Name:____ **Period:** __________

Accel. Pre-Calculus

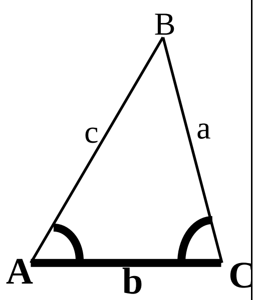
Unit 3 Packet

Trig Laws & Area of a Triangle

Trigonometry for Non-Right Triangles

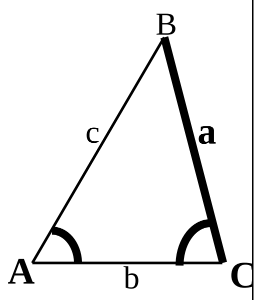
If you have …

Angle-Side-Angle



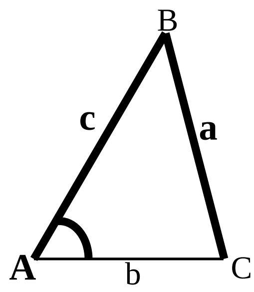
A + C < 180°
Otherwise 0∆s

Angle-Angle-Side



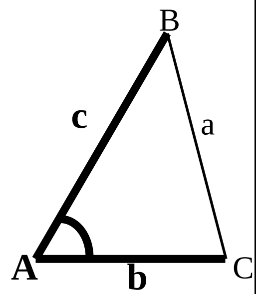
A + C < 180°
Otherwise 0∆s

Angle-Side-Side

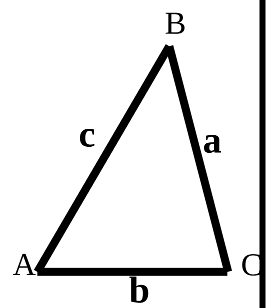


ambiguous case

Side-Angle-Side



Side-Side-Side



a + b > c
Otherwise 0∆s

Find angle measures and side lengths with …

Law of Sines

$$\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}$$

$$\frac{sinA}{a} = \frac{sinB}{b} = \frac{sinC}{c}$$

Law of Cosines

$a^2 = b^2 + c^2 - 2bc \cos A$
$b^2 = a^2 + c^2 - 2ac \cos B$
$c^2 = a^2 + b^2 - 2ab \cos C$

Ambiguous Case:

-Find your first missing angle (solution 1) using Law of Sines
-Check it to see if it makes sense
- given angle + solution 1 ≥ 180° means **NO TRIANGLE**
- given angle + solution 1 < 180° means at least **1 TRIANGLE**
-If you have 1 triangle, you may have two. Find your second possible angle.
-180° − solution 1 = second possible angle
- Check it to see if it makes sense
- given angle + solution 2 ≥ 180° means this solution is no good; only **1 TRIANGLE**; solve
- given angle + solution 2 < 180° means **2 TRIANGLES;** make sure you solve both

Just remember: if you're given ASS, the problem is going to be a pain in the _______. 😉

For SAS & SSS, find the largest angle either **first** (using Law of Cosines) or **last** (using angle sum of 180°)! May **not** use Law of Sines to find the largest angle!

Find the area of the triangle with …

$$\frac{1}{2}bc\ sinA$$

Heron's Formula:

$$\sqrt{s(s-a)(s-b)(s-c)}$$

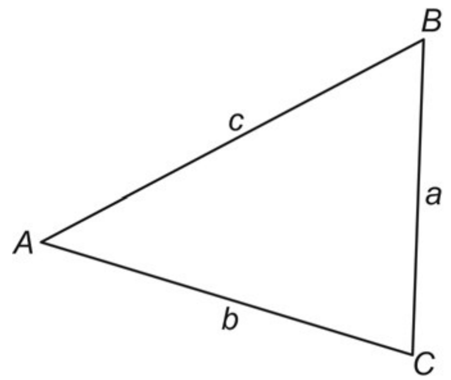
where $s = \frac{1}{2}(a+b+c)$

3.01 The Law of Sines

Name ______________________

Right triangle trigonometry can be used to solve problems involving right triangles. However, many interesting problems involve non-right triangles. In this lesson, you will use right triangle trigonometry to develop the *Law of Sines*. The law of sines is important because it can be used to solve problems involving non-right triangles as well as right triangles.

Consider oblique ΔABC shown to the right.



1. Sketch an altitude from vertex B.

2. Label the altitude k.

3. The altitude creates two right triangles inside ΔABC. Notice that $\angle A$ is contained in one of the right triangles, and $\angle C$ is contained in the other. Using right triangle trigonometry, write two equations, one involving $\sin A$, and one involving $\sin C$.

