**Name:**______________________________________________________________ **Period:** ___________

# Accel. Pre-Calculus

# Unit 5 Packet

# Matrices

| Properties of Matrix Addition | Properties of Matrix Multiplication |
|---|---|
| Given matrices A, B, C with the same dimensions | Given matrices A and B with the same inner dimensions |
| **Commutative Property**: A + B = B + A | **<u>Matrix Multiplication is not commutative</u>** |
| **Associative Property**: A + ( B + C) = (A + B) + C | Associative Property: (AB)C = A(BC) |
| **Scalar Distributive Property**: $k$ (A + B) = $k$A + $k$B | Associative Property of Scalar Multiplication: $k$(AB) = ($k$A)B = A($k$B) |

# Matrices Summary Sheet

**Identity matrix (I)** **A * I = I * A = A**

**2 x 2**

$$\begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{bmatrix}$$

**3 x 3**

$$\begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix}$$

**The product of two inverse matrices is equal to the identity matrix.**

**$(A)(A^{-1}) = (A^{-1})(A) = I$**

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a-e & b-f \\ c-g & d-h \end{bmatrix}$$

**<u>Scalar Multiplication:</u> for matrices of <u>any</u> dimension**

Given matrix M = $\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$ then $k$M = $\begin{bmatrix} ka & kb & kc \\ kd & ke & kf \end{bmatrix}$

**<u>Determinant:</u> <u>must</u> be a square matrix**

**2 x 2** $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$det(A) = |\,A\,| = ad - bc$

**3 x 3** B = $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ **repeat columns 1 and 2** $\begin{bmatrix} a & b & c & a & b \\ d & e & f & d & e \\ g & h & i & g & h \end{bmatrix}$

**Now multiply and combine products according to the pattern.**

$\begin{bmatrix} a & b & c & a & b \\ d & e & f & d & e \\ g & h & i & g & h \end{bmatrix}$ **and** $\begin{bmatrix} a & b & c & a & b \\ d & e & f & d & e \\ g & h & i & g & h \end{bmatrix}$

$$det(B) = |B| = (aei + bfg + cdh) - (ceg + afh + bdi)$$

**<u>Inverse:</u> <u>must</u> be a square matrix. If det(A) = 0, then A is a singular matrix (non-invertible).**
**For 2x2 find the inverse without a calculator. For matrices larger than 2x2 – find the inverse with an app.**

Given matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $A^{-1} = \frac{1}{det}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

**<u>Solving a System of Equations:</u>**

**C x V = A**

$\begin{cases} ax + by = c \\ dx + ey = f \end{cases}$ written as a **Matrix Equation**: $\begin{bmatrix} a & b \\ d & e \end{bmatrix} \text{x} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c \\ f \end{bmatrix}$ **Can be solved by: $V = C^{-1}A$**

where **C** is the **C**oefficient matrix, **V** is the **V**ariable matrix, and **A** is the **A**nswer matrix.

**5.01 Intro to Matrices Notes**

# Introduction to Matrices

Matrix: A rectangular array of numbers

Uses: to solve systems of equations (next week) and to record data (think Excel)

Example: $A = \begin{bmatrix} 5 & 0 & -1 \\ -3 & 1 & 9 \end{bmatrix}$

Dimensions

Elements

More examples:

$B = \begin{bmatrix} -2 \\ 4 \\ 0 \\ 7 \end{bmatrix}$

B is known as a "column matrix".
What are the dimensions of matrix B?

How about a "row matrix"?
Create a 1x3 matrix C.

How about a "square matrix"?
Create a 4x4 matrix D.

