Name:____ **Period:** __________

Accel. Pre-Calculus

Unit 4B (Part 2) Packet

Trigonometric Identities & Solving Trig Equations

Accelerated Pre-Calculus **Name**_____________________

4.12 Practice – Evaluating with Sum or Difference Identities **Date:** ________

Use Sum or Difference Identities to find the exact value of each expression. **Do not use a calculator.**

1. $\cos\frac{13\pi}{12}$

2. $\tan\frac{\pi}{12}$

3. $\sin\frac{\pi}{8}\cos\frac{7\pi}{8} - \cos\frac{\pi}{8}\sin\frac{7\pi}{8}$

4. $\frac{\tan 80° + \tan 55°}{1 - \tan 80° \tan 55°}$

Use identities to simplify.

5. $\sin(\pi + x)$

6. $\tan(\pi + x)$

7. $\cos\left(x - \frac{3\pi}{4}\right)$

8. $\tan\left(x + \frac{7\pi}{4}\right)$

Suppose that A and B are angles in standard position. Use the given information to find:
(a) $\sin(A + B)$, (b) $\tan(A + B)$, and (c) the quadrant of $(A + B)$. **Do not use a calculator.**

9. $\sin A = -\frac{3}{5}$, $\cos B = -\frac{12}{13}$, $\frac{3\pi}{2} < A < 2\pi, \pi < B < \frac{3\pi}{2}$

(a) sin (A + B) = ___________

(b) tan (A + B) = ___________

(c) Quadrant of (A + B) ____

10. $\cot A = \frac{2}{5}$, $\tan B = -\frac{4}{3}$, $\pi < A < \frac{3\pi}{2}$, $\frac{\pi}{2} < B < \pi$

(a) sin (A + B) = ___________

(b) tan (A + B) = ___________

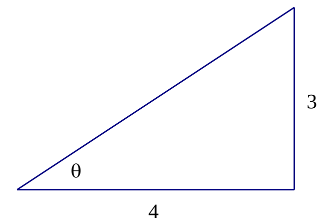
(c) Quadrant of (A + B) ____

Accelerated Precalculus **Name**______________________________
4.13 Practice- Evaluating with Double Angle Identity

Use the figure to find the exact value of the trigonometric function.

1. $\tan 2\theta$

2. $\csc 2\theta$



Find the exact values of **sin 2*u*, cos 2*u*,** and **tan 2*u*** using the double-angle formulas. In what quadrant does the angle **2*u*** have its terminal side?

3. $\tan u = -\frac{5}{12}, \frac{\pi}{2} < u < \pi$

sin 2*u* = __________

cos 2*u* = __________

tan 2*u* = __________

Quadrant of 2*u* _________

4. $\cos u = -\frac{1}{4}, \pi < u < \frac{3\pi}{2}$

sin 2*u* = __________

cos 2*u* = __________

tan 2*u* = __________

Quadrant of 2*u* _________

5. $\sin u = -\frac{\sqrt{5}}{7}$ and $\cos u > 0$

sin 2*u* = __________

cos 2*u* = __________

tan 2*u* = __________

Quadrant of 2*u* _________

Accelerated Precalculus Name ___________________________
4.14 Quiz Review

Do not use a calculator!

Find the exact values using angle sum or difference identities.

1. $\sin 105°$
2. $\tan\left(\frac{17\pi}{12}\right)$
3. $\cos 1005°$

4. If $\sin\theta = \frac{12}{13}$ and $\cos\theta < 0$, find $\tan 2\theta$.

5. If $\tan\theta = \frac{-5}{8}$ and $\sec\theta > 0$, find $\cos 2\theta$.

6. Find the exact value of each given that sin x = $\frac{15}{17}$, $\frac{\pi}{2} < x < \pi$ and cos y = $\frac{4}{5}$, $\frac{3\pi}{2} < y < 2\pi$.

