Morning Help session (Trig Unit)

Curve sketching

1) $f(x) = x - 2\sin x$ $\quad [-\pi, \pi]$

$f'(x) = 1 - 2\cos x$

$0 = 1 - 2\cos x$

$2\cos x = 1$

$\cos x = \frac{1}{2}$

$x = \boxed{\frac{\pi}{3}}, \frac{5\pi}{3}, -\frac{6\pi}{3}, -\frac{6\pi}{3}, -\frac{5\pi}{3}, \boxed{-\frac{\pi}{3}}$

$f'(x)$: + (increasing) on $(-\pi, -\frac{\pi}{3})$, − (decreasing) on $(-\frac{\pi}{3}, \frac{\pi}{3})$, + (increasing) on $(\frac{\pi}{3}, \pi)$

Test points: $-\frac{\pi}{2}$, 0, $\frac{\pi}{2}$

$f'(-\frac{\pi}{2}) = 1 - 2\cos(-\frac{\pi}{2})$

$f'(0) = 1 - 2\cos(0)$

$f'(\frac{\pi}{2}) = 1 - 2\cos(\frac{\pi}{2})$

$f''(x) = -2(-\sin x)$

$f''(x) = 2\sin x$

$0 = 2\sin x$

$2\sin x = 0$

$\sin x = 0$

$x = \boxed{0}, \pi, 2\pi, -\pi, -2\pi$

$f''(x)$: ∩ (concave down) on $(-\pi, 0)$, ∪ (concave up) on $(0, \pi)$

Test points: $-\frac{\pi}{2}$, $\frac{\pi}{2}$

$f''(-\frac{\pi}{2}) = 2\sin(-\frac{\pi}{2}) < 0$

$f''(\frac{\pi}{2}) = 2\sin(\frac{\pi}{2}) > 0$

1) (continued) * sketch graph $f(x) = x - 2\sin x$

$\sqrt{3} = 1.7$

$f(-\pi) = -\pi - 2\sin(-\pi) \approx -3$

$f(\pi) = \pi - 2\sin(\pi) \approx 3$

$f(-\pi/3) = -\frac{\pi}{3} - 2\sin(-\pi/3) \rightarrow -\frac{3}{3} - 2\left(-\frac{\sqrt{3}}{2}\right) = -1 + \frac{1.7}{1}$

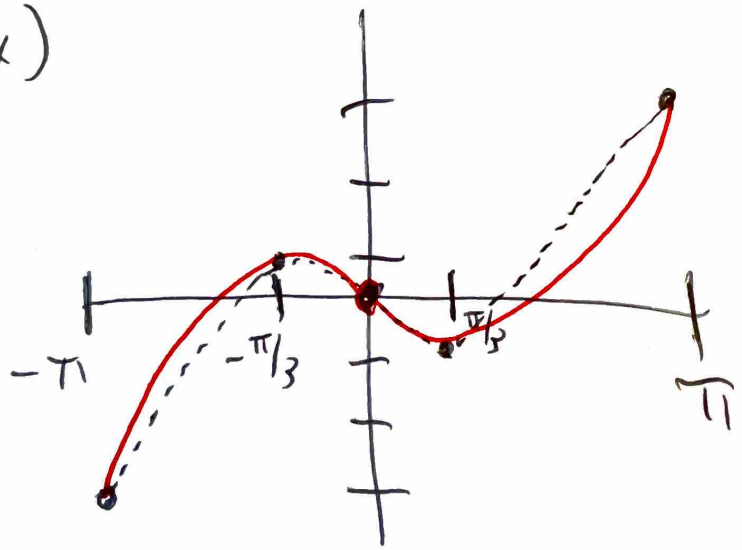
$\sin(5\pi/3)$

$-1 + 1.7 = \boxed{0.7}$

$f(\pi/3) = \frac{\pi}{3} - 2\sin(\pi/3) \rightarrow \frac{3}{3} - 2\left(\frac{\sqrt{3}}{2}\right)$ $\quad 1 - 1.7 = \boxed{-0.7}$

$f(0) = 0 - 2\sin(0) = 0$

$f(x)$



WS #1 Test Review & Trig Unit

4. An object's position is given by the F(t) = $2\sec^3\left(\frac{t}{6}\right)$, is continuous and differentiable in domain $0 \le t < 3\pi$ seconds. F(t) is given in meters.

a. Find the average velocity (avg. rate of change) from t = 0 to t = 2π (Include Units)

b. At what point in time does the instantaneous velocity equal the average velocity from part (a)? (Set up equation but do not solve) Use MVT.

c. What is the instantaneous velocity of the object when t = π seconds? (Include Units)

d. What is the instantaneous velocity of the object when t = 2π seconds? (Include Units)

e. Find the equation of the tangent line to the graph at t = 2π

a) Avg. rate of change means find slope between endpts.

$F(t) = 2\left[\sec\left(\frac{t}{6}\right)\right]^3$ or $2\left[\sec\left(\frac{1}{6}t\right)\right]^3$

$F(0) = 2\left[\sec\left(\frac{1}{6}\cdot 0\right)\right]^3 = 2[1]^3 = 2$

$F(2\pi) = 2\left[\sec\left(\frac{1}{6}\cdot 2\pi\right)\right]^3 = 2\left[\sec\left(\frac{\pi}{3}\right)\right]^3 = 2[2]^3 = 16$

Avg. rate of change $= \frac{16-2}{2\pi-0} = \frac{14}{2\pi} = \boxed{\frac{7}{\pi}\ m/s}$

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b) Find when instantaneous rate of change (ROC) = Avg. ROC.