## Operations

Addition and subtraction can only be done with matrices of the same size (dimensions).
Scalar multiplication can be done with a matrix of any size.

$$S = \begin{bmatrix} 2 & -1 & 3 \\ -4 & -2 & -8 \end{bmatrix} \quad T = \begin{bmatrix} -5 & -4 & 1 \\ 7 & -8 & 4 \end{bmatrix} \quad U = \begin{bmatrix} 9 & -2 \\ -3 & 0 \end{bmatrix}$$

Evaluate each:

S + U = 

S - T =

2T =

$^1/_2$ U =

Use matrices A, B, and C to solve for X.

$$A = \begin{bmatrix} 3 & 0 \\ -1 & 7 \end{bmatrix} \quad B = \begin{bmatrix} -4 \\ 12 \end{bmatrix} \quad C = \begin{bmatrix} -2 & 8 \\ 5 & -6 \end{bmatrix}$$

X = 2B

A + X = C

**5.01 Practice**

**State the dimensions of each matrix.**

1. $\begin{bmatrix} 1 & -8 \\ 6 & -2 \end{bmatrix}$

2. $\begin{bmatrix} -9 & -8 \\ 2 & 17 \\ 11 & -6 \end{bmatrix}$

**Find the value of each element in $A = \begin{bmatrix} -3 & 45 & 28 \\ 24 & 36 & -22 \\ -15 & 4 & 29 \end{bmatrix}$**

$a_{22}$

4. $a_{31}$

**Find each of the following for $Q = \begin{bmatrix} 13 & -6 \\ 2 & -10 \\ -4 & 8 \end{bmatrix}$, $R = \begin{bmatrix} 1 & -3 \\ -5 & 9 \\ 12 & 7 \end{bmatrix}$, $S = \begin{bmatrix} 5 & -2 & 1 \\ -6 & 14 & 8 \end{bmatrix}$, and $T = \begin{bmatrix} -11 & 3 & 7 \\ 4 & -9 & 16 \end{bmatrix}$.**

**If the matrix does not exist, write *impossible*.**

5. Q + R

6. $T - R$

7. $T - S$

8. $2R + Q$

9. $\frac{1}{2}(T + S)$

**Use matrices Q, R, S, and T to solve for X. If the matrix does not exist, write *impossible.***

10. $Q - R = \frac{1}{3}X$

11. $3S - X = T$

12. $2(Q - X) = -T$

13. Jessica took her two children to the community swimming pool once a week for six weeks. The daily admission fees are $4.50 for a child and $6.75 for an adult. Write a 1 x 3 matrix with a scalar multiple that represents the total cost of admission. What is the total cost?

**5.02 Matrix Multiplication Notes**

Scenario: Catherine and Meg spend their spare time working at Chick-Fil-A, babysitting, and working on homework. Chick-Fil-A pays them each $8.00/hour, they make $10/hour babysitting, and their homework will pay them in dividends when they're older (translation: they don't get paid). When working at Chick-Fil-A, they can't use their phones at all. While babysitting, they can send roughly 30 texts/hour. While working on homework, they can send about 40 texts/hour. Create hours/schedules for each student. How can we set up a matrix multiplication problem to calculate the pay and texts/hour for each activity for both Catherine and Meg?

To multiply matrices, multiply every element from one row in the 1st matrix by every element from one column in the 2nd matrix and find the sum of the products. This gives you the element in that row and column in the answer matrix. (It's easier than it sounds! Just watch)

Examples: $\begin{bmatrix} 4 & 8 \\ 0 & 2 \\ 1 & 6 \end{bmatrix} \cdot \begin{bmatrix} 5 & 2 \\ 9 & 4 \end{bmatrix} =$

$\begin{bmatrix} -2 & 8 & 4 \end{bmatrix} \cdot \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 5 & -6 \end{bmatrix} =$

5.02 Notes continued

$$\begin{bmatrix} 6 & 3 & 0 \\ 2 & 5 & 1 \\ 9 & 8 & 6 \end{bmatrix} \cdot \begin{bmatrix} 7 & 4 \\ 6 & 7 \\ 5 & 0 \end{bmatrix} =$$

Matrix Multiplication and Dimensions:

What happens if we switch the order of the matrices?

What happens if we switch the dimensions of one matrix?

So what has to be true for matrix multiplication to work?

