Name:____ **Period:** __________

Accel. Pre-Calculus

Unit 4A Packet

Trig Identities

Accelerated Pre-Calculus Name ________________________________

4.01 Trigonometric Identities – Simplifying Trig Expressions Date ____________________

Review – *What is an identity?*

Reciprocal Identities:

Examples:

a) Given $cosx = \frac{4}{5}$ $find\ secx$

b) Given $sinx = -\frac{\sqrt{8}}{4}$ $find\ cscx$

c) Given $tanx = -\frac{1}{4}$ $find\ cotx$

Quotient Identities:

Pythagorean Identities: *(Where do they come from?)*

Example: Given $cosx = \frac{3}{4}$ $find\ sinx$

a) Use the Pythagorean Theorem

b) Use the Pythagorean Identity

3 Types of Problems:

Simplifying (write in simplest terms)

Verifying (show why the identity is true by working from one side to the other)

Evaluating (use trig identities to solve)

Strategies for Simplifying and Verifying Trigonometric Identities

- **Use the correct Identity:** $sinxcotx =$
  - Rewrite everything in terms **sine** and **cosine**.
  - See $tanx$ or $cotx$? Think **Quotient Identity.**
  - See a trig function in the denominator? Think **Reciprocal Identity**. $\frac{1}{cot^2x} =$
  - See squares? Think **Pythagorean Identity**.

- **Use Algebra:** $cscxsecx - cotx =$
  - Are there multiple fractions? Bring them together. Adding requires common denominators! Multiplying does not!
  - Split a single fraction into two parts:
  - Factor out the common term(s): $tanxcsc^2x - tanx =$
  - Distribute conjugates: $(sec\,x + 1)(sec\,x - 1) =$
  - Is there a complex fraction? Multiply by a common denominator or by the reciprocal of the denominator - KCF! $\frac{\sec x}{\tan x} =$

4.01 Practice: Connect the Dots Activity

Directions: Simplify the following expressions and match them with their solution. Connect the number of the question with the letter of the solution to create a picture on the next page.

1. $\sec^2 x - 1$
2. $\frac{1}{\sin x}$
3. $\sin x \cot x$
4. $\frac{\sin^2 x}{1 + \cos x}$
5. $1 + \tan^2 x$
6. $\csc^2 x - \cot^2 x$
7. $\frac{\cos^2 x}{1+\sin x}$
8. $\frac{\sin x + \cos x}{\sin x \cos x}$
9. $\frac{\cot^2 x}{\csc x - 1}$
10. $\frac{1}{\tan x}$
11. $\frac{1}{\cot x}$
12. $\frac{\cos x}{\cot x}$
13. $\cot^2 x + 1$
14. $\frac{1}{\cos x}$
15. $\frac{\cot^2 x}{\csc x + 1}$
16. $\sin^2 x + \cos^2 x + 1$
17. $1 - (\sec^2 x - \tan^2 x)$
18. $\csc^2 x - 1$
19. $1 - \sin^2 x$
20. $\frac{\tan^2 x}{\sec x - 1}$

