"AP Live" Calculus AB Cumulative Course Review WS

Unit 1: Limits

1) Use Continuity Conditions to answer and justify the below question:

Is $f(x) = \begin{cases} \cos x, x < 0 \\ x^2 + 1, x \geq 0 \end{cases}$ continuous at $x = 0$?

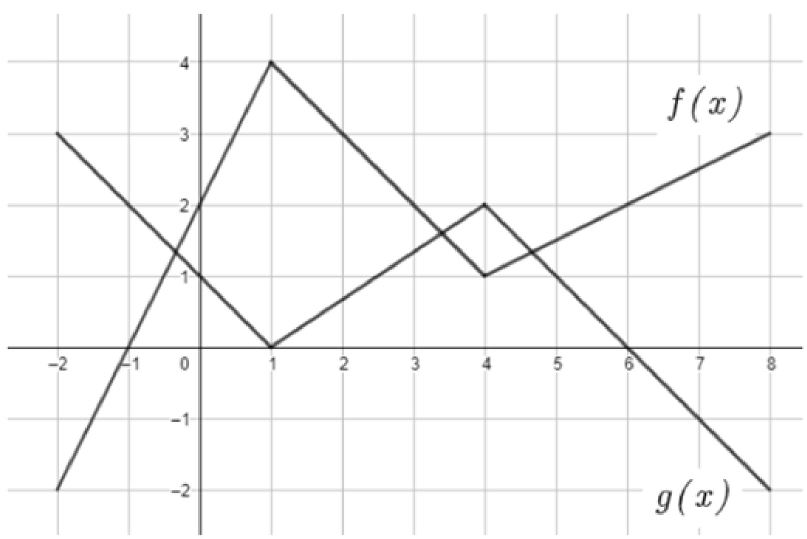
2) The function f is continuous at $x = 1$.

If $f(x) = \begin{cases} \dfrac{\sqrt{x+3} - \sqrt{3x+1}}{x-1} & \text{for } x \neq 1 \\ k & \text{for } x = 1 \end{cases}$ then $k =$

Unit 2-3: Derivatives / Derivatives of Composites

3) Given $h(x) = g(f(x))$

$h'(2) =$



4) Find $h'(1)$ $given\ h(x)$

$$h(x) = \frac{k(x)}{3x}$$

x	-1	1
$k(x)$	-3	2
$k'(x)$	4	-5

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5) $f(x) = \tan^2(3x^2)$, $f'(x) = ?$

6) Given that $p(x) = \sqrt[3]{2x-1}$ find $[p^{-1}]'(5)$.

7) $If\ y^2 - 3x = 7, then\ find\ \frac{d^2y}{dx^2}\ in\ terms\ of\ x\ and\ y$

8) The function f is defined on all the reals such that $f(x) = \begin{cases} x^2 + kx - 3 & for \quad x \leq 1 \\ 3x + b & for \quad x > 1 \end{cases}$

For which of the following values of k and b will the function f be both continuous and differentiable on its entire domain?

9) $If\ y = e^{kx}, then\ \frac{d^5y}{dx^5} =$

10) Consider the function $f(x) = \frac{6x}{a+x^3}\ for\ which\ f'(0) = 3$

Find the value of a.

Unit 4: Contextual Application of Differentiation

11) The positive variables **b** and **h** change with respect to time t. The relationship between b and h is given by the equation $h^3 = (4 - 2b)^2$. At the instant when $\frac{db}{dt} = 3$ and h = 4, what is the value of $\frac{dh}{dt}$?

12) Determine $\frac{dz}{dt}$ if you know that $z = xy^2,\ z = 3,\ y = \frac{1}{2},\ \frac{dx}{dt} = -2,$ and $\frac{dy}{dt} = 5.$

13) *The approximate value of* $y = \sqrt{3 + e^x}$ at x = 0.08, obtained from the tangent line to the graph at x = 0 is

14) ***The Function*** $\boldsymbol{C(x)}$ ***gives the dollar cost of digging a hole x feet deep.***

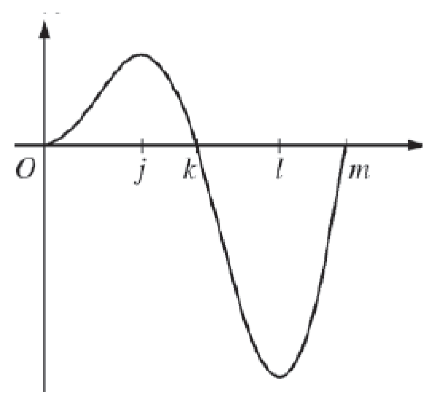
$\boldsymbol{C(20) = 140}$ means that a hole __________ deep costs ___________ to dig.

$\boldsymbol{C'(20) = 5}$ means that when the hole is ______________________, the cost of digging is _______________ at *a rate of* __________________________.

Unit 5: Analytic Applications of Derivative

15) $Let\ f(x) = x^4 + ax^2 + b.\ The\ graph\ of\ f\ has\ a\ relative\ maximum\ at\ (0{,}1)$ $and\ an\ inflection\ point\ when\ x = 1.\ The\ values\ of\ a\ and\ b\ are:$

16) The f ’(x) graph is shown . Answer the following:



