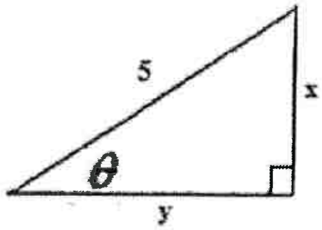
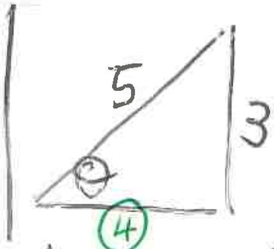
Trig Unit Test Review Session #1

1)



$\cos\theta = \frac{y}{5} = \frac{1}{5}y$

$-\sin\theta\left(\frac{d\theta}{dt}\right) = \frac{1}{5}\left(\frac{dy}{dt}\right)$



$a^2 + 3^2 = 5^2$

$a^2 = 16$

$a = 4$

$\sin\theta = \frac{3}{5}$

In the triangle shown above, if θ increases at a constant rate of 3 radians per minute, at what rate is y decreasing in units per minute when x equals 3 units?

$\frac{d\theta}{dt} = 3$ rad/min | $x = 3$

$\frac{dy}{dt} =$ ___

$-\left(\frac{3}{5}\right)\left(\frac{d\theta}{dt}\right) = \frac{1}{5}\left(\frac{dy}{dt}\right)$

$-\frac{3}{5}(3) = \frac{1}{5}\left(\frac{dy}{dt}\right)$

$-\frac{9}{5} = \frac{1}{5}\left(\frac{dy}{dt}\right)$

$\frac{5}{1} \cdot -\frac{9}{5} = \frac{dy}{dt}$

$\frac{dy}{dt} = -9$ units/min

2. A particle moves along a horizontal line so that at any time t its position is given by $x(t) = 2\pi t + \cos(2\pi t)$.

(a) Find the velocity at time t.

$v(t) = 2\pi + -\sin(2\pi t) \cdot 2\pi$

$v(t) = 2\pi - 2\pi\sin(2\pi t)$

(b) Find the acceleration at time t.

$a(t) = 0 - 2\pi\cos(2\pi t)2\pi$

$a(t) = -4\pi^2\cos(2\pi t)$

$\sin\theta = 1$

$v(t) = 0$

(c) What are all values of t for $0 \le t \le 3$, for which the particle is at rest. Justify your answer.

*set $v(t) = 0$

$2\pi - 2\pi\sin(2\pi t) = 0$

$-2\pi\sin(2\pi t) = -2\pi$

$\sin(2\pi t) = 1$

$2\pi t = \sin^{-1}(1)$ ← Add 2π $\left(+\frac{4\pi}{2}\right)$

$\frac{1}{2\pi}\left[2\pi t = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \frac{13\pi}{2}\right]\frac{1}{2\pi}$

$t = \frac{1}{4}, \frac{5}{4}, \frac{9}{4}$, ~~$\frac{13}{4}$~~ b/c $v(t) = 0$

d) When is particle moving left? Moving right? Justify with because statement

$v(t) = 2\pi(1 - \sin 2\pi t)$

$2\pi(1 - \sin(\frac{\pi}{4}))$

$v(t)$: + | + | + | + on the number line 0, 1/8, 1/4, 1, 5/4, 2, 9/4, 11/4, 3

moving right $(0, \frac{1}{4}) \cup (\frac{1}{4}, \frac{5}{4}) \cup (\frac{5}{4}, \frac{9}{4}) \cup (\frac{9}{4}, 3)$ b/c $v(t) > 0$

moving left: none.

3) Find dy/dx $\overset{f}{y}\overset{g}{\sec x} = 12 - 3y + 5x^2$

*product Rule
* implicit

$$\underbrace{\left(\frac{dy}{dx}\right)}_{f'}\underbrace{\sec x}_{g} + \underbrace{y}_{f} \cdot \underbrace{\sec x \tan x}_{g'} = 0 - 3\left(\frac{dy}{dx}\right) + 10x$$

$$\frac{dy}{dx}(\sec x) + 3\left(\frac{dy}{dx}\right) = 10x - y\sec x\tan x$$

$$\frac{dy}{dx}(\sec x + 3) = 10x - y\sec x\tan x$$

$$\boxed{\frac{dy}{dx} = \frac{10x - y\sec x\tan x}{\sec x + 3}}$$

MVT: $f(x) = x - 2\sin x \quad [-\pi, \pi]$

4) i) $f(x)$ continuous $[-\pi, \pi]$
$f(x)$ differentiable $(-\pi, \pi)$

MVT: $f'(c) = \frac{f(b) - f(a)}{b - a}$ find ordered pairs first

$f(-\pi) = -\pi - 2\sin(-\pi) = -\pi$

$f(\pi) = \pi - 2\sin(\pi) = \pi$

slope: $m = \frac{\pi - (-\pi)}{\pi - (-\pi)} = \frac{2\pi}{2\pi} = 1$

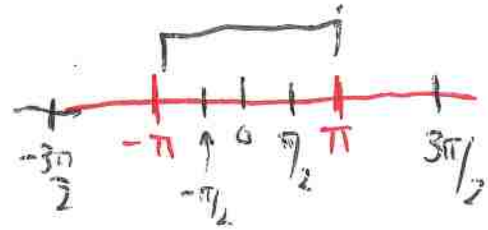
$f'(x) = 1 - 2\cos x$

$1 = 1 - 2\cos x$

$\frac{2\cos x}{2} = \frac{0}{2}$

$\cos x = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}, -\frac{\pi}{2}, -\frac{3\pi}{2}$



$$\boxed{c = -\frac{\pi}{2}, \frac{\pi}{2}}$$