Does matrix multiplication result in the same product when the order is reversed?

$$\begin{bmatrix} 2 & -3 \\ -5 & 1 \end{bmatrix} \begin{bmatrix} 6 & -2 \\ 1 & -1 \end{bmatrix} =$$

$$\begin{bmatrix} 6 & -2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ -5 & 1 \end{bmatrix} =$$

5.02 Practice

Find AB and BA, if possible.

1. $\boldsymbol{A} = \begin{bmatrix} 8 & 1 \end{bmatrix}; \boldsymbol{B} = \begin{bmatrix} 3 & -7 \\ -5 & 2 \end{bmatrix}$

2. $\boldsymbol{A} = \begin{bmatrix} 2 & 9 \\ -7 & 3 \end{bmatrix}; \boldsymbol{B} = \begin{bmatrix} 6 & -4 \\ 0 & 3 \end{bmatrix}$

3. $\boldsymbol{A} = \begin{bmatrix} 3 & -5 \end{bmatrix}; \boldsymbol{B} = \begin{bmatrix} 4 & 0 & -2 \\ 1 & -3 & 2 \end{bmatrix}$

4. $\boldsymbol{A} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}; \boldsymbol{B} = \begin{bmatrix} 6 & 1 & -10 & 9 \end{bmatrix}$

5. $\boldsymbol{A} = \begin{bmatrix} 2 \\ 5 \\ -6 \end{bmatrix}; \boldsymbol{B} = \begin{bmatrix} 6 & 0 & -1 \\ -4 & 9 & 8 \end{bmatrix}$

6. $\boldsymbol{A} = \begin{bmatrix} 2 & 0 \\ -4 & -3 \\ 1 & -2 \end{bmatrix}; \boldsymbol{B} = \begin{bmatrix} 0 & 6 & -5 \\ 2 & -7 & 1 \end{bmatrix}$

7. Different point values are awarded for different shots in basketball. Use the information to determine the total points scored by each player.

| Player | Free Throw | 2-pointer | 3-pointer |
|---|---|---|---|
| Rey | 44 | 32 | 25 |
| Chris | 37 | 24 | 31 |
| Jerry | 35 | 39 | 29 |

| Shots | Points |
|---|---|
| Free Throw | 1 |
| 2-pointer | 2 |
| 3-pointer | 3 |

**5.03 Notes: Introduction to Matrix Inverses**

What's created when you multiply inverses together?

$2\left(\frac{1}{2}\right) =$ $3\left(\frac{1}{3}\right) =$ $25\left(\frac{1}{25}\right) =$

Multiply matrix inverses together and you get…..

Identity Matrix: a square matrix with diagonal elements = 1 and all other elements = 0

$I_1 = [1]$ $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $I_3 =$ $I_4 =$

Multiplying any matrix by the identity matrix is like multiplying a number by 1

Example: $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 6 & -2 \\ 1 & -1 \end{bmatrix}$

Why?

Find the product of the given matrices to determine if they are inverses.

$\boldsymbol{A} = \begin{bmatrix} -7 & 2 \\ -8 & 2 \end{bmatrix}$ and $\boldsymbol{B} = \begin{bmatrix} 1 & -1 \\ 4 & -3.5 \end{bmatrix}$ $\boldsymbol{C} = \begin{bmatrix} 1 & 1 & 2 \\ -1 & 0 & 3 \\ -1 & -2 & -8 \end{bmatrix}$ and $\boldsymbol{D} = \begin{bmatrix} -6 & -4 & 3 \\ 11 & 6 & -5 \\ -2 & -1 & 1 \end{bmatrix}$

Determinant of a 2x2 Matrix: $\begin{bmatrix} a & b \\ c & d \end{bmatrix} =$

For $\boldsymbol{E} = \begin{bmatrix} 6 & 1 \\ -2 & 5 \end{bmatrix}$, find $\det(E)$

For $\boldsymbol{F} = \begin{bmatrix} 3 & 9 \\ -2 & -6 \end{bmatrix}$, find $|F|$

## 5.03 Practice

**Determine whether $A$ and $B$ are inverse matrices.**

**1.** $\boldsymbol{A} = \begin{bmatrix} 12 & -7 \\ -5 & 3 \end{bmatrix}$ and $\boldsymbol{B} = \begin{bmatrix} 3 & 7 \\ 5 & 12 \end{bmatrix}$