   $\sin A =$ ______ $\sin C =$ ______

4. Notice that each of the equations in Question 3 involves k. Why does this happen? Solve each equation for k.

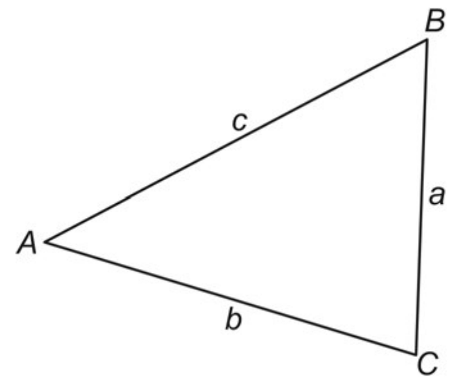
5. Since both equations in Question 4 are equal to k, they can be set equal to each other. Why is this possible? Set the expressions equivalent to k equal to each other to form a new equation.

6. Notice that the equation in Question 5 no longer involves k. Rewrite the equation in Question 5, regrouping a with $\sin A$ and c with $\sin C$.

Again, consider oblique ΔABC.

This time, sketch an altitude from vertex C.

Label the altitude k.



7. The altitude creates two right triangles inside ΔABC. Notice that $\angle A$ is contained in one of the right triangles and $\angle B$ is contained in the other. Using right triangle trigonometry, write two equations, one involving sin A and one involving sin B.

sin A = ______ sin B = ______

8. Again, each of the equations in Question 9 involves k. Solve each equation for k.

9. Since both equations in Question 10 are equal to k, they can be set equal to each other. Set the equations equal to each other to form a new equation.

10. Now equation in Question 11 no longer involves k. Rewrite the equation in Question 11, regrouping a with sin A and b with sin B.

11. Use the equations in Question 6 and Question 12 to write a third equation involving b, c, sin B, and sin C.

Together, the equations in Questions 6, 12, and 13 form the *Law of Sines*. The law of sines is important, because it can be used to solve problems involving both right and non-right triangles, because it involves only the sides and angles of a triangle.

3.01 Law of Sines Practice **Date:** ________________________

Draw and label each triangle. Then solve the triangle using the Law of Sines: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

1. A = 40° a = 20

   B = _________ b = ____________

   C = 70° c = ____________

2. A = _________ a = ____________

   B = 50° b = 30

   C = 100° c = ____________

3. D = 25° d = ____________

   E = 35° e = 12

   F = _________ f = ____________

4. R = 65° r = ____________

   S = 50° s = ____________

   T = _________ t = 12

5. X = _________ x = 8.2

   Y = 24.8° y = ____________

   Z = 61.3° z = ____________

6. A landscaper wants to plant begonias along the edges of a triangular plot of land in Wills Park. Two of the angles of the triangle measure 95° and 40°. The side between these two angles is 80 feet long.
   a) Find the measure of the third angle.

   b) Find the lengths of the other two sides of the triangle.

   c) What is the perimeter of the triangular plot?

7. A cable car transports passengers up and down a mountain. The track used by the cable car has an angle of elevation of 30°. The angle of elevation from a point 100 feet from the base of the track (away from the mountain) to the top of the track is about 26.8°. Find the length of the track.

3.02 Law of Sines – Ambiguous Case Notes **Name:** ________________

Remember: Law of Sines $\frac{sinA}{a} = \frac{sinB}{b} = \frac{sinC}{c}$

In some cases, you'll be given a consecutive angle, side, and side (ASS). For these problems you have what's called **The Ambiguous Case**- meaning: you don't know how many triangle solutions you'll have. There **could be no triangle, one triangle, or even two triangles** that work for your given measurements.

Here's what you do:

-Find your first missing angle (solution 1) using Law of Sines

-Check it to see if it makes sense

- given angle + solution 1 ≥ 180° means **NO TRIANGLE**

- given angle + solution 1 < 180° means at least **1 TRIANGLE**

-If you have 1 triangle, you may have two. Find your second possible angle.

-180° − solution 1 = second possible angle

- Check it to see if it makes sense

- given angle + solution 2 ≥ 180° means this solution is no good; only **1 TRIANGLE**; solve

- given angle + solution 2 < 180° means **2 TRIANGLES;** make sure you solve both

Just remember: if you're given ASS, the problem is going to be a pain in the _______. 😊

Examples:

1. $a = 2, c = 1, C = 50°$

2. $a = 3, b = 2, A = 40°$

3. $a = 6, b = 8, A = 35°$

APC 3.02 Law of Sines – Ambiguous Case Practice **Date: ________________________**

a) Determine the number of solutions for triangle ABC. b) **<u>If there is only 1 solution</u>,** solve the triangle.