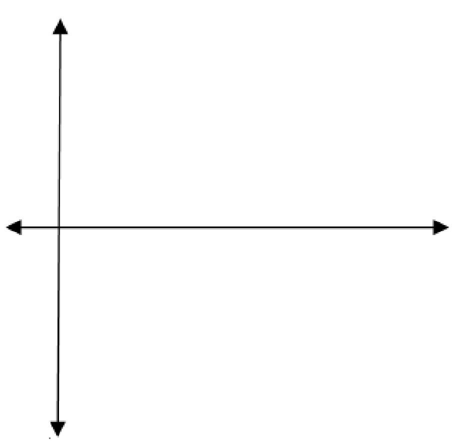
a) Find the x-value where absolute minimum occurs

b) Find the x-value where the absolute maximum occurs

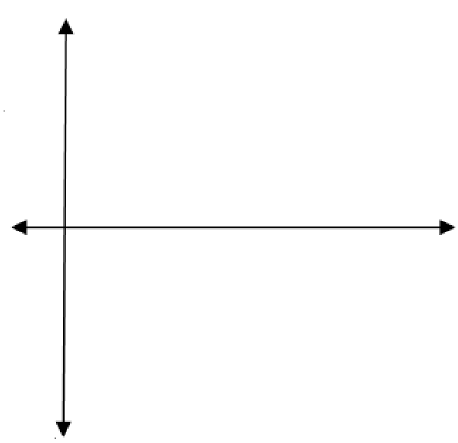
c) Sketch a possible f(x) graph given that f(0) = -2. (below)

d) Sketch a possible f ’’(x) graph (below)

c) $f(x)$



d) $f''(x)$



17) *The table below gives selected values for the differentiable function g.*

x	0	2	6	8	11	12
g(x)	-4	5	2	5	10	20

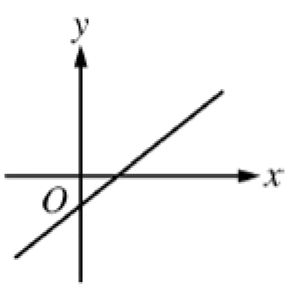
a) What's the least number of times $g(x) = 3$ in the given interval above? Justify your answer.

b) What can be concluded with Mean Value Theorem on the interval [0, 12]?

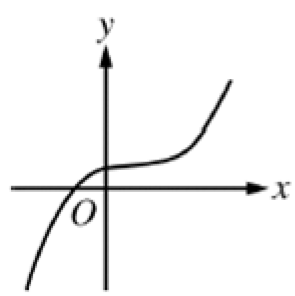
c) Can Rolle's Theorem be applicable in the interval [2, 8]? Justify your answer.

18) *The function g is differentiable and increasing for all real numbers x, and the graph of f has exactly 2 points of inflections. Of the following, which could be the graph of g', the derivative of g?*

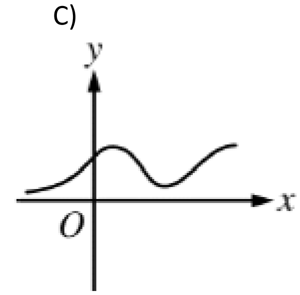
A)



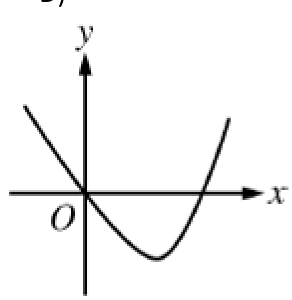
B)



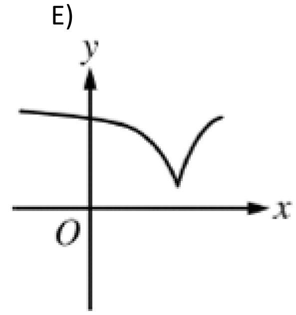
C)



D)

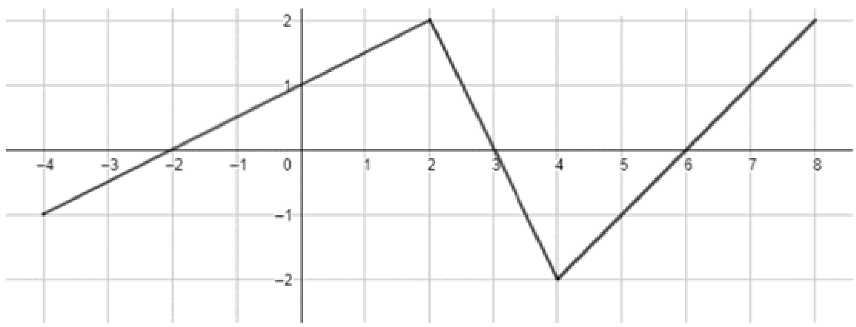


E)



Unit 6: Integration and Accumulation of Change

19) *Let f be the function defined by* $f(x) = \int_0^x (2t^3 - 15t^2 + 36t)dt$. *On which of the following interval is the graph of* $y = f(x)$ *concave down?*



20) a) *The graph of the piecwise linear function f is shown. What is the value of* $\int_{-4}^{8} f(x)dx$?

b) *The graph of the piecwise linear function f is shown. What is the value of* $\int_{-4}^{8} f'(x)dx$?

c) *The graph of the piecwise linear function f is shown. What is the value of* $\int_{-3}^{7} f''(x)dx$?

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21) *Let f and g be continuous function such that* $\int_0^8 f(x)dx = 12$, $\int_0^8 2g(x)dx = 4$,

and $\int_5^8 \big(f(x) - g(x)\big)dx = 3$. *What is the value of* $\int_0^5 (f(x) - g(x))dx$

22) *The function f is continuous and* $\int_0^8 f(u)du = 6$. *What is the value of* $\int_1^3 xf(x^2 - 1)dx$?

23) *If* $\int_0^b (4bx - 2x^2)\,dx = 36$, *then* $b =$

24) *If* $\int_{-2}^{2} (x^7 + k)dx = 16$, *then* $k =$

Unit 7: Differentiation Equations

25) Consider the differential equation $\frac{dy}{dx} = \left(1 - \frac{4}{x^3}\right)(y-1)^2$. Let $y = f(x)$ be the particular solution to the differential equation with initial condition $f(2) = -1$. Find $f(1)$.