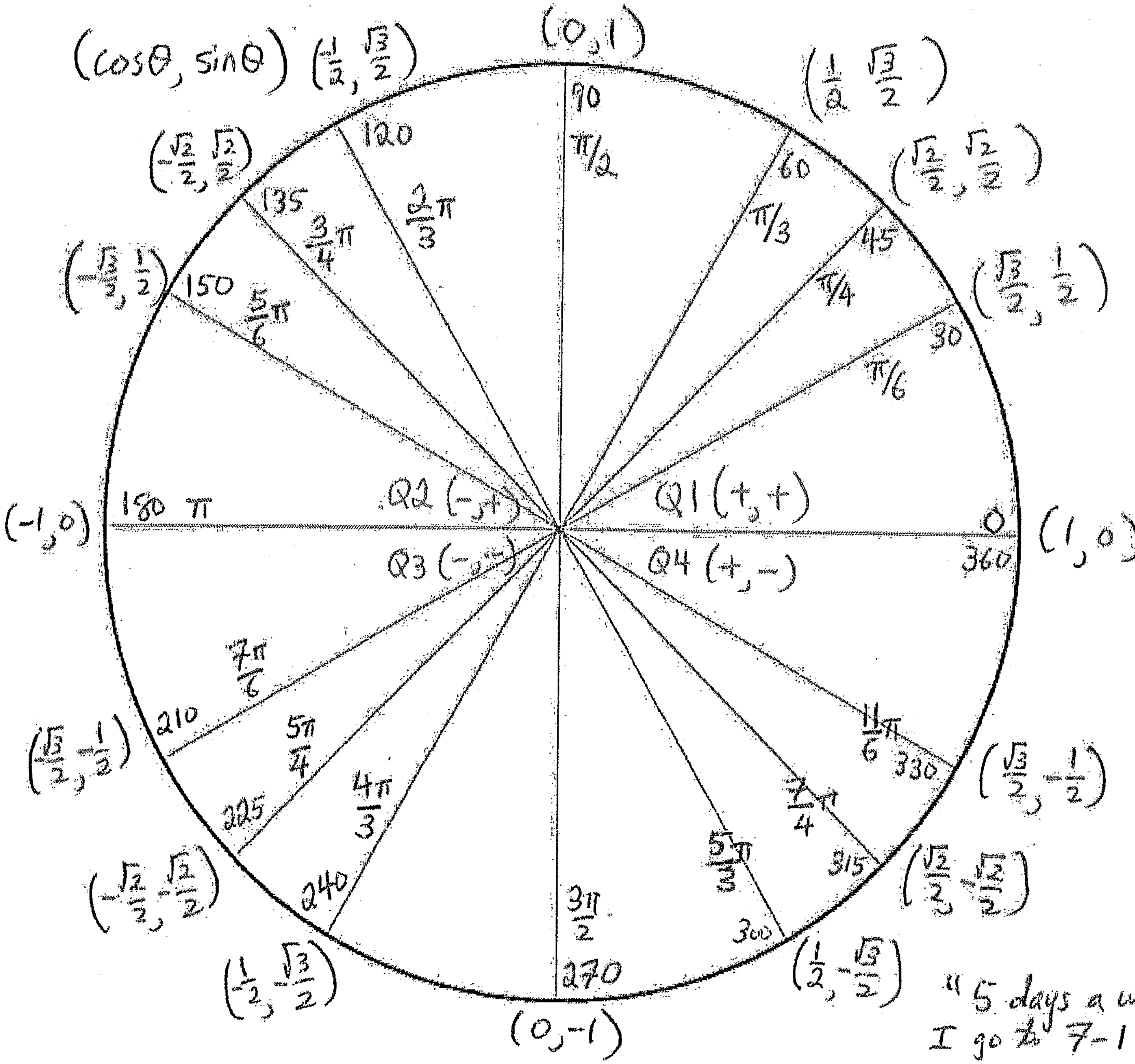
6d

Unit Circle – Left-Hand Memorization Method

(cos θ, sin θ)







"5 days a week I go to 7-11"

PreCalc Trig Review: Solving Trig Equations

In Exercises 31–38, solve the equation for θ $(0 \le \theta < 2\pi)$.

31. $2\sin^2\theta = 1$

32. $\tan^2\theta = 3$

33. $\tan^2\theta - \tan\theta = 0$

34. $2\cos^2\theta - \cos\theta = 1$

35. $\sec\theta\csc\theta = 2\csc\theta$

36. $\sin\theta = \cos\theta$

37. $\cos^2\theta + \sin\theta = 1$

38. $\cos\dfrac{\theta}{2} - \cos\theta = 1$

(8)

Key

PreCalc Trig Review: Solving Trig Equations

In Exercises 31–38, solve the equation for θ ($0 \le \theta < 2\pi$).