a. cos (x + y)

b. sin (y – x)

c. tan (x + y)

d. cos (y – x)

e. What quadrant is (x + y) in? ______

f. What quadrant is (y – x) in? ______

g. sin 2x

h. cos 2x

i. tan 2x

j. What quadrant is 2x in? _______

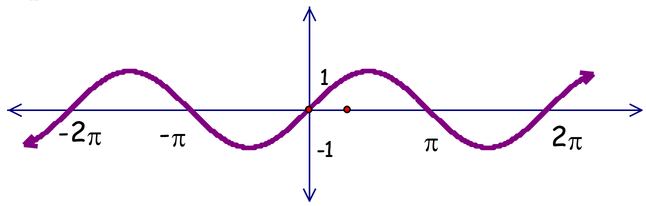
4.16 Trig Inverses and Principal Values

Recall that for a graph to be considered a function, it must pass the vertical line test. For a function to have an inverse, it also must pass the horizontal line test (meaning if you switch the x's and y's, the inverse would pass the vertical line test)

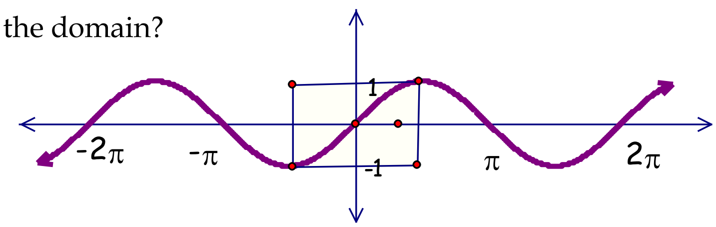
Recall the properties of an inverse function: 1) x and y switch, so the domain and range switch

2) the graph is reflected over y=x.

Does the graph of sin(x) pass the vertical line test? The horizontal line test?



What if we restrict the domain?

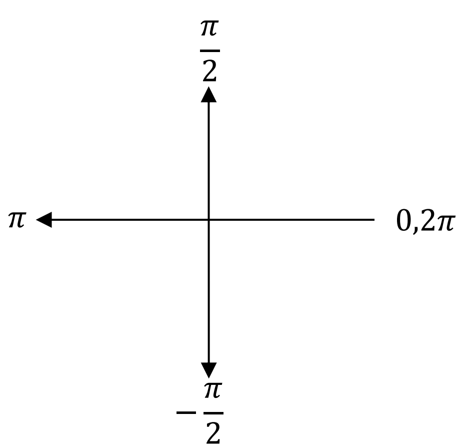


If we restrict the domain of sin(x) from $-\frac{\pi}{2}\ to\ \frac{\pi}{2}$, the restricted portion <u>would</u> pass the horizontal line test, so the inverse is a function. We refer to these restricted values of x as the <u>Principal Values</u> of sinx.

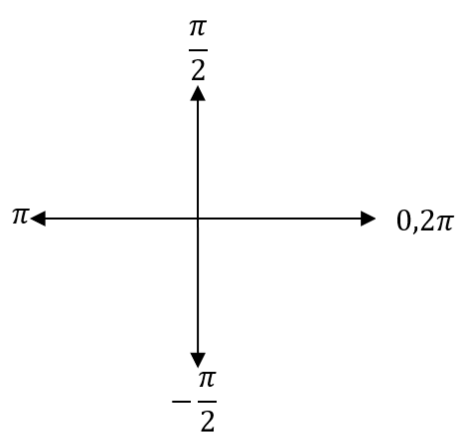
Notations for inverses with principal values: $Sin^{-1}x\ or\ Arcsinx$, $Cos^{-1}x\ or\ Arccosx$, and $Tan^{-1}x\ or\ Arctanx$

Notice the capital letters at the beginning; they indicate you're finding principal values

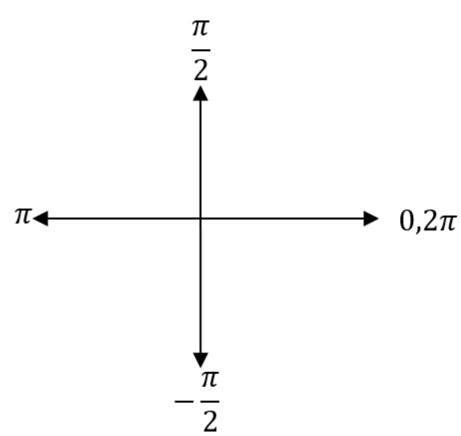
Here's how the domain restrictions look with respect to the unit circle:



$Sin^{-1}x\ or\ Arcsinx$

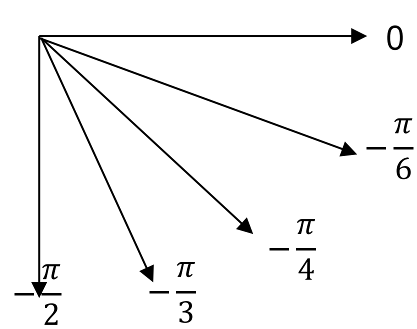


$Cos^{-1}x\ or\ Arccosx$



$Tan^{-1}x\ or\ Arctanx$

Pay special attention to QIV as these are not the values you see on the unit circle



<u>Example 1:</u> Find $Arcsin\left(-\frac{1}{2}\right)$

a) Where do we look for $Arcsin$?

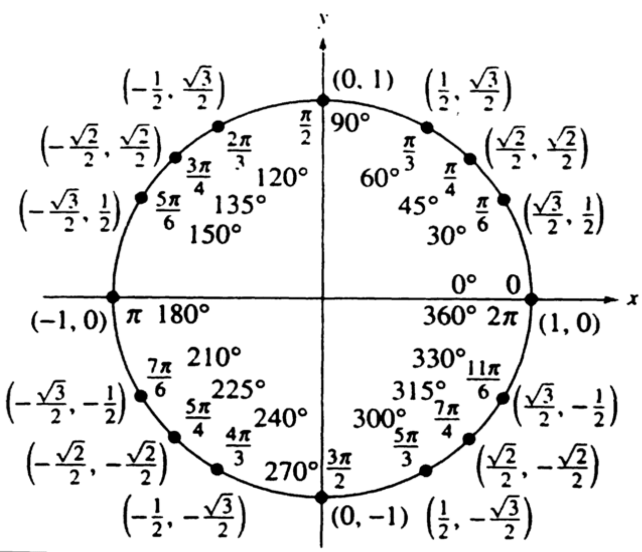
b) What angle has $sin = -\frac{1}{2}$?

<u>Example 2:</u> Find $Tan^{-1}(0)$

a) Where do we look for Tan^{-1}?

b) What angle has $tan = 0$?

Refer to the unit circle to find Principal Values in degrees and radians.



a. $Arcsin(-1)$

This problem asks us to find the angle that has a sine value of -1

$Sin\theta = -1$

$\theta =$

b. $Arccos\left(-\frac{\sqrt{2}}{2}\right)$

This problem asks us to find the angle that has a cosine value of $-\frac{\sqrt{2}}{2}$

$Cos\theta = -\frac{\sqrt{2}}{2}$

$\theta =$

c. $Arctan(1)$

This problem asks us to find the angle that has a tangent value of 1

$Tan\theta = 1$

$\theta =$

d. $Sin^{-1}(0)$

e. $Cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

f. $Tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$

g. $Arcsin\left(\frac{\sqrt{2}}{2}\right)$

h. $Arccos\left(-\frac{1}{2}\right)$

i. $Arctan(-\sqrt{3})$

4.16 Practice- Trig Inverses & Principal Values **Name: ______________________**

Find the exact principal value for each trig inverse function. **Date: ___________**

1. $Arcsin\left(-\frac{\sqrt{3}}{2}\right)$
2. $Cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$
3. $Arctan(1)$
4. $Arccos\left(\frac{\sqrt{3}}{2}\right)$
5. $Sin^{-1}(-1)$
6. $Arctan(\sqrt{3})$
7. $Arctan(0)$
8. $Sin^{-1}(3)$
9. $Arccos(-1)$
10. $Arctan\left(-\frac{\sqrt{3}}{3}\right)$
11. $Arcsin(1)$
12. $Cos^{-1}\left(-\frac{1}{2}\right)$

4.17 Practice- Evaluating with Trigonometric Inverses **Date: ________________**

Find the exact value for each expression.