1. $A = 57°$, $a = 11$, $b = 19$ a) ________ Triangles

2. $A = 30°$, $a = 13$, $b = 26$ a) ________ Triangles

3. $A = 100°$, $b = 18$, $a = 12$ a) ________ Triangles

4. $A = 58°$, $C = 94°$, $b = 17$ a) ________ Triangles

5. $A = 37°$, $a = 27$, $b = 32$ a) ________ Triangles

6. $A = 65°$, $a = 55$, $b = 57$ a) ________ Triangles

3.03 Ambiguous Case Worksheet

Date: _______________

Determine the number of possible solutions. If a solution exists, solve the triangle(s).

1. $C = 30°$, $b = 20$, $c = 10$

2. $a = 32$, $b = 38$, $A = 47°$

3. $a = 16.5$, $c = 10.1$, $A = 140°$

4. $A = 52°$, $B = 61°$, $c = 25$

5. $B = 58°$, $b = 11$, $c = 12$

6. $a = 29$, $b = 35$, $B = 90°$

Explain why the set of measurements given in #7 & #8 do not create any triangles.

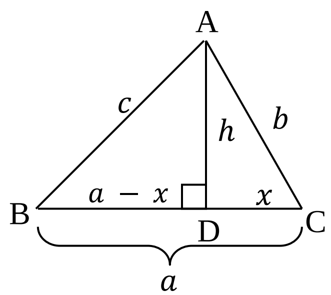
7. $C = 23°$, $a = 8$, $c = 2$

8. $a = 5$, $b = 9$, $A = 100°$

9. Charlie is standing near a river and wants to calculate the distance across the river. He measures the angle made between his line of sight to a tree on the edge of his side of the river (further downriver from where he is) and to a boat ramp directly on the other side of the river to be 28°. The distance between him and the tree can be measured and is 300 feet. The angle formed by him, the tree, and the boat ramp is 128°. What is the distance across the river from the tree to the boat ramp?

Accel Pre-Calculus
3.04 Notes: Law of Cosines

Let's look at that non-right triangle with the altitude again. The altitude breaks the side it intersects into two lengths, x and $a - x$. Apply the Pythagorean Theorem to ΔADB:



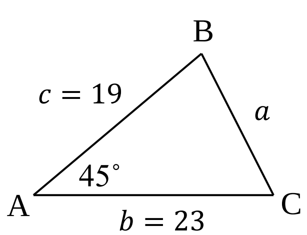
$$c^2 = (a - x)^2 + h^2$$

<u>Law of Cosines:</u> **$c^2 = a^2 + b^2 - 2ab\cos C$**
$b^2 = a^2 + c^2 - 2ac\cos B$
$a^2 = b^2 + c^2 - 2bc\cos A$

The law of cosines allows us to solve for oblique (non-right) triangles when we have 3 side lengths (SSS) or 2 side lengths and the included angle's measure (SAS).

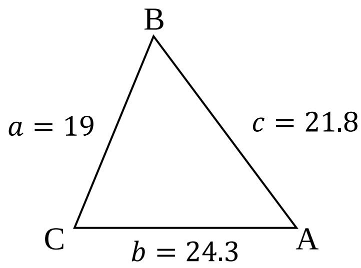
Examples: Solve ΔABC. Round all answers to the nearest tenth.

1. $A = 45^\circ, b = 23, and\ c = 19.$



To find an angle, use Law of Sines:

2. $a = 19, b = 24.3, and\ c = 21.8$

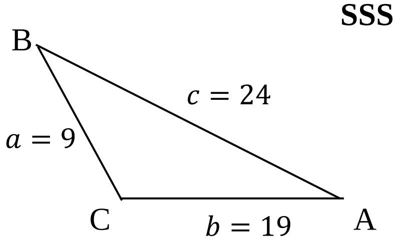


SSS information – ***find the angle opposite the longest side:***

To find another angle use the Law of Sines:

To find the last angle, subtract from 180˚:

3. $a = 9, b = 19, c = 24$



SSS information – ***find the angle opposite the longest side:***

To find another angle use the Law of Sines:

To find the last angle, subtract from 180˚:

3.04 Law of Cosines Practice **Date:** ____________________

$$a^2 = b^2 + c^2 - 2bc\cos A \qquad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Solve each triangle using the Law of Cosines and the Law of Sines.