31. $2\sin^2\theta = 1$

$\sin^2\theta = \frac{1}{2}$

$\sqrt{\sin^2\theta} = \sqrt{1/2}$

$\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\sin\theta = \pm\frac{1}{\sqrt{2}}$

$\sin\theta = \pm\frac{\sqrt{2}}{2}$

$\boxed{\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}}$

32. $\tan^2\theta = 3$

$\sqrt{\tan^2\theta} = \pm\sqrt{3}$

$\tan\theta = \pm\sqrt{3}$

$\boxed{\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}}$

$*\tan\frac{\pi}{3} = \frac{\sin(\pi/3)}{\cos(\pi/3)} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$

33. $\tan^2\theta - \tan\theta = 0$

$\tan\theta(\tan\theta - 1) = 0$

$\tan\theta = 0$ | $\tan\theta - 1 = 0$

$\boxed{\theta = 0, \pi}$ | $\tan\theta = 1$

$\boxed{\theta = \pi/4, 5\pi/4}$

34. $2\cos^2\theta - \cos\theta = 1$

$2\cos^2\theta - \cos\theta - 1 = 0$

$(2\cos\theta + 1)(\cos\theta - 1) = 0$

$2\cos\theta + 1 = 0$ | $\cos\theta - 1 = 0$

$\cos\theta = -1/2$ | $\cos\theta = 1$

$\boxed{\theta = \frac{2\pi}{3}, \frac{4\pi}{3}}$ | $\boxed{\theta = 0}$

35. $\sec\theta\csc\theta = 2\csc\theta$

$\frac{1}{\cos\theta} \cdot \frac{1}{\sin\theta} = \frac{2}{\sin\theta}$

$\frac{1}{\cos\theta\sin\theta} = \frac{2}{\sin\theta}$

$2\sin\theta\cos\theta = \sin\theta$

$2\sin\theta\cos\theta - \sin\theta = 0$

$\sin\theta(2\cos\theta - 1) = 0$

$\sin\theta(2\cos\theta - 1) = 0$

$\sin\theta = 0$ | $2\cos\theta - 1 = 0$

$\cancel{\theta = 0}$ | $\cos\theta = 1/2$

Extraneous solution | $\boxed{\theta = \frac{\pi}{3}, \frac{5\pi}{3}}$

36. $\sin\theta = \cos\theta$

$\frac{\sin\theta}{\cos\theta} = \frac{\cos\theta}{\cos\theta}$

$\tan\theta = 1$

$\boxed{\theta = \pi/4, 5\pi/4}$

37. $\cos^2\theta + \sin\theta = 1$

*Express in terms of 1 trig function

$*\cos^2\theta = 1 - \sin^2\theta$

$(1 - \sin^2\theta) + \sin\theta = 1$

$1 - \sin^2\theta + \sin\theta - 1 = 0$

$\sin\theta - \sin^2\theta = 0$

$\sin\theta(1 - \sin\theta) = 0$

$\sin\theta = 0$ | $1 - \sin\theta = 0$

$\boxed{\theta = 0, \pi}$ | $\sin\theta = 1$

$\boxed{\theta = \pi/2}$

38. $\cos\frac{\theta}{2} - \cos\theta = 1$

$*$ Let $x = \frac{\theta}{2}$

$\cos x - \cos(2x) = 1$

$\cos x - (2\cos^2 x - 1) = 1$

$\cos x - 2\cos^2 x + 1 - 1 = 0$

$\cos x - 2\cos^2 x = 0$

$\cos x(1 - 2\cos x) = 0$

$\cos x = 0$ | $1 - 2\cos x = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ | $\cos x = 1/2$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

$\frac{\theta}{2} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{3}, \frac{5\pi}{3}$

$\theta = 2 \cdot \left[\frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{3}, \frac{5\pi}{3}\right]$