A. $\csc x$

B. 1

C. $\sec x + \csc x$

D. $\sin x$

E. $\sec x$

F. 2

G. -1

H. $\cos^2 x$

I. 3

J. $\csc x - 1$

K. $\sec x - 1$

L. $\cot^2 x$

M. $\sec x - \csc x$

N. $\sec x + 1$

O. 0

P. $\tan^2 x$

Q. $\cos x$

R. $-\cot^2 x$

S. $\sec^2 x$

T. $1 - \sin x$

21. $\frac{\tan^2 x}{\sec x + 1}$

22. $-(\sin^2 x + \cos^2 x)$

23. $\frac{\sin x - \cos x}{\sin x \cos x}$

24. $1 - \sec^2 x$

25. $1 - \csc^2 x$

26. $3(\sin^2 x + \cos^2 x)$

U. $\csc x + 1$

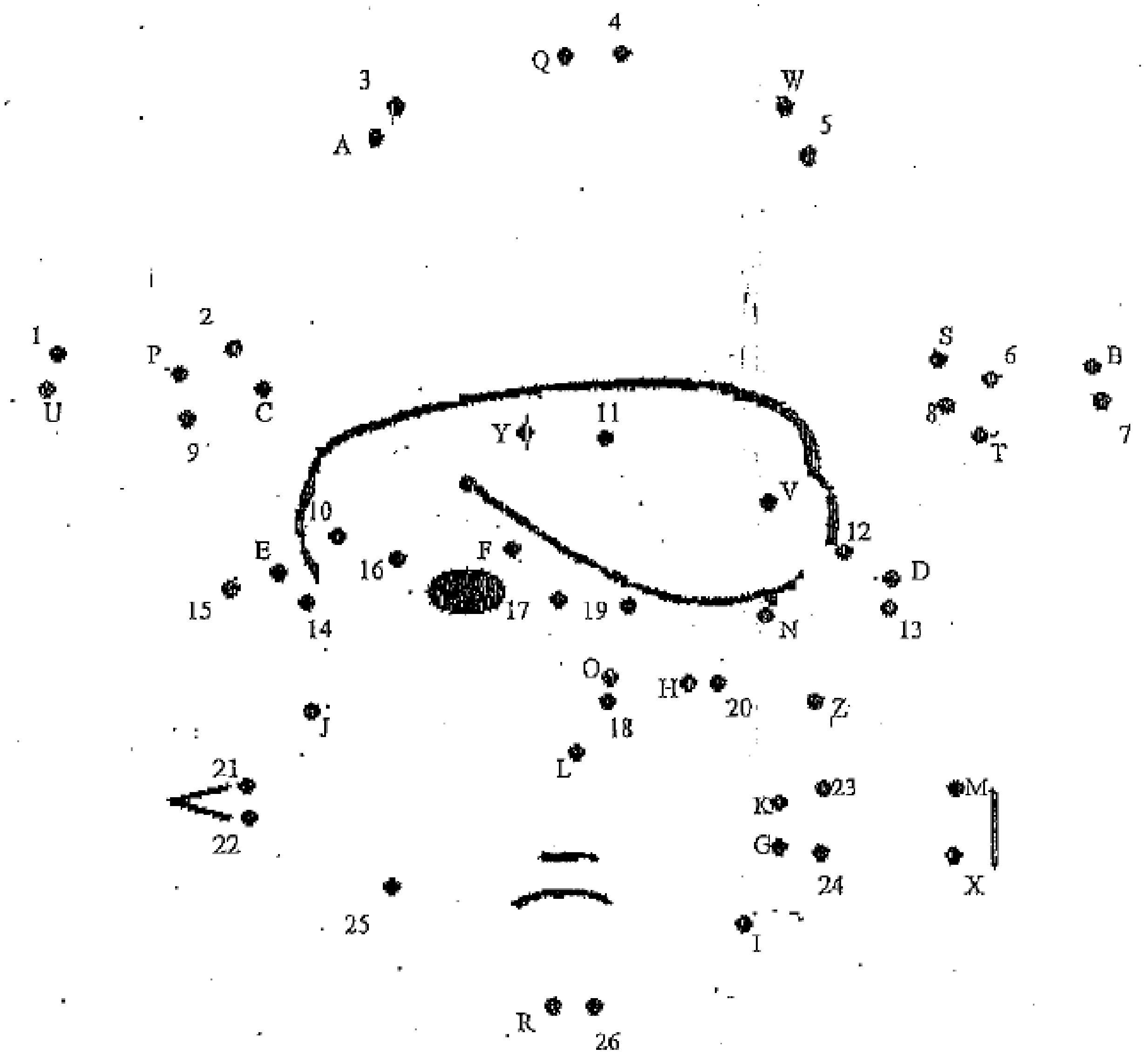
V. $\tan x$

W. $1 - \cos x$

X. $-\tan^2 x$

Y. $\cot x$

Z. $\csc^2 x$



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4.02 Simplifying Trig Identities Cont'd — Date: ____________ Period: __

Cofunction Identities:

$sin\left(\frac{\pi}{2}-\theta\right)=cos\theta$ $tan\left(\frac{\pi}{2}-\theta\right)=cot\theta$ $sec\left(\frac{\pi}{2}-\theta\right)=csc\theta$

$cos\left(\frac{\pi}{2}-\theta\right)=sin\theta$ $cot\left(\frac{\pi}{2}-\theta\right)=tan\theta$ $csc\left(\frac{\pi}{2}-\theta\right)=sec\theta$

Cofunctions are translations of the same graph shape

Even Odd Identities:

$\cos(-\theta)=cos\theta$

$\sec(-\theta)=sec\theta$

negative goes away

$\sin(-\theta)=-sin\theta$

$\csc(-\theta)=-csc\theta$

$\tan(-\theta)=-tan\theta$

$\cot(-\theta)=-cot\theta$

negative goes to the front

Even Functions have symmetry with respect to the y-axis f(-x) = f(x)	**Odd Functions** have symmetry with about the origin f(-x) = -f(x)

Use trig identities to simplify the expression.

