Accelerated Pre-Calculus
January 2022
Unit 5 – Matrices

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Matrices

Properties of Matrix Addition	**Properties of Matrix Multiplication**
Given matrices A, B, C with the same dimensions	Given matrices A and B with the same inner dimensions
Commutative Property: A + B = B + A	**<u>Matrix Multiplication is not commutative</u>**
Associative Property: A + (B + C) = (A + B) + C	Associative Property: (AB)C = A(BC)
Scalar Distributive Property: k (A + B) = kA + kB	Associative Property of Scalar Multiplication: k(AB) = (kA)B = A(kB)

Identity matrix (I) **A * I = I * A = A**

2 x 2

$$\begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{bmatrix}$$

3 x 3

$$\begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix}$$

The product of two inverse matrices is equal to the identity matrix.

(A)(A^{-1}) = (A^{-1})(A) = I

<u>Addition and Subtraction:</u> matrices <u>must</u> have the same dimensions

$$\begin{array}{ccccc} \textbf{A} & \textbf{+} & \textbf{B} & \textbf{=} & \textbf{A + B} \\ \begin{bmatrix} a & b \\ c & d \end{bmatrix} & + & \begin{bmatrix} e & f \\ g & h \end{bmatrix} & = & \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix} \end{array}$$

$$\begin{array}{ccccc} \textbf{A} & \textbf{–} & \textbf{B} & \textbf{=} & \textbf{A – B} \\ \begin{bmatrix} a & b \\ c & d \end{bmatrix} & - & \begin{bmatrix} e & f \\ g & h \end{bmatrix} & = & \begin{bmatrix} a-e & b-f \\ c-g & d-h \end{bmatrix} \end{array}$$

<u>Scalar Multiplication:</u> for matrices of <u>any</u> dimension

Given matrix M = $\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$ then kM = $\begin{bmatrix} ka & kb & kc \\ kd & ke & kf \end{bmatrix}$

<u>Determinant:</u> <u>must</u> be a square matrix

2 x 2 $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

det(A) = $|\,A\,| = ad - bc$

3 x 3 B = $\begin{bmatrix} \mathrm{a} & \mathrm{b} & \mathrm{c} \\ \mathrm{d} & \mathrm{e} & \mathrm{f} \\ \mathrm{g} & \mathrm{h} & \mathrm{i} \end{bmatrix}$ **repeat columns 1 and 2** $\begin{bmatrix} \mathrm{a} & \mathrm{b} & \mathrm{c} & \mathrm{a} & \mathrm{b} \\ \mathrm{d} & \mathrm{e} & \mathrm{f} & \mathrm{d} & \mathrm{e} \\ \mathrm{g} & \mathrm{h} & \mathrm{i} & \mathrm{g} & \mathrm{h} \end{bmatrix}$

Now multiply and combine products according to the pattern.

$\begin{bmatrix} \mathrm{a} & \mathrm{b} & \mathrm{c} & \mathrm{a} & \mathrm{b} \\ \mathrm{d} & \mathrm{e} & \mathrm{f} & \mathrm{d} & \mathrm{e} \\ \mathrm{g} & \mathrm{h} & \mathrm{i} & \mathrm{g} & \mathrm{h} \end{bmatrix}$ **and** $\begin{bmatrix} \mathrm{a} & \mathrm{b} & \mathrm{c} & \mathrm{a} & \mathrm{b} \\ \mathrm{d} & \mathrm{e} & \mathrm{f} & \mathrm{d} & \mathrm{e} \\ \mathrm{g} & \mathrm{h} & \mathrm{i} & \mathrm{g} & \mathrm{h} \end{bmatrix}$

$$det(B) = |B| = (aei + bfg + cdh) - (ceg + afh + bdi)$$

<u>Inverse:</u> <u>must</u> be a square matrix. If det(A) = 0, then A is a singular matrix (non-invertible).
For 2x2 find the inverse without a calculator. For matrices larger than 2x2 – find the inverse with an app.

Given matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $A^{-1} = \frac{1}{det}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

<u>Solving a System of Equations:</u>

$\begin{cases} ax + by = c \\ dx + ey = f \end{cases}$ written as a **Matrix Equation**: $\begin{array}{ccccc} \textbf{C} & \textbf{x} & \textbf{V} & \textbf{=} & \textbf{A} \\ \begin{bmatrix} a & b \\ d & e \end{bmatrix} & \mathrm{x} & \begin{bmatrix} x \\ y \end{bmatrix} & = & \begin{bmatrix} c \\ f \end{bmatrix} \end{array}$ **Can be solved by: V = C^{-1}A**

where **C** is the **C**oefficient matrix, **V** is the **V**ariable matrix, and **A** is the **A**nswer matrix.