1. A = 51° a = ____________

   B = ____________ b = 7

   C = ____________ c = 10

2. A = ____________ a = 4

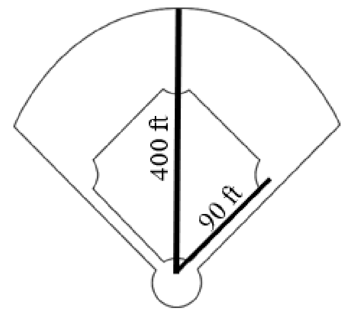
   B = ____________ b = 5

   C = ____________ c = 7

3. The sides of a triangle measure 14.9 cm, 23.8 cm and 36.9 cm. Find the angle with the least measure.

4. The lengths of two sides of a parallelogram are 48 inches and 30 inches. One angle measures 120°. Find the length of the longer diagonal.

5. In baseball, dead center field is the farthest point in the outfield on the straight line through home plate and second base. The distance between consecutive bases is 90 feet. In Wrigley Field in Chicago, dead center field is 400 feet from home plate. How far is dead center field from first base?



Solve the following triangles using the law of sines and/or the law of cosines. First, draw and label each triangle. Then, determine the number of solutions. Finally, solve each triangle.

6. $P = 48°$, $Q = 96°$, $r = 12.1$

7. $A = 132°$, $a = 33$, $b = 50$

8. $k = 21$, $m = 12$, $n = 28$

9. $B = 99°$, $a = 10$, $c = 6$

10. $A = 23°$, $a = 120$, $b = 171$

11. $a = 1.5$, $b = 2.3$, $c = 5.9$

3.06 Law of Sines and Cosines Review **Date: ________________**

State the number of possible triangles that can be formed using the given measurements.

1. In $\Delta RST, m\measuredangle R = 61°, t = 35, r = 31$

2. In $\Delta CAB, m\measuredangle A = 95°, c = 9, a = 19$

Solve each triangle.

3. In $\Delta STR, t = 18, r = 28, m\measuredangle S = 114°$

R_1 = ________	R_2 = ________
T_1 = ________	T_2 = ________
s_1 = ________	s_2 = ________

4. In $\Delta PKH, m\measuredangle P = 19°, h = 34, p = 26$

H_1 = ________	H_2 = ________
K_1 = ________	K_2 = ________
k_1 = ________	k_2 = ________

5. In $\Delta CAB, a = 24.1, b = 28.9, c = 28.1$

A_1 = ________	A_2 = ________
B_1 = ________	B_2 = ________
C_1 = ________	C_2 = ________

6. In $\Delta ZXY, m\measuredangle Z = 48°, m\measuredangle X = 78°, y = 24$

Y_1 = ________	Y_2 = ________
z_1 = ________	z_2 = ________
x_1 = ________	x_2 = ________

7. In $\Delta PQR, m\measuredangle P = 26°, r = 19, p = 16$

R_1 = _________	R_2 = _________
Q_1 = _________	Q_2 = _________
q_1 = _________	q_2 = _________

8. Determine which law you would use to solve and how many triangles there are given A = 62°, b = 24, and a = 20

9. Determine which law you would use to solve and how many triangles there are given B = 138°, c = 15, and a = 12

Accel Pre-Calculus Name ________________________

3.08 Area of Triangles Notes Date _______________

For SAS:

For SSS:

Find the area of each triangle. Round answers to the nearest tenth.

1. $A = 52°,\ b = 12,\ c = 18$

2. $a = 13,\ b = 8,\ c = 15$

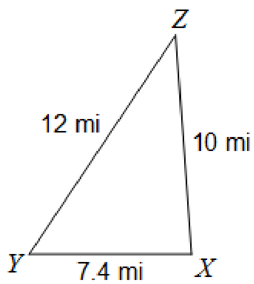
3. $a = 24,\ b = 18,\ c = 21$

4. $a = 20,\ b = 33,\ C = 98°$

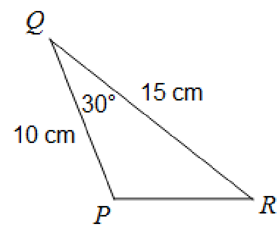
3.08 Area of a Triangle **Date: ____________**

Find the area of each triangle.