**2.** $\boldsymbol{A} = \begin{bmatrix} 2 & 3 & -4 \\ 3 & 6 & -5 \\ -2 & -8 & 1 \end{bmatrix}$ and $\boldsymbol{B} = \begin{bmatrix} -34 & 29 & 9 \\ 7 & -6 & -2 \\ -12 & 10 & 3 \end{bmatrix}$

**Find the determinant of each matrix.**

**3.** $\begin{bmatrix} 6 & -5 \\ 3 & -2 \end{bmatrix}$

**4.** $\begin{bmatrix} -2 & 7 \\ 1 & 8 \end{bmatrix}$

**5.** $\begin{bmatrix} -4 & -7 \\ 6 & 9 \end{bmatrix}$

**6.** $\begin{bmatrix} 12 & -9 \\ -4 & 3 \end{bmatrix}$

**5.04 Matrix Inverses Notes**

Things to Know:

Matrix Multiplicative Identity: $\mathrm{X} \cdot \mathrm{I} = \mathrm{X}$ and $\mathrm{I} \cdot \mathrm{X} = \mathrm{X}$

Determinants: Value used to find the inverse

Inverses: (notation: $A^{-1}$) $\quad \mathrm{A} \cdot A^{-1} = \mathrm{I} \qquad A^{-1} \cdot \mathrm{A} = \mathrm{I}$

Finding the inverse of a matrix:

-A matrix is considered non-singular if it has an inverse and singular if it does not have an inverse

-A matrix is singular and has no inverse if the determinant = 0

If $\mathrm{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $A^{-1} =$

Find the determinant. Then find the inverse of the matrix, if it exists.

1. $A = \begin{bmatrix} 2 & -3 \\ 4 & 4 \end{bmatrix}$

2. $B = \begin{bmatrix} 6 & 4 \\ 9 & 6 \end{bmatrix}$

3. $C = \begin{bmatrix} -4 & 6 \\ 8 & -12 \end{bmatrix}$

4. $D = \begin{bmatrix} 2 & -3 \\ 2 & 2 \end{bmatrix}$

Given $A = \begin{bmatrix} 2 & 6 \\ 1 & 4 \end{bmatrix}$ and $AB = \begin{bmatrix} 30 & -20 \\ 21 & -14 \end{bmatrix}$ find B. (Hint: remember that the product of inverses is the identity matrix)

5.04 Practice

In #1–4, use the determinants you found yesterday in #3–6 to find the inverse of the given matrix, if it exists.

**1.** $\begin{bmatrix} 6 & -5 \\ 3 & -2 \end{bmatrix}$

**2.** $\begin{bmatrix} -2 & 7 \\ 1 & 8 \end{bmatrix}$

**3.** $\begin{bmatrix} -4 & -7 \\ 6 & 9 \end{bmatrix}$

**4.** $\begin{bmatrix} 12 & -9 \\ -4 & 3 \end{bmatrix}$

5. Given A and AB, find B. $\boldsymbol{A} = \begin{bmatrix} 8 & -4 \\ 3 & 6 \end{bmatrix}$, and $\boldsymbol{AB} = \begin{bmatrix} 36 & 48 \\ -24 & 48 \end{bmatrix}$

**5.05 Solving a 2x2 System of Equations Notes**

Ways to solve a 2-variable system of equations:

What does your solution represent?

Review: Solve the system using the elimination method.

$$\begin{cases} -4x + 9y = 9 \\ x - 3y = -6 \end{cases}$$

**This year, a NEW way to solve!**

How can matrices help to solve a system of equations?

Matrix equations should take the form AX = B; where A = the coefficient matrix, X = the variable matrix, and B = the constant matrix

Example:

As a system: $\begin{cases} -4x + 9y = 9 \\ x - 3y = -6 \end{cases}$ As a matrix equation:

If A·X = B, how do we solve for X?

Solve $\begin{cases} -4x + 9y = 9 \\ x - 3y = -6 \end{cases}$ using a matrix equation.