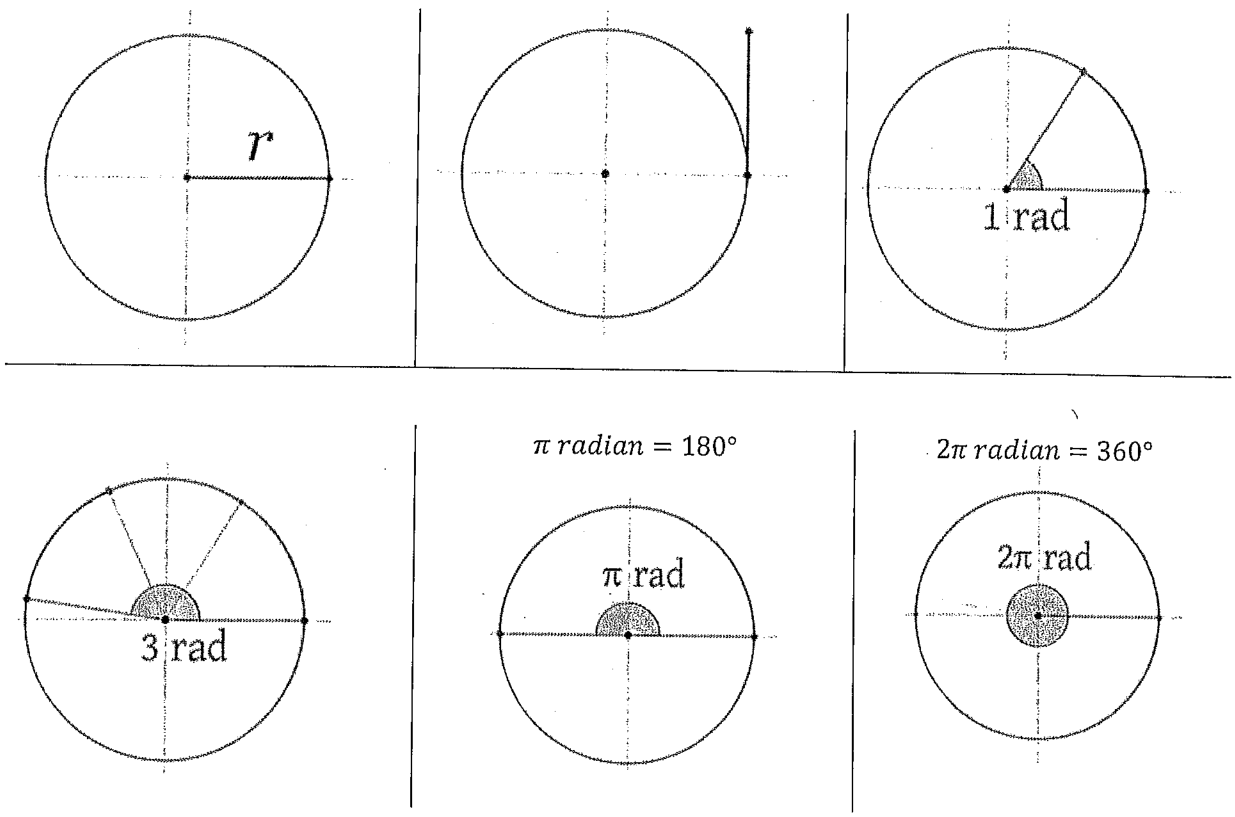
$\theta = \pi, \cancel{3\pi}, \frac{2\pi}{3}, \cancel{\frac{10\pi}{3}}$

$\boxed{\theta = \pi, \frac{2\pi}{3}}$

What is Radians?

One radian is the angle measured at the center of a circle by an arc that is equal in length to the radius of the circle.

1 radian $\approx 57.3°$



Conversions between Degrees and Radians

$$Angle\ in\ \mathbf{degrees} = angle\ in\ radians \times \frac{180°}{\pi}$$

$$Angle\ in\ \mathbf{radians} = angle\ in\ degree \times \frac{\pi}{180°}$$

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A.P. Calculus AB Trigonometry Review

Note: Almost all problems involving trigonometry in calculus will be in radians

RIGHT TRIANGLES (SOH-CAH-TOA):

$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$ $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$ $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{\sin\theta}{\cos\theta}$

$\csc\theta = \dfrac{1}{\sin\theta}$ $\sec\theta = \dfrac{1}{\cos\theta}$ $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{\cos\theta}{\sin\theta}$

IDENTITIES

<u>Pythagorean</u>

$\sin^2\theta + \cos^2\theta = 1$

divide by $\sin^2\theta$ to get: $1 + \cot^2\theta = \csc^2\theta$

divide by $\cos^2\theta$ to get: $\tan^2\theta + 1 = \sec^2\theta$

<u>Double Angle</u>

$\sin(2\theta) = 2\sin\theta\cos\theta$

$\cos(2\theta) = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

ODD/EVEN

sinx is an odd function, so $\sin(-\theta) = -\sin\theta$

cosx is an even function, so $\cos(-\theta) = \cos\theta$

OTHER STUFF

$\sin(\pi - \theta) = \sin\theta$ $\cos(\pi - \theta) = -\cos\theta$ $\cos\left(x - \frac{\pi}{2}\right) = \cos\left(\frac{\pi}{2} - x\right) = \sin x$ $\sin\left(\frac{\pi}{2} - x\right) = \cos x$