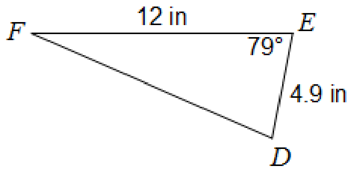
1)



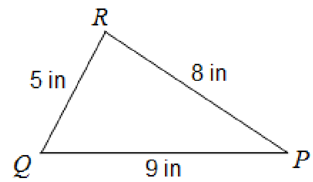
2)



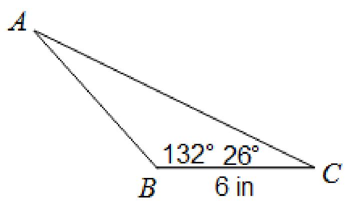
3)



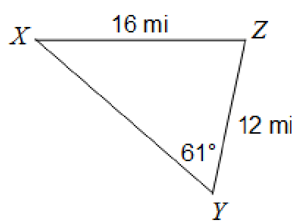
4)



5)



6)



7) In $\triangle DEF$, $d = 11$ m, $e = 12$ m, $f = 15.6$ m

8) In $\triangle ABC$, $a = 8.3$ yd, $c = 6$ yd, $m\angle A = 79°$

9) In $\triangle ZXY$, $z = 4$ mi, $m\angle X = 21°$, $m\angle Z = 121°$

10) In $\triangle ABC$, $c = 5$ cm, $m\angle A = 59°$, $b = 4$ cm

3.09 Applications of Law of Sines and Cosines, Area of Triangles Date __________

Law of Sines:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Law of Cosines:

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

Area:

SAS: $\frac{1}{2}ab\sin C = \frac{1}{2}bc\sin A = \frac{1}{2}ac\sin B$

SSS: $\sqrt{s(s-a)(s-b)(s-c)}, where\ s = \frac{(a+b+c)}{2}$

1. The Alpharetta Community Garden Network wants to plant vegetables in a triangular plot of land in Wills Park. Two of the angles of the triangle measure 95º and 40º. The side between these two angles is 80 feet long.

   a) Find the measure of the third angle.

   b) Find the lengths of the other two sides of the triangle.

   c) What is the perimeter of the triangular plot?

2. A cable car transports passengers up and down a mountain. The track used by the cable car has an angle of elevation of 30º. The angle of elevation from a point 100 feet from the base of the track (away from the mountain) to the top of the track is about 26.8º. Find the length of the track.

3. The sides of a triangle measure 14.9 cm, 23.8 cm and 36.9 cm. Find the angle with the least measure.

4. The lengths of two sides of a parallelogram are 48 inches and 30 inches. One angle measures 120 º. Find the length of the longer diagonal.

5. The adjacent sides of a parallelogram measure 14 cm and 20 cm and one angle measures 57°. Find the area of the parallelogram.

6. The side of a rhombus is 15 cm long and the length of the longer diagonal is 24.6 cm. Find the area of the rhombus.

7. The roof on a house has one side that is in the shape of an isosceles triangle. The sides of this part of the roof are 18 feet long and the angle at the peak is 50°. Find the area of this part of the roof.

3.10 Test Review: Laws and Area Date _______________

For each, a) state how many triangles and b) solve the triangle(s) if possible.

1. $A = 79°$, $B = 33°$, $a = 7$

2. $b = 5$, $a = 8$, $A = 110°$

3. $b = 8$, $a = 3$, $A = 30°$

4. $a = 24.1$, $b = 27$, $C = 18°$

5. $A = 34°$, $B = 74°$, $c = 5$

6. $c = 41$, $A = 22.9°$, $C = 55.1°$

7. $a = 4.1$, $b = 12$, $c = 8.7$

8. $A = 47°$, $a = 25$, $b = 34$

Find the area of each.

9. $a = 3$, $b = 5$, $c = 6$

10. $a = 10$, $b = 6$, $C = 50°$

11. $A = 34°$, $B = 74°$, $c = 5$

12. If b = 14, A = 58°, and a = 9, which law would you use first to solve and how many triangle solutions are there?

13. When a hockey player attempts a shot, he is 20 feet from the left post of the goal and 24 feet from the right post. If a regulation hockey goal is 6 feet wide, what is the player's shot angle to the nearest degree?

14. A lamppost tilts toward the sun at a 2° angle from the vertical and casts a 25-foot shadow. The angle from the tip of the shadow to the top of the lamppost is 45°. Find the length of the lamppost.

15. To estimate the height of Milton High School all the way to the top of the eagle weathervane, two students stand on the front lawn looking up at it. Jack looks up with a 35° angle of elevation. From a point 45 feet closer to the building, Emily looks up with a 51° angle of elevation. Find the height to the top of the school.

16. Find the area of a regular decagon inscribe in a circle with radius 10 cm.