Calculus AB Course Review: Unit 4 Antidifferentiation MC WS

Integral Methods Priority Order Checklist:

1) **Expand/Power Rule** $ex: \int \frac{(x^2-2)^2}{\sqrt{x}}dx$ *only one term in the denominator *Always check **first** to see if problem can be expanded BEFORE attempting U-substitution	2) **U-Substitution:** $ex: \int \frac{3x}{(x^2-2)^2}dx$ *u-value is expression inside parentheses *rewrite problem using parentheses to identify u-value *Most Integral Problems fall in this category
3) **U-Sub/Change of Variable**: $\int 5x\sqrt{3-x}dx$ *initial u-value not enough to remove x *re-arrange assigned u-value equation to solve for x in terms of u	4) **Long/Synthetic Division** $\int \frac{3x^2-2x+1}{x-4}dx$ *condition: numerator degree is same or higher than denominator
5) **Arc-Trig U-Sub**: $\int \frac{4}{\sqrt{1-x^2}}dx$ *Condition: Denominator degree is 2 or more degrees greater than numerator(*condition applies only for rational expressions, not trig or exponential*)	6) 6) **Arc-Trig U-Sub**: $\int \frac{4x}{x^4-4x^2+19}dx$ *Condition: Denominator degree is 2 or more degrees greater than numerator (*for rational expressions*) * Complete the square in denominator to match Arc-Trig Integral Rules

1. $\int \left(x - \frac{1}{2x}\right)^2 dx =$

(A) $\frac{1}{3}\left(x - \frac{1}{2x}\right)^3 + C$ (B) $x^2 - 1 + \frac{1}{4x^2} + C$ (C) $\frac{x^3}{3} - 2x - \frac{1}{4x} + C$

(D) $\frac{x^3}{3} - x - \frac{4}{x} + C$ (E) none of these

2. $\int \frac{1-3y}{\sqrt{2y-3y^2}} dy =$

(A) $4\sqrt{2y-3y^2} + C$ (B) $\frac{1}{4}(2y-3y^2)^2 + C$ (C) $\frac{1}{2}\ln\sqrt{2y-3y^2} + C$

(D) $\frac{1}{4}(2y-3y^2)^{1/2} + C$ (E) $\sqrt{2y-3y^2} + C$

3. $\int \frac{x\,dx}{1+4x^2} =$

(A) $\frac{1}{8}\ln(1+4x^2)+C$ (B) $\frac{1}{8(1+4x^2)^2}+C$ (C) $\frac{1}{4}\sqrt{1+4x^2}+C$

(D) $\frac{1}{2}\ln\left|1+4x^2\right|+C$ (E) $\frac{1}{2}\tan^{-1}2x+C$

4. $\int \frac{x}{(1+4x^2)^2}\,dx =$

(A) $\frac{1}{8}\ln(1+4x^2)^2+C$ (B) $\frac{1}{4}\sqrt{1+4x^2}+C$ (C) $-\frac{1}{8(1+4x^2)}+C$

(D) $-\frac{1}{3(1+4x^2)^3}+C$ (E) $-\frac{1}{(1+4x^2)}+C$

5. $\int \frac{dy}{\sqrt{4-y^2}} =$

(A) $\frac{1}{2}\sin^{-1}\frac{y}{2}+C$ (B) $-\sqrt{4-y^2}+C$ (C) $\sin^{-1}\frac{y}{2}+C$

(D) $-\frac{1}{2}\ln\sqrt{4-y^2}+C$ (E) $-\frac{1}{3(4-y^2)^{3/2}}+C$

6. $\int \frac{2x+1}{2x}\,dx =$

(A) $x + \frac{1}{2}\ln|x| + C$ (B) $1 + \frac{1}{2}x^{-1} + C$ (C) $x + 2\ln|x| + C$

(D) $x + \ln|2x| + C$ (E) $\frac{1}{2}\left(2x - \frac{1}{x^2}\right) + C$

7. $\int \frac{\cos x\,dx}{\sqrt{1+\sin x}} =$

(A) $-\frac{1}{2}(1+\sin x)^{1/2} + C$

(B) $\ln\sqrt{1+\sin x} + C$

(C) $2\sqrt{1+\sin x} + C$

(D) $\ln|1+\sin x| + C$

(E) $\frac{2}{3(1+\sin x)^{3/2}} + C$

8. $\int \sec\frac{t}{2}\,dt =$

(A) $\ln\left|\sec\frac{t}{2} + \tan\frac{t}{2}\right| + C$ (B) $2\tan^2\frac{t}{2} + C$ (C) $2\ln\cos\frac{t}{2} + C$