UNIT CIRCLE VALUES

Must know the exact values of $\sin\theta$ and $\cos\theta$ for $\theta = \dfrac{\pi}{6}, \dfrac{\pi}{4}, \dfrac{\pi}{3}$ and all of their multiples

GRAPHS

Must also be able to identify the domain, range, and locations of the asymptotes (if any) for all trigonometric functions. Must be able to use the features below to sketch the graph.

For $y = a\sin bx$ or $y = a\cos bx$, period $= \dfrac{2\pi}{|b|}$, amplitude $= |a|$

For $y = a\tan bx$ or $y = a\cot bx$, period $= \dfrac{\pi}{|b|}$, amplitude is not applicable

For $y = a\sec bx$ or $y = a\csc bx$, period $= \dfrac{2\pi}{|b|}$, amplitude is not applicable

For all functions $y = a\sin b(x + c) + d$, phase shift $= c$ (left or right), vertical shift $= d$ (up or down)

For all functions $y = a\sin(bx + c) + d$, phase shift $= \dfrac{c}{b}$ (left or right), vertical shift $= d$ (up or down)

AP Calculus AB **Trig Day 3: Trig Derivatives** Sections 1-3, 1-4, 3-5

Trig Derivative Rules: $\frac{d}{dx}\sin x = \cos x$ $\frac{d}{dx}\cos x = -\sin x$

Ex. 1: Show that the following are true using the above rules and product/quotient rules:

a) $\frac{d}{dx}\tan x = \sec^2 x$

b) $\frac{d}{dx}\cot x = -\csc^2 x$

c) $\frac{d}{dx}\sec x = \sec x \tan x$

d) $\frac{d}{dx}\csc x = -\csc x \cot x$

Ex 2: if y = sin(5x), find y'

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Ex 3: if y = $\sqrt{\cos(x^2)}$, find y'

Ex. 4: if = $\sqrt[3]{\tan(6x^4 + 3x - 2)}$, find y '

Ex. 5: if y = $\sec^2(\pi x)$, find y''

Ex. 6: if sin(x + y) = 3x + 2y, find $\frac{dy}{dx}$

AP Calculus AB **Trig Day 3: Trig Derivatives** Sections 1-3, 1-4, 3-5 $\boxed{\text{KEY}}$ 11

Trig Derivative Rules: $\frac{d}{dx}\sin x = \cos x$ $\frac{d}{dx}\cos x = -\sin x$

Ex. 1: Show that the following are true using the above rules and product/quotient rules:

a) $\frac{d}{dx}\tan x = \sec^2 x$ *use quotient rule $\frac{f'g - fg'}{g^2}$

$\frac{d}{dx}\frac{\sin x}{\cos x}$

$f'(x) = \frac{(\cos x)(\cos x) - (\sin x)(-\sin x)}{\cos^2 x}$

$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \boxed{\sec^2 x}$

b) $\frac{d}{dx}\cot x = -\csc^2 x$

$\frac{d}{dx}\left(\frac{\cos x}{\sin x}\right) = \frac{(-\sin x)(\sin x) - (\cos x)(\cos x)}{\sin^2 x}$

$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}$

$= \frac{-1}{\sin^2 x} = \boxed{-\csc^2 x}$

c) $\frac{d}{dx}\sec x = \sec x \tan x$

$f(x) = \frac{1}{\cos x} = (\cos x)^{-1}$

$f'(x) = -1(\cos x)^{-2}(-\sin x)$

$= \frac{+\sin x}{\cos^2 x} = \frac{\sin x}{(\cos x)(\cos x)}$

$= \frac{\sin x}{\cos x}\cdot\frac{1}{\cos x} = \tan x \sec x$

$= \boxed{\sec x \tan x}$

d) $\frac{d}{dx}\csc x = -\csc x \cot x$

$f(x) = \frac{1}{\sin x} = (\sin x)^{-1}$

$f'(x) = -1(\sin x)^{-2}(\cos x)$

$= \frac{-\cos x}{\sin^2 x} = \frac{-\cos x}{\sin x \cdot \sin x}$

$= -\frac{\cos x}{\sin x}\cdot\frac{1}{\sin x} = -\cot x(\csc x)$

$= \boxed{-\csc x \cot x}$

Ex 2: if y = sin(5x), find y'