1. $cotxtan(-x)$

2. $cos\theta\sec\left(\frac{\pi}{2}-\theta\right)$

Practice: Complete #5-23 odd

In Exercises 5–8, use identities to find the value of the expression.

5. If $\sin\theta = 0.45$, find $\cos(\pi/2 - \theta)$.

6. If $\tan(\pi/2 - \theta) = -5.32$, find $\cot\theta$.

7. If $\sin(\theta - \pi/2) = 0.73$, find $\cos(-\theta)$.

8. If $\cot(-\theta) = 7.89$, find $\tan(\theta - \pi/2)$.

In Exercises 9–16, use basic identities to simplify the expression.

9. $\tan x \cos x$

10. $\cot x \tan x$

11. $\sec y \sin(\pi/2 - y)$

12. $\cot u \sin u$

13. $\dfrac{1 + \tan^2 x}{\csc^2 x}$

14. $\dfrac{1 - \cos^2\theta}{\sin\theta}$

15. $\cos x - \cos^3 x$

16. $\dfrac{\sin^2 u + \tan^2 u + \cos^2 u}{\sec u}$

In Exercises 17–22, simplify the expression to either 1 or −1.

17. $\sin x \csc(-x)$

18. $\sec(-x)\cos(-x)$

19. $\cot(-x)\cot(\pi/2 - x)$

20. $\cot(-x)\tan(-x)$

21. $\sin^2(-x) + \cos^2(-x)$

22. $\sec^2(-x) - \tan^2 x$

In Exercises 23–26, simplify the expression to either a constant or a basic trigonometric function. Support your result graphically.

23. $\dfrac{\tan(\pi/2 - x)\csc x}{\csc^2 x}$

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4.03 Verifying Identities Practice Day 1

Name: ___________________

Date: __________ Period: ____

Tell whether or not f(x) = sin(x) is an identity.

1. $f(x) = \frac{tanx}{secx}$

Prove the following identities.

2. $(sinx)(cotx + cosxtanx) = cosx + sin^2x$

3. $(cosx - sinx)^2 = 1 - 2sinxcosx$

4. $tanx + secx = \frac{cosx}{1-sinx}$

5. $\frac{sec^2\theta-1}{sin\theta} = \frac{sin\theta}{1-sin^2\theta}$

6. $\frac{1}{1-cosx} + \frac{1}{1+cosx} = 2csc^2x$

7. $sin^2\alpha - cos^2\alpha = 1 - 2cos^2\alpha$

8. $\frac{1}{tan\beta} + tan\beta = sec\beta csc\beta$

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4.04 Verifying Identities Classwork Day 2

Name: ___________________

Date: __________ Period: ____

Prove the following identities.

1. $\frac{secx+1}{tanx} = \frac{sinx}{1-cosx}$

2. $\frac{cotv-1}{cotv+1} = \frac{1-tanv}{1+tanv}$

3. $tan^2\theta - sin^2\theta = tan^2\theta sin^2\theta$

4. $tan^4t + tan^2t = sec^4t - sec^2t$

5. $\frac{1-cos\theta}{sin\theta} = \frac{sin\theta}{1+cos\theta}$

6. $\frac{\sin t}{1+\cos t} + \frac{1+\cos t}{\sin t} = 2\csc t$

7. $\frac{1+cosx}{1-cosx} = \frac{secx+1}{secx-1}$

8. $\frac{sinAcosB+cosAsinB}{cosAcosB-sinAsinB} = \frac{tanA+tanB}{1-tanAtanB}$

4.04 Verifying Identities Practice

Verify the following identities.