## Using a Matrix Equation to Solve a System

1. Write the equation, AX = B, made of the coefficient matrix A times the variable matrix X equal to the constant matrix B.

2. Solve for the variable matrix by multiplying the inverse of the coeffiecient matrix times the answer matrix. $X = A^{-1}B$

3. If $A^{-1}$ does not exist, then there are either:
   - NO SOLUTIONS
   - or INFINITELY MANY SOLUTIONS

Examples Cont'd: Solve the following systems using matrix equations.

2. $\begin{cases} x - 4y = 12 \\ 3x + 2y = 8 \end{cases}$

3. $\begin{cases} 2x + 12y = 20 \\ 5x + 30y = -7 \end{cases}$

4. $\begin{cases} 5x + 4y = -30 \\ 3x - 9y = -18 \end{cases}$

**5.06 Practice**

**Solve each system of equations using the elimination method.**

**1.** $5x = -3y - 31$
$2y = -4x - 22$

**2.** $4y + 17 = -7x$
$8x + 5y = -19$

**3.** $12x = 21 - 3y$
$2y = -4x - 22$

**4.** $4y = 12x - 3$
$9x = 20y - 2$

**Write each system of equations as a matrix equation. Solving using an inverse matrix.**

**5.** $\begin{aligned} 5x - 2y &= 11 \\ -4x + 7y &= 2 \end{aligned}$

**6.** $\begin{aligned} 2x + 3y &= 2 \\ x - 4y &= -21 \end{aligned}$

**7.** $\begin{aligned} -3x + 5y &= 33 \\ 2x - 4y &= -26 \end{aligned}$

**8.** $\begin{aligned} -4x + y &= 19 \\ 3x - 2y &= -18 \end{aligned}$

Matrices on the Calculator

**Entering matrices into your TI calculator:**

1. Access the "Matrix" window by pressing [2nd] and the button showing "Matrix" in blue.
2. The first menu, "NAMES", is how you select matrices to work with. The second menu, "MATH", is where you will find operations to perform on matrices (like *determinant* and *transpose*!). The third menu, "EDIT", is where you will enter matrices.
3. Select a matrix under the "EDIT" menu. Enter the dimensions of the matrix (number of rows first and then number of columns). Enter all elements of the matrix. When you reach the end of the matrix, double check all elements are entered correctly.
4. If you need to enter another matrix, start the process all over, [2nd], "Matrix", go to the "EDIT" menu, select a matrix that is NOT the one you just entered, set its dimensions, and enter its elements.
5. Press [2nd] followed by [mode] to quit to the home screen.

**Using matrices on your TI calculator:**

- Find the <u>determinant</u> of A: det [A] or |A|
  1. Store the matrix in the "EDIT" menu (described above). Be sure to "quit" at the end.
  2. Press [2nd], "Matrix", go to the "MATH" menu. Select "determinant" from the "MATH" list of the matrix window. You should see "det(" on the home screen.
  3. Press [2nd], "Matrix". Select the matrix from the "NAMES" menu that was entered.
  4. Press [ENTER]. The calculator returns the determinant.

- Find the <u>inverse</u> of A: $A^{-1}$
  1. Store the matrix in the "EDIT" menu (described above). Be sure to "quit" at the end.
  2. Press [2nd], "Matrix". Select the matrix from the "NAMES" menu that was entered.
  3. Enter the 'inverse' exponent of -1. You should see "$[A]^{-1}$" on the home screen
     a. Graphing Calc use the button [$x^{-1}$]
     b. TI36xPro: inverse is in the matrix "MATH" menu OR use [$x^{\square}$] and enter -1.
  4. [ENTER] = WHOA! Ugly decimals. Convert that to fractions!
     a. Graphing Calc: [MATH], "1:►Frac", [ENTER]
     b. TI-36xPro: [◄►≈] is found right above [enter].