***MVT:** $f'(c) = \frac{f(b)-f(a)}{b-a}$

out: $2[\ ]^3$

in: $\sec(\ )$

inner: $\frac{1}{6}t$

* $F(t) = 2\left[\sec\left(\frac{1}{6}t\right)\right]^3$

$F'(t) = \cancel{6}\left[\sec\left(\frac{1}{6}t\right)\right]^2 \cdot \sec\left(\frac{1}{6}t\right)\tan\left(\frac{1}{6}t\right)\cdot \frac{1}{\cancel{6}}$

$F'(t) = \left[\sec\left(\frac{1}{6}t\right)\right]^3 \tan\left(\frac{1}{6}t\right)$

b) *set up: $\left[\sec\left(\frac{1}{6}t\right)\right]^3 \tan\left(\frac{1}{6}t\right) = \frac{7}{\pi}$

c) Find instantaneous ROC at $t=\pi$

$$F'(t)=\left[\sec\left(\tfrac{1}{6}t\right)\right]^3\tan\left(\tfrac{1}{6}t\right)$$

$$F'(\pi)=\left[\sec\left(\tfrac{1}{6}\pi\right)\right]^3\tan\left(\pi/6\right)$$

$$\cos(\pi/6)=\frac{\sqrt{3}}{2}$$

$$\sec(\pi/6)=\frac{2}{\sqrt{3}}$$

$$\tan(\pi/6)=\frac{1}{\sqrt{3}}$$

$$\downarrow$$

$$\left[\frac{2}{\sqrt{3}}\right]^3\cdot\frac{1}{\sqrt{3}}$$

$$F'(\pi)=\frac{8}{3\sqrt{3}}\cdot\frac{1}{\sqrt{3}}\rightarrow\frac{8}{3\cdot3}=\boxed{\frac{8}{9}}$$

3) $y^2\sec(2x) - e^{3\pi x} = \arctan(3y) - 4\pi$ Find $\frac{dy}{dx}$

* implicit
* product Rule
* Arctrig

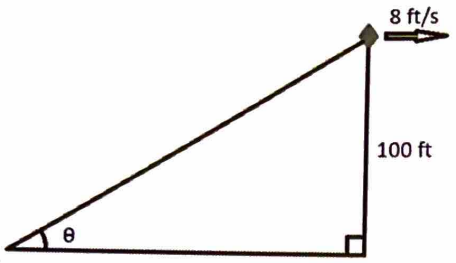
$2y\left(\frac{dy}{dx}\right)\sec(2x) + y^2 \cdot \sec(2x)\tan(2x)\cdot 2 - e^{3\pi x}\cdot(3\pi) = \frac{3\left(\frac{dy}{dx}\right)}{1+(3y)^2} - 0$

$2y\sec(2x)\frac{dy}{dx} - \frac{3}{1+9y^2}\left(\frac{dy}{dx}\right) = 3\pi e^{3\pi x} - 2y^2\sec(2x)\tan(2x)$

$\frac{dy}{dx}\left[2y\sec(2x) - \frac{3}{1+9y^2}\right] = 3\pi e^{3\pi x} - 2y^2\sec(2x)\tan(2x)$

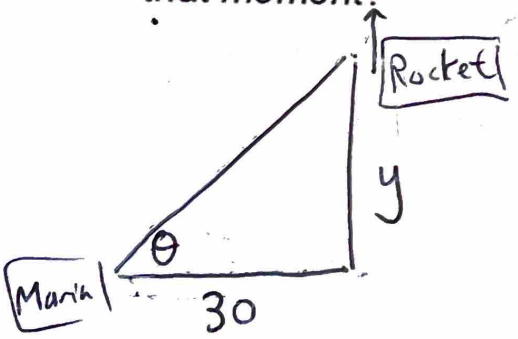
$$\frac{dy}{dx} = \frac{3\pi e^{3\pi x} - 2y^2\sec(2x)\tan(2x)}{2y\sec(2x) - \frac{3}{1+9y^2}}$$

9) A kite 100 ft above the ground moves horizontally at a speed of 8 ft/s. At what rate is the angle between the string and the horizontal decreasing when 200 ft of string has been let out?



10)

A model rocket is launched 30 feet from Maria, and is rising vertically at a constant rate of 20 ft/s when the rocket has an elevation of 40 feet. How fast is the angle of elevation from Maria to the rocket changing at that moment?



$\frac{dy}{dt} = 20 ft/s$

$y = 40$

$\frac{d\theta}{dt} =$ ___

$\tan\theta = \frac{y}{30} \rightarrow \frac{1}{30}y$

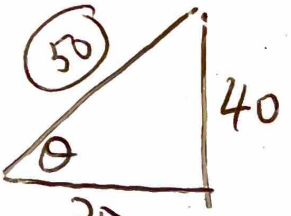
$\tan\theta = \frac{1}{30}y$

$\sec^2\theta\left(\frac{d\theta}{dt}\right) = \frac{1}{30}\left(\frac{dy}{dt}\right)$

$\left(\frac{5}{3}\right)^2\left(\frac{d\theta}{dt}\right) = \frac{1}{30}(20)$

$\frac{25}{9}\left(\frac{d\theta}{dt}\right) = \frac{2}{3}$

$\frac{d\theta}{dt} = \frac{2}{3}\cdot\frac{9}{25} = \frac{6}{25}$ rad/sec



$30^2 + 40^2 = c^2$

$c = 50$

$\sec\theta = \frac{H}{A} =$

$\sec\theta = \frac{50}{30} = \frac{5}{3}$