1. $csc\theta tan\theta = sec\theta$

2. $\frac{sec^2\theta - tan^2\theta + tan\theta}{sec\theta} = cos\theta + sin\theta$

3. $\frac{\cot(-\theta)}{csc\theta} = -cos\theta$

4. $(sec\theta - tan\theta)(csc\theta + 1) = cot\theta$

5. $\frac{sin\theta}{csc\theta} + \frac{cos\theta}{sec\theta} = 1$

6. $\frac{sin\theta + cos\theta}{sin\theta} - \frac{cos\theta - sin\theta}{cos\theta} = sec\theta csc\theta$

Accelerated Pre-Calculus

4.05 Verifying Identities Classwork Day 3

Name: ___________________

Date: __________ Period: ____

Prove the following identities.

1. $sin^5xcos^2x = (cos^2x - 2cos^4x + cos^6x)(sinx)$

2. $sin^3xcos^3x = (sin^3x - sin^5x)(cosx)$

3. $\frac{\cos x}{1+\sin x} + \frac{\cos x}{1-\sin x} = 2secx$

4. $\frac{1-3cos\theta-4cos^2\theta}{sin^2\theta} = \frac{1-4cos\theta}{1-cos\theta}$

5. $sec^4x = (1 + tan^2x)(sec^2x)$

4.05 Verifying Identities Practice

Factor the expression and use fundamental identities to simplify.

1. $cot^2x - cot^2xcos^2x$

2. $sin^2\text{x}sec^2\text{x} - sin^2\text{x}$

3. $tan^4\text{x} + 2tan^2\text{x} + 1$

4. $sin^4x - cos^4x$

Perform the operation and use fundamental identities to simplify.

5. $(\sin x + \cos x)^2$

6. $(\sec x + 1)(\sec x - 1)$

7. $\frac{1}{1+\cos x} + \frac{1}{1-\cos x}$

8. $\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x}$

Rewrite the expression so that it is <u>not</u> in fractional form.

9. $\frac{sin^2x}{1-\cos x}$

10. $\frac{3}{\sec x-\tan x}$

Accel Pre-Calculus Trig Identities Cont'd- Classwork Date: ____________

Simplify the trig expressions to find the words that complete the singing telegram.

ANSWERS

OH	$\sin^2 x$
HUNDRED	$\sin x$
SEE	$2\sin x$
GUESSED	$\sin x + \cos x$
WHO	$\cos x$
TWO	$\cos^2 x$
WHEN	$3\cos^2 x$
BOY	$\cos x - \sin x$
KNOW	$\cos^2 x - \sin^2 x$
WEIRD	$1 - \tan x$
FIVE	$\tan x$
BASE	$\tan^2 x$
TRIPLE	0
THINGS	1
COMPUTATIONS	2
BINARY	$\csc x$
MANY	$1 + \csc^2 x$
STRANGE	$\sec x$
AN	$2\sec^3 x$
CHORES	$\sec^2 x - 1$
NEVER	$\sec^2 x$
SYSTEM	$\cot x$
ONE	$\cot^2 x$
WAY	$\cot^2 x - 1$

Singing telegram

The ______(A)________ ______(B)________ is fun

For with it ______(C)______ ______(D)______ can be done.

And ______(E)________ as you ______(F)_________

Is a _____(G)_______ and _____(H)_____ _____(I)_______

And ___(J)____ is ___(G)___ _____(K)_____ and ____(G)____.

Simplify:

(A) $\sin x + \cos x \cot x$

(B) $\cos x \csc x$

(C) $\tan x(\sin x + \cot x \cos x)$

(D) $\sec^2 x(1 - \sin^2 x)$

(E) $\dfrac{\sin^2 x - \cos^2 x}{\tan^2 x - 1}$

(F) $\cos^4 x - \sin^4 x$

(G) $\dfrac{1 - \sin^2 x}{1 - \cos^2 x}$

(H) $\dfrac{\sec x}{1 - \sin x} + \dfrac{\sec x}{1 + \sin x}$

(I) $\cos^2 x \tan^2 x$

(J) $\sin x \sec x$

(K) $\sqrt{\cos^2 x \sec^2 x - \cos^2 x}$

Accelerated Precalculus **Name:** ________________________________

4.06 HW Simplifying Trigonometric Expressions

Simplify each trigonometric expression. Match the result to an expression in the list to the right.