1. $\sin\left(Arccos\ \frac{\sqrt{3}}{2}\right)$
2. $\cos(Arctan\ 0)$
3. $\tan\left[2 * Cos^{-1}\left(-\frac{1}{2}\right)\right]$
4. $Arcsin\left(\sin\frac{\pi}{3}\right)$
5. $Cos^{-1}\left(\cos\frac{3\pi}{2}\right)$
6. $Arctan\left(\tan\frac{11\pi}{6}\right)$
7. $Sin^{-1}\left(\cos\frac{2\pi}{3}\right)$
8. $Cos^{-1}\left(\tan\frac{5\pi}{4}\right)$
9. $Arctan(\sin 7\pi)$

Find the exact value for each expression.

10. $\sin\left(Arctan\ \frac{4}{3}\right)$
11. $\cos\left(Sin^{-1}\ \frac{24}{25}\right)$
12. $\tan\left[Cos^{-1}\left(-\frac{8}{17}\right)\right]$
13. $\cos\left[Tan^{-1}\ \left(-\frac{2}{3}\right)\right]$
14. $\cot\left[Sin^{-1}\ \left(-\frac{1}{4}\right)\right]$
15. $\csc(Arccos\ 2)$
16. $\sin(Arctan\ x)$
17. $\tan\left(Arccos\ \frac{2x}{7}\right)$
18. $\cos\left(Sin^{-1}\ \frac{\sqrt{11}}{3x}\right)$
19. $\sec\left(Sin^{-1}\ \frac{4}{x}\right)$
20. $\cot(Arctan\ 5x)$
21. $\csc(Cos^{-1}\ 3x)$

Accelerated Precalculus **Name: ______________________________**
4.18 Review Worksheet: Trig Inverses & Principal Values **Date:**
Find the exact value for each expression. Use radian measures for angles. Use principal values for inverses.

1. $Arctan\ (-1)$
2. $Cos^{-1}(-2)$
3. $Arcsin\ (1)$
4. $Tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$
5. $Arccos\ (1)$
6. $Sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$
7. $Arcsin\left[\sin\left(\frac{3\pi}{4}\right)\right]$
8. $\cos[Tan^{-1}(-\sqrt{3})]$
9. $\text{Arccos}\left[cos\ \left(\frac{7\pi}{4}\right)\right]$
10. $Sin^{-1}\left[\tan\left(\frac{5\pi}{4}\right)\right]$
11. $\tan\left[2\,Sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]$
12. $\csc[Arctan\ (1)]$
13. $\tan\left[Sin^{-1}\left(\frac{3}{8}\right)\right]$
14. $\cot\left[Arccos\ \left(-\frac{12}{13}\right)\right]$
15. $\sec[Arctan\ (4)]$
16. $\cos[Arctan(3x)]$
17. $\tan[Sin^{-1}(x-1)]$
18. $\csc\left[Cos^{-1}\left(\frac{x}{\sqrt{7}}\right)\right]$

19. Amy's family went to an amusement park while they are at the beach. She decides to ride the Ferris wheel so she can look out at the ocean. She was disappointed to find out that a 100 foot building blocked her view for part of the ride. Amy's height in feet above the ground as she travels around the Ferris wheel can be modeled using the following equation where t = time in minutes from the beginning of Amy's ride: $h(t) = -60\cos\left(\frac{2\pi}{3}t\right) + 70$.

a) How long it will take until Amy can see over the building?

b) How long will it take Amy to reach the top of the ride?

c) How long into Amy's ride will the building again obstruct her view of the ocean?

20. The area of an isosceles triangle can be found using the formula $Area = \frac{1}{2}x^2 \sin\theta$, where x is the length of the legs and θ is the vertex angle. If an isosceles triangle has a leg length of 4, then what measures for the vertex angle will produce an area of 4?

Accelerated Pre-Calculus Name ______________________________

4.19 Inverses with Calculator Activity Date _____________________

Give answers in degrees.