(D) $\ln|\sec t + \tan t| + C$ (E) $2\ln\left|\sec\frac{t}{2} + \tan\frac{t}{2}\right| + C$

9. $\int \frac{dx}{x^2+2x+2} =$

(A) $\ln(x^2+2x+2)+C$ (B) $\ln|x+1|+C$ (C) $\arctan(x+1)+C$

(D) $\frac{1}{\frac{1}{3}x^3+x^2+2x}+C$ (E) $-\frac{1}{x}+\frac{1}{2}\ln|x|+\frac{x}{2}+C$

10. $\int \frac{(2-y)^2}{4\sqrt{y}}\,dy =$

(A) $\frac{1}{6}(2-y)^3\sqrt{y}+C$

(B) $2\sqrt{y}-\frac{2}{3}y^{3/2}+\frac{8}{5}y^{5/2}+C$

(C) $\ln|y|-y+2y^2+C$

(D) $2y^{1/2}-\frac{2}{3}y^{3/2}+\frac{1}{10}y^{5/2}+C$

(E) none of these

11. $\int \frac{e^x}{1+e^{2x}}\,dx =$

(A) $\tan^{-1} e^x + C$ (B) $\frac{1}{2}\ln(1+e^{2x})+C$ (C) $\ln(1+e^{2x})+C$

(D) $\frac{1}{2}\tan^{-1} e^x + C$ (E) $2\tan^{-1} e^x + C$

Definite Integrals and Applications:

12. $\int_1^2 \frac{x^2+6x+6}{x+1}\,dx =$

(A) $1+\ln\frac{3}{2}$

(B) 6.5

(C) $6.5+\ln\frac{3}{2}$

(D) $6.5+\ln 6$

Definite Integrals and Applications:

13. $\int_{-3}^{2} |x+1| \, dx =$

(A) $\frac{5}{2}$ (B) $\frac{7}{2}$ (C) 5 (D) $\frac{11}{2}$ (E) $\frac{13}{2}$

14. $\frac{d}{dt}\int_{0}^{t} \sqrt{x^3+1}\, dx =$

(A) $\sqrt{t^3+1}$ (B) $\frac{\sqrt{t^3+1}}{3t^2}$ (C) $\frac{2}{3}(t^3+1)(\sqrt{t^3+1}-1)$

(D) $3x^2\sqrt{x^3+1}$ (E) none of these

15. $\frac{d}{dx}\int_{\pi/2}^{x^2} \sqrt{\sin t}\, dt =$

(A) $\sqrt{\sin t^2}$ (B) $2x\sqrt{\sin x^2}-1$ (C) $\frac{2}{3}(\sin^{3/2} x^2 - 1)$

(D) $\sqrt{\sin x^2}-1$ (E) $2x\sqrt{\sin x^2}$

16. $\int_{0}^{1} xe^{x^2}\, dx =$

(A) $e-1$ (B) $\frac{1}{2}(e-1)$ (C) $2(e-1)$ (D) $\frac{e}{2}$ (E) $\frac{e}{2}-1$

17. $\int_0^{\pi} \cos^2\theta \sin\theta \, d\theta =$

(A) $-\frac{2}{3}$ (B) $\frac{1}{3}$ (C) 1 (D) $\frac{2}{3}$ (E) 0

18. $\int_0^1 \frac{e^{-x}+1}{e^{-x}}\,dx =$

(A) e (B) $2+e$ (C) $\frac{1}{e}$ (D) $1+e$ (E) $e-1$

19. If the substitution $u = \sqrt{x+1}$ is used, then $\int_0^3 \frac{dx}{x\sqrt{x+1}}$ is equivalent to

(A) $\int_1^2 \frac{du}{u^2-1}$ (B) $\int_1^2 \frac{2\,du}{u^2-1}$ (C) $2\int_0^3 \frac{du}{(u-1)(u+1)}$

(D) $2\int_1^2 \frac{du}{u(u^2-1)}$ (E) $2\int_0^3 \frac{du}{u(u-1)}$