*chain rule

$y' = \cos(5x)\cdot 5$

$\boxed{y' = 5\cos(5x)}$

1) $\frac{d}{dx}\sin u = \cos u \cdot u'$ 2) $\frac{d}{dx}\cos u = -\sin u \cdot u'$

3) $\frac{d}{dx}\tan u = \sec^2 u \cdot u'$ 4) $\frac{d}{dx}\cot u = -\csc^2 u \cdot u'$

5) $\frac{d}{dx}\sec u = \sec u \tan u \cdot u'$ 6) $\frac{d}{dx}\csc u = -\csc u \cot u \cdot u'$

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Ex 3: if y = $\sqrt{\cos(x^2)}$, find y'

$$y = [\cos(x^2)]^{1/2}$$

$$y' = \frac{1}{2}[\cos(x^2)]^{-1/2} \cdot -\sin(x^2) \cdot 2x$$

$$y' = \frac{-x\sin(x^2)}{[\cos(x^2)]^{1/2}}$$

$$= \boxed{\frac{-x\sin(x^2)}{\sqrt{\cos(x^2)}}}$$

Ex. 4: $y = \sqrt[3]{\tan(6x^4 + 3x - 2)}$, find y '

$$y = [\tan(6x^4+3x-2)]^{1/3}$$

$$y' = \frac{1}{3}[\tan(6x^4+3x-2)]^{-2/3} \sec^2(6x^4+3x-2) \cdot (24x^3+3)$$

$$y' = \frac{(8x^3+1)(\sec^2(6x^4+3x-2))}{[\tan(6x^4+3x-2)]^{2/3}}$$

Ex. 5: if y = $\sec^2(\pi x)$, find ~~y''~~ $\boxed{y'}$

$$y = [\sec(\pi x)]^2$$

$$y' = 2[\sec(\pi x)] \cdot \sec(\pi x)\tan(\pi x) \cdot (\pi)$$

$$\boxed{y' = 2\pi\sec^2(\pi x)\tan(\pi x)}$$

Ex. 6: if sin(x + y) = 3x + 2y, find $\frac{dy}{dx}$

$$\cos(x+y) \cdot \left[1 + \frac{dy}{dx}\right] = 3 + 2\left(\frac{dy}{dx}\right)$$

$$\cos(x+y) + \cos(x+y)\left(\frac{dy}{dx}\right) = 3 + 2\left(\frac{dy}{dx}\right)$$

$$\frac{dy}{dx}\cos(x+y) - 2\left(\frac{dy}{dx}\right) = 3 - \cos(x+y)$$

$$\frac{dy}{dx}\left(\cos(x+y) - 2\right) = 3 - \cos(x+y)$$

$$\boxed{\frac{dy}{dx} = \frac{3-\cos(x+y)}{\cos(x+y) - 2}}$$

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AP Calculus AB **Trig Quiz #2 Review Worksheet 1**

1) If $y = \cos^3(\pi x^2)$, find $\frac{dy}{dx}$

2) Given $y = \frac{\sin(\pi x)}{\cos(2\pi x)}$ Evaluate $y'(0)$

3) If $\frac{\sin(x)}{3\cos(y)} = 2x$ find $\frac{dy}{dx}$

4) If $[\sin(\pi x) + \cos(\pi y)]^2 = 2$ find $\frac{dy}{dx}$

5) If velocity of a particle is $v(t) = \sin(2t)\tan(t)$,

a) Find a(t)

b) find acceleration at $t = \pi$

6. Find the equation of the tangent line at $x = \frac{\pi}{4}$ for $y = 2tan^3x$

(14)

AP Calculus AB **Trig Quiz #2 Review** Name: Key

* chain rule
* quotient rule $f'g - fg'$

1) If $y = \cos^3(\pi x^2)$, find $\frac{dy}{dx}$

$$y = [\cos(\pi x^2)]^3$$

$$y' = 3[\cos(\pi x^2)]^2 \cdot (-\sin(\pi x^2)) \cdot \pi(2x)$$

$$y' = -6\pi x[\cos(\pi x^2)]^2 \sin(\pi x^2)$$

or

$$y' = -6\pi x \cos^2(\pi x^2)\sin(\pi x^2)$$

2) Given $y = \frac{\sin(\pi x)}{\cos(2\pi x)}$ Evaluate $y'(0)$

$$y' = \frac{\underbrace{\cos(\pi x)\cdot\pi}_{f'}\,\underbrace{\cos(2\pi x)}_{g} - \underbrace{\sin(\pi x)}_{f}\cdot\underbrace{-\sin(2\pi x)\cdot 2\pi}_{g'}}{\underbrace{[\cos(2\pi x)]^2}_{g^2}}$$