- <u>Solve a system</u> using a matrix equation:
  1. Write the matrix equation on paper: CV = A.
     - Not here, but if an equation in the system is missing variables, fill in 0 for the coefficient in the <u>correct spot</u> in the row (x is first, then y, and z). Also, if an equation is not in standard form (x first, then y, followed by z), then rearrange the equation so that it is.
  2. Store the *coefficient matrix* and the *answers matrix* in the matrix "EDIT" menu. Be sure to "quit" at the end of the process.
  3. Multiply the *inverse of the coefficient matrix times the answers matrix.* You should see something similar to this "$[A]^{-1}[B]$" on the home screen. The calculator will return a 3x1 matrix with the solution for (x, y, z).

**5.07 Matrix Inverses (3x3) Notes**

Determinant of a 3x3: If $\mathrm{A} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ then $\det[A] = (aei + bfg + cdh) - (bdi + afh + ceg)$

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = \begin{matrix} a & b & c & a & b \\ d & e & f & d & e \\ g & h & i & g & h \end{matrix}$$

Examples: Find the determinant of the following matrices, then use a calculator to find the inverse, if it exists.

1. $E = \begin{bmatrix} -3 & 2 & 4 \\ 1 & -1 & 2 \\ -1 & 4 & 0 \end{bmatrix}$

2. $F = \begin{bmatrix} 3 & 1 & 2 \\ 1 & 2 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

3. $G = \begin{bmatrix} -1 & -2 & 1 \\ 4 & 0 & 3 \\ -3 & 1 & -2 \end{bmatrix}$

4. Given A and AB, find B.

$A = \begin{bmatrix} 1 & -2 & 3 \\ -1 & 0 & 2 \\ 4 & 1 & -2 \end{bmatrix}$ and $AB = \begin{bmatrix} 17 & -2 \\ 3 & 3 \\ 12 & -4 \end{bmatrix}$

5.07 Practice

Find the determinant of each matrix. Then, find the inverse of the matrix, if it exists.

**1.** $\begin{bmatrix} 3 & 1 & -2 \\ 8 & -5 & 2 \\ -4 & 3 & -1 \end{bmatrix}$

**2.** $\begin{bmatrix} 1 & -1 & -2 \\ 5 & 9 & 3 \\ 2 & 7 & 4 \end{bmatrix}$

**3.** $\begin{bmatrix} 9 & 3 & 7 \\ -6 & -2 & -5 \\ 3 & 1 & 4 \end{bmatrix}$

**4.** $\begin{bmatrix} 2 & 3 & -1 \\ -4 & -5 & 2 \\ 6 & 1 & 3 \end{bmatrix}$

**5.** $\begin{bmatrix} -1 & 3 & 2 \\ 3 & -5 & -3 \\ 4 & 2 & 6 \end{bmatrix}$

**6.** $\begin{bmatrix} 6 & -1 & 2 \\ 1 & -2 & -4 \\ -3 & 1 & -5 \end{bmatrix}$

7. Given A and AB, find B. $\boldsymbol{A} = \begin{bmatrix} 5 & 0 & 1 \\ 2 & -3 & 2 \\ 1 & -1 & 4 \end{bmatrix}, \boldsymbol{AB} = \begin{bmatrix} 1 & 4 \\ -16 & -6 \\ -2 & -5 \end{bmatrix}$

Find the determinant of each matrix.

**8.** $\begin{bmatrix} r & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & t \end{bmatrix}$

**9.** $\begin{bmatrix} c & c & c \\ 0 & c & c \\ 0 & 0 & c \end{bmatrix}$

**5.08 Notes: Solving a 3x3 System of Equations**

Solving a 3-variable system of equations: you have 3 equations and can still use the previous methods from 2-variable systems. Mainly, we use the **elimination method** when solving by hand. For this unit, we can use a **matrix equation** and use **technology!**