Inverse Sine

1. Find Sin-1 $\left(\frac{1}{2}\right)$, a) without a calculator _________ b) with a calculator (check your mode!) __________

2. How would you solve sin-1 $\left(\frac{1}{2}\right)$ for all angles $0° \leq \theta < 360°$? Where else is sinθ positive? ___________

 How do you find this angle using the angle measurement found in #1? _________________

 State the two answers to sin-1 $\left(\frac{1}{2}\right) = \theta$, where $0° \leq \theta < 360°$ ____________________________

3. Find Sin-1 $\left(-\frac{1}{2}\right)$, a) without a calculator __________ b) with a calculator __________

 Why did your calculator give you that answer instead of 330o?

____

4. How would you solve sin-1 $\left(-\frac{1}{2}\right)$ if $0° \leq \theta < 360°$? Where else is sinθ negative? __________________

 How do you find this angle using the angle measurement found in #3? _________________

 State the two answers to sin-1 $\left(-\frac{1}{2}\right) = \theta$, where $0° \leq \theta < 360°$ _________________________

5. Write a formula that uses the principal value solution for $\theta = \sin^{-1} x$ to find the second answer in [0°, 360°).

____

Inverse Tangent

1. Find Tan-1 (1), a) without a calculator __________b) with a calculator (check your mode!) __________

2. How would you solve tan-1 (1) if $0° \leq \theta < 360°$? Where else is tanθ positive? _____________________

 How do you find this angle using the angle measurement found in #1? ___________________

 State the two answers to tan-1 $(1) = \theta$, where $0° \leq \theta < 360°$ ____________________________

3. Find Tan-1 (− 1), a) without a calculator ___________ b) with a calculator ___________

 Why did your calculator give you that answer instead of 315o?

____

4. How would you solve tan-1 (-1) if $0° \leq \theta < 360°$? Where else is tanθ negative? ___________________

 How do you find this angle using the angle measurement found in #3? _________________

 State the two answers to tan-1 $(-1) = \theta$, where $0° \leq \theta < 360°$ __________________________

5. Write a formula that uses the principal value solution for $\theta = \tan^{-1} x$ to find the second answer in [0°, 360°).

____

Inverse Cosine

1. Find $\text{Cos}^{-1}\left(\frac{1}{2}\right)$, a) without a calculator __________b) with a calculator (check your mode!) __________

2. How would you solve $\cos^{-1}\left(\frac{1}{2}\right)$ if $0° \leq \theta < 360°$? Where else is cosθ positive? ____________________

   How do you find this angle using the angle measurement found in #1? __________________
   State the two answers to $\cos^{-1}\left(\frac{1}{2}\right) = \theta$, where $0° \leq \theta < 360°$ ____________________________

3. Find $\text{Cos}^{-1}\left(-\frac{1}{2}\right)$, a) without a calculator ___________ b) with a calculator ___________
   Why did your calculator give you that answer instead of -60°? ____________________________________

4. How would you solve $\cos^{-1}\left(-\frac{1}{2}\right)$ if $0° \leq \theta < 360°$? Where else is cosθ negative? ___________________

   How do you find this angle using the angle measurement found in #3? __________________
   State the two answers to $\cos^{-1}\left(-\frac{1}{2}\right) = \theta$, where $0° \leq \theta < 360°$ ____________________________

5. Write a formula that uses the principal value solution for $\theta = \cos^{-1} x$ to find the second answer in [0°, 360°).

____

<u>Examples</u>: Use your formulas to find each for $0° \leq \theta < 360°$ using a calculator. Round to the nearest degree.

a. $\sin^{-1}$ (0.1736) b. $\cos^{-1}$ (−0.6427) c. $\tan^{-1}$ (-2.7475)

Check your answers with the key found at the bottom of the page. Did your formulas work?

<u>Practice</u>: Use a calculator to find two values of θ, where $0° \leq \theta < 360°$. Round to the nearest degree.