$$y'(0) = \frac{\pi\cos 0\cdot\cos 0 - \sin(0)\cdot(-\sin 0)\cdot 2\pi}{(\cos 0)^2}$$

$$= \frac{\pi}{1} = \boxed{\pi}$$

3) If $\frac{\sin(x)}{3\cos(y)} = 2x$ find $\frac{dy}{dx}$

* quotient
* implicit

$$\frac{\cos x\cdot 3\cos y - \sin x\cdot(-3\sin y)\left(\frac{dy}{dx}\right)}{(3\cos y)^2} = 2$$

$$3\cos x\cos y + 3\sin x\sin y\left(\frac{dy}{dx}\right) = 2\cdot 9\cos^2 y$$

$$\frac{dy}{dx}(3\sin x\sin y) = 18\cos^2 y - 3\cos x\cos y$$

$$\frac{dy}{dx} = \frac{18\cos^2 y - 3\cos x\cos y}{3\sin x\sin y} = \boxed{\frac{6\cos^2 y - \cos x\cos y}{\sin x\sin y}}$$

4) If $[\sin(\pi x) + \cos(\pi y)]^2 = 2$ find $\frac{dy}{dx}$

* chain
* implicit

$$\cancel{2[\sin(\pi x) + \cos(\pi y)]}\cdot\left[\cos(\pi x)\cdot\pi - \sin(\pi y)\cdot\pi\left(\frac{dy}{dx}\right)\right] = 0$$

$$\pi\cos(\pi x) - \frac{dy}{dx}(\pi\sin(\pi y)) = 0$$

$$-\frac{dy}{dx}(\pi\sin(\pi y)) = -\pi\cos(\pi x)$$

$$\frac{dy}{dx} = \frac{-\pi\cos(\pi x)}{-\pi\sin(\pi y)} = \boxed{\frac{\cos(\pi x)}{\sin(\pi y)}}$$

5) If velocity of a particle is $v(t) = \sin(2t)\tan(t)$,

* product rule $f'g + fg'$
* chain rule

a) Find a(t)

$$a(t) = \cos(2t)\cdot 2\tan(t) + \sin(2t)\sec^2(t)\cdot(1)$$

$$a(t) = 2\cos(2t)\tan(t) + \sin(2t)\sec^2(t)$$

b) find acceleration at $t = \pi$

$$a(\pi) = 2\cos(2\pi)\tan(\pi) + \sin(2\pi)\sec^2(\pi)$$

$$= 2(1)(0) + (0)(-1)^2 = \boxed{0}$$

6) Find equation of tangent line at $x = \pi/4$ for $y = 2\tan^3 x$

$$y = 2[\tan x]^3$$

$$y' = 2\cdot 3[\tan x]^2\sec^2 x$$

$$y' = 6[\tan x]^2[\sec x]^2$$

$$y'(\pi/4) = 6[\tan(\pi/4)]^2[\sec(\pi/4)]^2$$

$$= 6(1)(\sqrt{2})^2 = 12$$

point: $y(\pi/4) = 2[\tan(\pi/4)]^3 = 2(1)^3 = 2$

point: $(\pi/4, 2)$ slope: $m = 12$

$$\boxed{y - 2 = 12(x - \pi/4)}$$

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AP Calculus AB | **Trig Quiz #2 (Chapter 2)** **REVIEW** #2 | **Name:** ____________

No Calculators. | Leave all answers reasonably simplified

1. If $y = 4x^3 \tan\left(4x^3\right)$ find $\frac{dy}{dx}$.

2. If $y = \frac{\csc(4x)}{\cot(3x)}$ find $\frac{dy}{dx}$.

3. If $\cot(y) = 3x\sin(5x)$ find $\frac{dy}{dx}$.

4. $y = \frac{4}{3}\tan(4\theta)$ find $\frac{dy}{d\theta}$. And find $\frac{d^2y}{d\theta^2}$

5. Find the equation of the tangent line to the curve $y = \csc x$ at $x = \frac{2\pi}{3}$

6. The position of a particle moving along the x-axis is given by $x(t) = \cot 2t$ find the acceleration of the particle at $t = \frac{3\pi}{4}$

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AP Calculus AB Trig Quiz #2 (Chapter 2) **REVIEW** #2 **Name:** Key