1. $\frac{\sin x}{1-\cos x} + \cot(-x)$

2. $\csc\left(\frac{\pi}{2} - x\right) - \cos(-x) - \sin(-x)\tan(-x)$

Simplified expressions:

A. $2\tan x$

B. $\cos^2 x$

C. $\csc x$

D. $\tan^4 x + \tan^2 x$

E. 0

F. $\sec x$

3. $-\cot\left(x - \frac{\pi}{2}\right) + \frac{\cos x}{1+\sin x}$

4. $\frac{\cos^2(-x)-\sin^2(-x)}{1-\tan^2 x}$

5. $\frac{\sec x}{\csc x} + \frac{\cos\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)}$

6. $\sec^4 x - \sec^2 x$

Trigonometric Identities Magic Square

Directions: Simplify each of the expressions and then find the letter which matches the answer from the choices below. Write the letter which corresponds to the correct answer in the box which corresponds to the number of the problem. For example, if the answer to question 1 corresponded to the letter A you would put an A in box 1. When you are done you should have a *Magic Square.*

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

Accel Pre-Calculus 4.07 HW

Questions

1) $\dfrac{1-\sin x}{\cos x} =$

2) $\dfrac{2\sin^2 x - 1}{\sin x - \cos x} =$

3) $(\sin x - 1)(\tan x + \sec x) =$

4) $\dfrac{\cot^2 x}{\csc x + 1} =$

5) $\sin x \cos x \tan x + \cos^2 x + 1 =$

6) $\dfrac{\csc x}{\cot x} =$

7) $\dfrac{1}{\cot x} - \dfrac{\sec x}{\csc x} =$

8) $\dfrac{\cot x + 1}{\csc x} =$

9) $\dfrac{1+\sin x}{\cos x} + \dfrac{\cos x}{1+\sin x} =$

10) $\cot x \sec x \sin x =$

11) $\dfrac{\cot x - \tan x}{\sin x \cos x} =$

12) $\sin x + \cos x \cot x =$

13) $\dfrac{\csc x}{1+\csc x} - \dfrac{\csc x}{1-\csc x} =$

14) $(\sec x + \cos x)(\sec x - \cos x) =$

15) $\cos x(\cos x - \sec x) =$

16) $\cos x \csc x =$

Answers

$A = \tan^2 x + \sin^2 x$	$K = \csc^2 x - \sec^2 x$	$H = \cot x$	$O = \sin x + \cos x$
$C = \sec x - \tan x$	$L = \csc x - 1$	$I = \sec x$	$O = -\cos x$
$D = 0$	$L = 2\sec x$	$I = 1$	$S = \sin x + \cos x$
$E = \csc x$	$M = 2\sec^2 x$	$K = 2$	$T = -\sin^2 x$

Accelerated Pre-Calculus Name ______________________________

4.08 Verify using Sum/Difference Identities Date ______________________ Per ______

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$ $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

Use sum or difference identities to find the exact value.

1. $\cos 75°$

2. $\sin 40°\cos 20° + \cos 40°\sin 20°$

Use sum or difference identities to verify the following:

1. $\sin(x - y) + \sin(x + y) = 2\sin x\cos y$

2. $\cos(x - y) + \cos(x + y) = 2\cos x\cos y$

4.08 Practice:

3. $\tan(x + y)\tan(x - y) = \dfrac{\tan^2 x - \tan^2 y}{1 - \tan^2 x\tan^2 y}$

4. $\dfrac{\sin(x - y)}{\sin(x + y)} = \dfrac{\tan x - \tan y}{\tan x + \tan y}$

5. $\dfrac{\sin(A-B)}{\sin B}+\dfrac{\cos(A-B)}{\cos B}=\dfrac{\sin A}{\sin B\cos B}$

6. $\cos^2\beta - \sin^2\beta = 1 - 2\sin^2\beta$

7. $\tan\left(\frac{\pi}{2} - x\right)\sec x = \csc x$

8. $\frac{\sec(-x)}{\csc(-x)} = -\tan x$

9. $\frac{\cos(-\theta)}{1+\sin(-\theta)} = \sec\theta + \tan\theta$

10. $2 + \cos^2 x - 3\cos^4 x = \sin^2 x(2 + 3\cos^2 x)$

11. $\csc^4 x - 2\csc^2 x + 1 = \cot^4 x$

Accelerated Pre-Calculus Name ___________________________________

4.09 Verifying with Double Angle Identities Date ___________Per ___

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= 2\cos^2\theta - 1$

$= 1 - 2\sin^2\theta$

$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

Use double angle identities to find the exact value.

1. $2\sin 15°\cos 15° =$

Use double angle identities to verify the following identities.