1. arccos 0.8746 2. $\tan^{-1}$ 19.0811 3. sin θ = 0.8290

4. arctan (-28.6363) 5. $\sin^{-1}$(-0.2250) 6. cos θ = -0.3907

7. arcsin (0.6691) 8. $\cos^{-1}$ (-0.9511) 9. tan θ = -0.6249

Example Answers: (a.) 10° and 170° (b.) 130° and 230° (c.) 110° and 290°

Accelerated Pre-Calculus Name ______________________________
4.20 Extension for Inverses with Calculator Activity Date _____________________

Give answers in radians.

Inverse Sine: From the previous handout: Write a formula that uses the principal value solution for $\theta = \sin^{-1} x$ to find the second answer in [0°, 360°).

____

Now, convert your formula to **radians**. __________________________

Inverse Tangent: From the previous handout: Write a formula that uses the principal value solution for $\theta = \tan^{-1} x$ to find the second answer in [0°, 360°). __________________________

Now, convert your formula to **radians**. __________________________

Inverse Cosine: From the previous handout: Write a formula that uses the principal value solution for $\theta = \cos^{-1} x$ to find the second answer in [0°, 360°). __________________________

Now, convert your formula to **radians**. __________________________

Caution: Remember that the principal values for the inverse of sine and the inverse of tangent will return *negative angle values* for Quadrant 4. How do you adjust this result to be a value in [0°, 360°)? _______________

Now, convert this formula to **radians**. __________________________

Examples: Find each for $0 \le \theta < 2\pi$ using a calculator. Round to the nearest thousandth of a radian.

a. $\sin^{-1}$ (0.2847) b. $\cos^{-1}$ (0.7538) c. $\tan^{-1}$ (-3.8565)

Practice: Use a calculator to find two values of θ, where $0 \le \theta < 2\pi$. Round to the nearest thousandth of a radian.

1. arccos 0.9857
2. $\tan^{-1}$ 20.1923
3. sin θ = 0.7301
4. arctan (-39.7447)
5. $\sin^{-1}$(-0.3360)
6. cos θ = -0.4018
7. arcsin (0.7723)
8. $\cos^{-1}$ (-0.1689)
9. tan θ = -0.7350

Example Answers: (a.) 0.289 and 2.853 (b.) 0.717 and 5.566 (c.) 1.825

4.21 Solving Trig Equations Worksheet **Date: ____________**

Find all solutions to the equation in the interval $[\mathbf{0}, \mathbf{2\pi})$. Do not use a calculator.

1. $2\cos x \sin x - \cos x = 0$

2. $\sqrt{2}\tan x \cos x - \tan x = 0$

3. $\tan x \sin^2 x = \tan x$

4. $\sin x \tan^2 x = \sin x$

5. $\tan^2 x = 3$

6. $2\sin^2 x = 1$

Find <u>all solutions</u> to the equation. Do not use a calculator.

7. $4\cos^2 x - 4\cos x + 1 = 0$

8. $2\sin^2 x + 3\sin x + 1 = 0$

9. $\sin^2 x - 2\sin x = 0$

10. $3\sin x = 2\cos^2 x$

11. $2\sin^2 x + 3\sin x = 2$

12. $\sin^4 x + 2\sin^2 x - 3 = 0$

4.22 Solving Trig Equations with Double Angles Worksheet **Date: ____________**

Solve the equation for the variable on the interval $[0, 2\pi)$.

1. $\sqrt{2}\tan x = 2\sin x$

2. $3\tan^2 y - 1 = 0$

3. $\cos^3\theta = \cos\theta$

4. $5\sin\theta - 2\cos^2\theta = 1$

5. $\cos 2y = \frac{1}{2}$

6. $3\tan^4 x - 10\tan^2 x + 3 = 0$

Solve the equation for all values of the variable.

7. $\sin 2x = 2\sin x$

8. $\cos 2x = 3\sin x - 1$

9. $\sin 2x - \tan x = 0$

10. $\cos 2x + \cos x = 0$

11. $\cos x + \cos 3x = 0$

12. $\sin^2 x = 4 - 2\cos^2 x$

4.23 Solving Trig Equations with Sum/Difference Identities Wksht **Date: __________**

Solve the equation for the variable on the interval $[0, 2\pi)$.