No Calculators. Leave all answers reasonably simplified

1. If $y = 4x^3\tan(4x^3)$ find $\frac{dy}{dx}$. *product rule $f'g + fg'$

$y' = \underbrace{12x^2}_{f'} \cdot \underbrace{\tan(4x^3)}_{g} + \underbrace{4x^3}_{f} \cdot \underbrace{\sec^2(4x^3)\cdot 12x^2}_{g'}$

$\boxed{y' = 12x^2\tan(4x^3) + 48x^5\sec^2(4x^3)}$

OR

$y' = 12x^2\left[\tan(4x^3) + 4x^3\sec^2(4x^3)\right]$

2. If $y = \frac{\csc(4x)}{\cot(3x)}$ find $\frac{dy}{dx}$. *quotient rule $\frac{f'g - fg'}{g^2}$

$\frac{\overbrace{-\csc(4x)\cot(4x)\cdot 4}^{f'}\overbrace{(\cot(3x))}^{g} - \overbrace{\csc(4x)}^{f}\cdot\overbrace{-\csc^2(3x)\cdot 3}^{g'}}{[\cot(3x)]^2}$

$y' = \frac{-4\csc(4x)\cot(4x)\cot(3x) + 3\csc(4x)\csc^2(3x)}{[\cot(3x)]^2}$

3. If $\cot(y) = 3x\sin(5x)$ find $\frac{dy}{dx}$. *implicit *product rule

$-\csc^2 y\left(\frac{dy}{dx}\right) = \overbrace{3}^{f'}\overbrace{\sin(5x)}^{g} + \overbrace{3x}^{f}\cdot\overbrace{\cos(5x)\cdot 5}^{g'}$

$\boxed{\frac{dy}{dx} = \frac{3\sin(5x) + 15x\cos(5x)}{-\csc^2 y}}$

4. $y = \frac{4}{3}\tan(4\theta)$ find $\frac{dy}{d\theta}$.

$y' = \frac{4}{3}\sec^2(4\theta)\cdot 4$

$\boxed{y' = \frac{16}{3}\sec^2(4\theta)}$

Find $y''(\theta)$

$y' = \frac{16}{3}[\sec(4\theta)]^2$

$y'' = \frac{16}{3}\cdot 2[\sec 4\theta]^1 \cdot \sec 4\theta\tan 4\theta \cdot 4$

$\boxed{y'' = \frac{128}{3}\sec^2 4\theta\tan 4\theta}$

5. Find the equation of the tangent line to the curve $y = \csc x$ at $x = \frac{2\pi}{3}$

*find point
*find slope: $y'\left(\frac{2\pi}{3}\right)$
*equation: $y - y_1 = m(x - x_1)$

point: $y\left(\frac{2\pi}{3}\right) = \csc\left(\frac{2\pi}{3}\right) = \frac{2\sqrt{3}}{3}$

$= \frac{1}{\sin\frac{2\pi}{3}} = \frac{1}{\sqrt{3}/2}$

point: $\left(\frac{2\pi}{3}, \frac{2\sqrt{3}}{3}\right)$

$y' = -\csc x\cot x \cdot 1$

$y'\left(\frac{2\pi}{3}\right) = -\csc\left(\frac{2\pi}{3}\right)\cot\left(\frac{2\pi}{3}\right)$

$= -\left(\frac{2\sqrt{3}}{3}\right)\left(-\frac{\sqrt{3}}{3}\right) = \frac{2}{3}$

slope: $m = 2/3$

$\boxed{y - \frac{2\sqrt{3}}{3} = \frac{2}{3}\left(x - \frac{2\pi}{3}\right)}$

6. The position of a particle moving along the x-axis is given by $x(t) = \cot 2t$ find the acceleration of the particle at $t = \frac{3\pi}{4}$.

$v(t) = -\csc^2(2t)\cdot 2 = -2\csc^2(2t)$

$= -2[\csc(2t)]^2$ chain rule

$a(t) = -2\cdot 2[\csc(2t)]\cdot -\csc(2t)\cot(2t)\cdot 2$

$a(t) = 8\csc^2(2t)\cot(2t)$

$a\left(\frac{3\pi}{4}\right) = 8\csc^2\left(2\left(\frac{3\pi}{4}\right)\right)\cot\left(2\left(\frac{3\pi}{4}\right)\right)$

$= 8(-1)^2(0) = \boxed{0}$

AP Calculus AB   Trig Quiz #2 Morning Review (WS #3)   Name:______________

1) If $y = \sqrt{csc^3(7 - \pi x)}$, find $\frac{dy}{dx}$

2) Given $y = \frac{\cos(x^5)}{\csc(ex)}$ Find y'

3) Given $y = -\csc(4 - \pi x)$ Find y'