1. $\csc 2x = \frac{1}{2}\sec x \csc x$

2. $\cos A - \sin A = \frac{\cos 2A}{\cos A + \sin A}$

4.09 Practice:

3. $(\sin x + \cos x)^2 - 1 = \sin 2x$

4. $\cos x - 1 = \frac{\cos 2x - 1}{2(\cos x + 1)}$

5. sec 2x = $\frac{\cos^2 x + \sin^2 x}{\cos^2 x - \sin^2 x}$

6. sin 3x = 3 sin x – 4 $\sin^3$x

7. cos 3x = 4 $\cos^3$x – 3 cos x

8. $\frac{2}{\cot\theta - \tan\theta} = \tan 2\theta$

9. $\sin x(1 - 2\cos^2 x + \cos^4 x) = \sin^5 x$

10. $4\tan^4 x + \tan^2 x - 3 = \sec^2 x(4\tan^2 x - 3)$

11. $\sec^4 x - \tan^4 x = 1 + 2\tan^2 x$

12. $\frac{\cot\alpha}{\csc\alpha - 1} = \frac{\csc\alpha + 1}{\cot\alpha}$

Accelerated Pre-Calculus Name ________________________________

APC 4.10 Unit 4A Test Review

Simplify each expression and match it to one of the expressions in the given answer bank.

a. sinx	b. cosx	c. tanx	d. $\sin^2$x	e. $\cos^2$x	f. $\sin^4$x	g. $\tan^2$x

1. $\frac{1+\tan^2 x}{\csc^2 x}$

2. $\sin x - \tan x \cos x + \cos\left(\frac{\pi}{2} - x\right)$

3. $1 - 2\cos^2 x + \cos^4 x$

Verify each.

4. $\frac{\cos^2 x}{1-\sin x} = 1 + \sin x$

5. $\frac{\tan^2 x(\sin^2 x - \cos^2 x)}{(\sin x - \cos x)(\sec^2 x - 1)} = \sin x + \cos x$

6. $\frac{\cos(-x)}{\sec(-x)+\tan(-x)} = 1 + \sin x$

7. $2\cos^2\left(\frac{x}{2}\right) - \cos x = 1$

8. $\cos x \sin x \tan x + \cos x \sin x \cot x = 1$

9. $\frac{\tan^3 x + \tan x}{\sec^2 x} = \tan x$

10. $\frac{\csc x + 1}{\cot x} = \frac{\cot x}{\csc x - 1}$

11. $\sec^2 x \cot x - \cot x = \tan x$

12. $(\sin x + \cos x)^2 - 1 = \sin 2x$

13. $\cos B \cot B = \csc B - \sin B$

Trigonometric Identities

Reciprocal Identities:

$\sin\theta = \frac{1}{\csc\theta}$ $\quad$ $\cos\theta = \frac{1}{\sec\theta}$ $\quad$ $\tan\theta = \frac{1}{\cot\theta}$

$\csc\theta = \frac{1}{\sin\theta}$ $\quad$ $\sec\theta = \frac{1}{\cos\theta}$ $\quad$ $\cot\theta = \frac{1}{\tan\theta}$

Quotient Identities:

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$

Pythagorean Identities:

$\sin^2\theta + \cos^2\theta = 1$ $\quad$ $\tan^2\theta + 1 = \sec^2\theta$ $\quad$ $1 + \cot^2\theta = \csc^2\theta$

Cofunction Identities:

$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$ $\quad$ $\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta$ $\quad$ $\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$

$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$ $\quad$ $\sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta$ $\quad$ $\cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$

Even/Odd Identities:

$\sin(-\theta) = -\sin\theta$ $\quad$ $\cos(-\theta) = \cos\theta$ $\quad$ $\tan(-\theta) = -\tan\theta$

$\csc(-\theta) = -\csc\theta$ $\quad$ $\sec(-\theta) = \sec\theta$ $\quad$ $\cot(-\theta) = -\cot\theta$

Sum & Difference Identities:

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$ $\quad$ $\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

Double-Angle Identities:

$\sin 2\theta = 2\sin\theta\cos\theta$ $\quad$ $\cos 2\theta = \cos^2\theta - \sin^2\theta$ $\quad$ $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$= 2\cos^2\theta - 1$

$= 1 - 2\sin^2\theta$

Half-Angle Identities:

$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$ $\quad$ $\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$ $\quad$ $\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$

$= \frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$