1. $\sin x + \sqrt{2} = -\sin x$
2. $\sin 2x \cos x = \sin x$
3. $\cos(x - \pi) = \frac{\sqrt{2}}{2}$
4. $\sin(x - \pi) = -\frac{\sqrt{3}}{2}$
5. $\sin\left(x + \frac{\pi}{4}\right) + 1 = \sin\left(\frac{\pi}{4} - x\right)$
6. $\tan\left(x + \frac{\pi}{4}\right) = 1$

Solve the equation for all values of the variable.

7. $3\sec^2 x = 4$

8. $2\sin^2 x + 3\cos x = 3$

9. $\sin\left(\frac{\pi}{2} - x\right) = -1$

10. $\cos\left(x + \frac{\pi}{6}\right) = \cos\left(x - \frac{\pi}{6}\right) + 1$

11. $\tan(x + \pi) + 2\sin(x + \pi) = 0$

4.24 Solving Trig Equations with Functions of Multiple Angles Wksht **Date ____________**

Solve the equation for the variable on the interval $[0, 2\pi)$.

1. $\sin 3x = 1$
2. $\cos 4x = 1$
3. $\sec 2x = -2$
4. $\cos x + \sin x \tan x = 2$
5. $\cos\left(x + \frac{\pi}{2}\right) = \tan\frac{\pi}{4}$
6. $2\sec^2 x + 2\tan^2 x = 14$

Solve the equation for all values of the variable.

7. $2\sin^2 2x = 1$

8. $-2\cos 2x = \sqrt{3}$

9. $3\tan^2 2x = 1$

10. $\csc^2 x + 3\csc x = -2$

11. $\tan(x + \pi) + \cos\left(x + \frac{\pi}{2}\right) = 0$

12. sinx – 1 = cosx

4.25 Solving Trig Equations Review Date: _______________

Solve each equation for the principal values.

1. $\sqrt{2}\sin x - 1 = 0$
2. $2\cos x + 1 = 0$
3. $\sin 2x - 1 = 0$
4. $\cos^2 x = \cos x$

Solve each equation for x if 0º ≤ x < 360º

5. $2\cos^2 x + 3\cos x - 2 = 0$
6. $\cos x \tan x = \frac{1}{2}$
7. $\sin x = 1 + \cos^2 x$

Solve each equation for x if $0 \le x < 2\pi$ **.**

8. $\cot^2 x - \csc x = 1$
9. $\sin x = \cos 2x - 1$
10. $\sin 2x = -\sin x$

11. Solve $\sin x + \cos x = 0 \quad 0 \le x < \pi$.

Solve each equation for all values of x.

12. $-1 - 3\sin x = \cos 2x$

13. $\cos x \tan x - 2\cos^2 x = -1$

14. $2\cos^2 x = 3\sin x$

15. $3\tan^2 x = \sqrt{3}\tan x$

Trigonometric Identities

Reciprocal Identities:

$\sin\theta = \frac{1}{\csc\theta}$ $\cos\theta = \frac{1}{\sec\theta}$ $\tan\theta = \frac{1}{\cot\theta}$

$\csc\theta = \frac{1}{\sin\theta}$ $\sec\theta = \frac{1}{\cos\theta}$ $\cot\theta = \frac{1}{\tan\theta}$

Quotient Identities:

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$

Pythagorean Identities:

$\sin^2\theta + \cos^2\theta = 1$ $\tan^2\theta + 1 = \sec^2\theta$ $1 + \cot^2\theta = \csc^2\theta$

Sum & Difference Identities:

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$ $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$ $\tan(\alpha \pm \beta) =$

$\frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$

Double-Angle Identities:

$\sin 2\theta = 2\sin\theta\cos\theta$ $\cos 2\theta = \cos^2\theta - \sin^2\theta$ $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$= 2\cos^2\theta - 1$

$= 1 - 2\sin^2\theta$

Half-Angle Identities:

$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$ $\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$ $\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$

$= \frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$