4) $x cos y = tan y - 3x$ find $\frac{dy}{dx}$

5) If velocity of a particle is v(t) = $2tan^3 x$

a) Find a(t)

b) find acceleration at t = $\pi/4$

6. Find the tangent line equation for $f(x) = 2sec^2(2x)\ at\ x = \frac{\pi}{6}$

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Quiz Review Trig Quiz #2 Morning Session

1) $y = \sqrt{\csc^3(7-\pi x)}$ Find y'

$$y = \left[\csc^3(7-\pi x)\right]^{1/2} = \left[\csc(7-\pi x)\right]^{3/2}$$

$$y' = \frac{3}{2}\left[\csc(7-\pi x)\right]^{1/2} \cdot -\csc(7-\pi x)\cot(7-\pi x) \cdot -\pi$$

$$= \boxed{\frac{3\pi}{2}\left[\csc(7-\pi x)\right]^{3/2}\cot(7-\pi x)}$$

2) $y = \dfrac{\cos(x^5)}{\csc(ex)} = \cos(x^5)\sin(ex)$ Find y'

$$y' = \underbrace{\left(-\sin(x^5)\cdot 5x^4\right)}\underbrace{\left(\sin(ex)\right)} + \underbrace{\cos(x^5)}\cdot \underbrace{\cos(ex)\cdot e}$$

$$= \boxed{-5x^4\sin(x^5)\sin(ex) + e\cos(x^5)\cos(ex)}$$

3) $y = -\csc(4-\pi x)$ Find y' and y''

$$y' = -\left(-\csc(4-\pi x)\cot(4-\pi x)\right)\cdot(-\pi) = \boxed{-\pi\csc(4-\pi x)\cot(4-\pi x)}$$

$$y'' = \underbrace{-\pi\cdot -\csc(4-\pi x)\cot(4-\pi x)\cdot(-\pi)}_{f'} \cdot \underbrace{\cot(4-\pi x)}_{g} + \underbrace{-\pi\csc(4-\pi x)}_{f} \cdot \underbrace{-\csc^2(4-\pi x)\cdot -\pi}_{g'}$$

$$= \boxed{-\pi^2\csc(4-\pi x)\cot^2(4-\pi x) - \pi^2\csc^3(4-\pi x)}$$

4) $x\cos y = \tan y - 3x$ Find $\frac{dy}{dx}$

$$(1)(\cos y) + (x)(-\sin y)\left(\frac{dy}{dx}\right) = \sec^2 y\left(\frac{dy}{dx}\right) - 3$$

$$\frac{dy}{dx}(-x\sin y) - \frac{dy}{dx}(\sec^2 y) = -3 - \cos y$$

$$\frac{dy}{dx}(-x\sin y - \sec^2 y) = -3 - \cos y$$

$$\frac{dy}{dx} = \frac{-3-\cos y}{-x\sin y - \sec^2 y} = \frac{-(3+\cos y)}{-(x\sin y + \sec^2 y)} = \boxed{\frac{3+\cos y}{x\sin y + \sec^2 y}}$$

5) $y = 2\tan^3 x$ Evaluate $y'(\pi/4)$

$$y = 2[\tan x]^3$$

$$y' = 2\cdot 3[\tan x]^2 \sec^2 x = 6\tan^2 x \sec^2 x$$

$$y'(\pi/4) = 6[\tan(\pi/4)]^2[\sec(\pi/4)]^2$$

$$= 6(1)^2(\sqrt{2})^2$$

$$= 6(1)(2) \quad \boxed{y'(\pi/4) = 12}$$

6) Find tangent line equation at $x = \pi/6$ for $f(x) = 2\sec^2(2x)$

$f(x) = 2[\sec(2x)]^2$ $\quad f'(x) = 2\cdot 2[\sec(2x)]^1 \sec(2x)\tan(2x)\cdot 2 = \underline{8\sec^2(2x)\tan(2x)}$

$f'\left(\frac{\pi}{6}\right) = 8\left[\sec\left(\frac{2\pi}{6}\right)\right]^2 \tan\left(\frac{2\pi}{6}\right) = 8(2)^2(\sqrt{3}) = \underline{32\sqrt{3}}$ $\quad$ point: $f(\pi/6) = 2\left[\sec\left(\frac{2\pi}{6}\right)\right]^2 = \underline{8}$

point: $(\pi/6, 8)$

slope: $m = 32\sqrt{3}$

$$y - y_1 = m(x - x_1)$$

$$\boxed{y - 8 = 32\sqrt{3}(x - \pi/6)}$$