Key

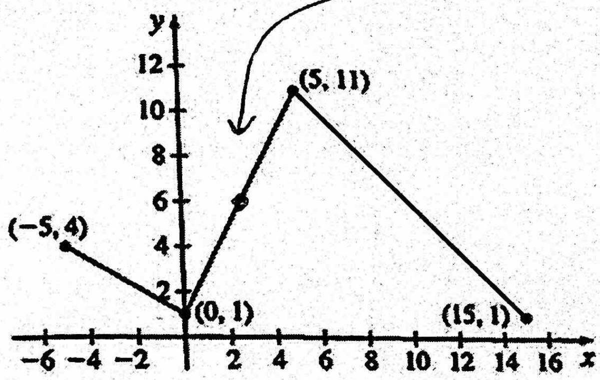
2.1 AP Practice Problems (p.171) – Rates of Change and the Derivative

1. The line $x + y = 5$ is tangent to the graph of $y = f(x)$ at the point where $x = 2$. The values $f(2)$ and $f'(2)$ are:

(A) $f(2) = 2;\ f'(2) = -1$ $\boxed{\text{(B)}}$ $f(2) = 3;\ f'(2) = -1$
(C) $f(2) = 2;\ f'(2) = 1$ (D) $f(2) = 3;\ f'(2) = 2$

$x + y = 5$
$y = 5 - x$ $y' = -1$
$\boxed{y(2) = 5 - 2 = 3}$ $\boxed{y'(2) = -1}$

2. The graph of the function f, given below, consists of three line segments. Find $f'(3)$.



Graph of f

(A) 1 $\boxed{\text{(B)}}$ 2 (C) 3 (D) $f'(3)$ does not exist

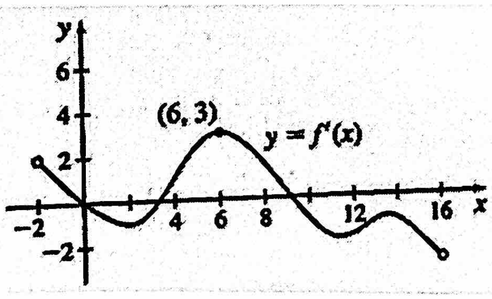
$f'(3) = \frac{11-1}{5-0} = \frac{10}{5} = 2$

3. What is the instantaneous rate of change of the function $f(x) = 3x^2 + 5$ at $x = 2$?

(A) 5 (B) 7 $\boxed{\text{(C)}}$ 12 (D) 17

$f'(x) = 6x$
$f'(2) = 6(2) = \boxed{12}$

4. The function f is defined on the closed interval $[-2, 16]$. The graph of the derivative of f, $y = f'(x)$, is given below.



The point $(6, -2)$ is on the graph of $y = f(x)$. An equation of the tangent line to the graph of f at $(6, -2)$ is

(A) $y = 3$ (B) $y + 2 = 6(x + 3)$
(C) $y + 2 = 6x$ $\boxed{\text{(D)}}$ $y + 2 = 3(x - 6)$

$f'(6) = 3$
$m = 3$

point: $(6, -2)$
slope: $m = 3$

$y - y_1 = m(x - x_1)$
$\boxed{y + 2 = 3(x - 6)}$

perpendicular (opposite reciprocal)

5. If $x - 3y = 13$ is an equation of the $\boxed{\text{normal line}}$ to the graph of f at the point $(2, 6)$, then $f'(2) =$

(A) $-\frac{1}{3}$ (B) $\frac{1}{3}$ $\boxed{\text{(C)}}$ -3 (D) $-\frac{13}{3}$

$x - 3y = 13$

$-3y = -1x + 13$

$y = \frac{-1x}{-3} + \frac{13}{-3}$

$y = \frac{1}{3}x - \frac{13}{3}$

$m = 1/3$

$m_2 = \frac{-3}{1}$

$\boxed{f'(2) = -3}$

6. If f is a function for which $\lim_{x \to -3} \frac{f(x) - f(-3)}{x + 3} = 0$, then which of the following statements must be true?

(A) $x = -3$ is a vertical asymptote of the graph.

$\boxed{\text{(B)}}$ The derivative of f at $x = -3$ exists.

(C) The function f is continuous at $x = 3$.

(D) f is not defined at $x = -3$.

* Alternate limit definition of derivative

$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$

$f'(-3) = \frac{f(x) - f(-3)}{x - (-3)} = 0$

$\boxed{f'(-3) = 0}$

*slope of the graph at $x = -3$ is 0

7. If the position of an object on the x-axis at time t is $4t^2$, then the average velocity of the object over the interval $0 \le t \le 5$ is

(A) 5 $\boxed{\text{(B)}}$ 20 (C) 40 (D) 100

position function $s(t) = 4t^2$

Avg. velocity $= \frac{\text{change in position}}{\text{change in time}}$

Avg. velocity $= \frac{s(5) - s(0)}{5 - 0}$

$s(5) = 100$ $s(0) = 0$

Avg. velocity $= \frac{100 - 0}{5 - 0} = \frac{100}{5}$

$= \boxed{20}$

8. A tank is filled with 80 liters of water at 7 a.m. ($t = 0$). Over the next 12 hours the water is continuously used and no water is added to replace it. The table below gives the amount of water $A(t)$ (in liters) remaining in the tank at selected times t, where t measures the number of hours after 7 a.m.

t	0	2	5	7	9	12
$A(t)$	80	71	66	60	54	50

Use the table to approximate $A'(5)$.

$m_{sec} \approx \frac{-5}{3}$ or -3 or $-11/5$

* Any of the 3 approximations are acceptable

$A'(5) \approx \frac{A(7) - A(5)}{7 - 5} = \frac{60 - 66}{2} = -3$ liters/hr

OR

$\frac{A(5) - A(2)}{5 - 2} = \frac{66 - 71}{5 - 2} = \frac{-5}{3}$ liters/hr

OR

$\frac{A(7) - A(2)}{7 - 2} = \frac{60 - 71}{7 - 2} = \frac{-11}{5}$ liters/hr