**Solve each system using the elimination method.**

**1.**
$$\begin{aligned} -4x + 2y + 4z &= 4 \\ -6x + 2y - 2z &= -30 \\ x - 3y - 6z &= -16 \end{aligned}$$

**2.**
$$\begin{aligned} 3x + 2y - 6z &= 6 \\ 3x + y + 4z &= -20 \\ 6x + 2y - z &= -22 \end{aligned}$$

**Write each system as a matrix equation and then solve with technology.**

**3.**
$$\begin{aligned} -x - 9y + 2z &= -13 \\ 6y - 5z &= -6 \\ 6z + 5 &= 2y - 3x \end{aligned}$$

**4.**
$$\begin{aligned} 2x - 3y + 2z &= 16 \\ -4x + 6y - 4z &= 9 \\ 8x + z &= -11 \end{aligned}$$

5.08 Practice

**Solve each system of equations using the elimination method.**

**1.**
$$\begin{gathered}-3x + y + 6z = 15\\ 2x + 2y - 5z = 9\\ 4x - 5y + 2z = -3\end{gathered}$$

**2.**
$$\begin{gathered}8x - 24y = -7 - 16z\\ 40x + 2z = 9y + 10\\ 32x + 8y - z = -2\end{gathered}$$

Write each system of equations as a matrix equations. Solving using an inverse matrix and technology, if possible.

**3.**
$$\begin{aligned} 2x + y - z &= -13 \\ 3x + 2y - 4z &= -36 \\ x + 6y - 3z &= 12 \end{aligned}$$

**4.**
$$\begin{aligned} 3x - 2y + 6z &= 38 - 2z \\ 6x + 3y - 9z &= -12 \\ 4y + 20z &= -4x \end{aligned}$$

**5.**
$$\begin{aligned} -x + 2y - z &= 2 - 2x \\ 2x + 3z &= y + 4 \\ 3x + y + 2z &= 6 \end{aligned}$$

5.09 Matrix Applications Notes Date _______________

**For each, determine if you need to multiply matrices or write a matrix equation and solve using an inverse.**

1. In football a touchdown is 6 points, a field goal is 3 points, a touchdown extra point is 1 point, and a two-point conversion is 2 points. The number of each for the top 3 teams in a high school league are given below. Use this information to determine the team that scored the most points.

| **TEAM** | **TD** | **FG** | **EP** | **2EP** |
|---|---|---|---|---|
| Tigers | 21 | 14 | 12 | 9 |
| Rams | 24 | 12 | 18 | 3 |
| Eagles | 27 | 7 | 21 | 2 |

2. A restaurant manager wants to purchase 200 sets of dishes. One design costs $25 per set, while another costs $45 per set. If she only has $7400 to spend, how many of each design should she order?

3. A local coffee shop specializes in espresso drinks. The table shows the cups of each drink sold throughout the day. Determine the price of each espresso drink.

| **Hours** | **Cappuccino** | **Latte** | **Macchiato** | **Earnings** |
|---|---|---|---|---|
| 8-11 | 103 | 86 | 79 | 1040.25 |
| 11-2 | 48 | 32 | 26 | 406.50 |
| 2-5 | 45 | 25 | 18 | 334.00 |

4. Claire purchased 25 total pounds of dog food, bird seed, and cat food for $100. The dog food costs $4.00/lb, the bird seed $7.00/lb, and the cat food $3.00/lb. She purchased 10 pounds more dog food than bird seed. Determine the number of pounds of each type of food Claire purchased.

5.09 Practice

**For each, determine if you need to multiply matrices or write a matrix equation and solve using an inverse.**

1. Kelly, Joelle, and Emily are competitive skaters. Their routines are judged on skating skill (SS), choreography (C), and interpretation (I). In a recent competition, they received the following scores shown below. One of two weighted systems is used. System A weights SS 20%, C 50%, and I 30%. System B weights SS 40%, C 30%, and I 30%. Determine which systems favors each skater.

| **Skater** | **SS** | **C** | **I** |
|---|---|---|---|
| Kelly | 6 | 4 | 2 |
| Joelle | 3 | 5 | 1 |
| Emily | 2 | 4 | 6 |

2. A recently retired couple needs $8,000 per year to supplement their Social Security. They have $100,000 to invest to obtain this income. They have decided on two investment options: AA bonds yielding 10% per annum and a Bank Certificate yielding 5%. How much money should they invest in AA bonds and how much in Bank Certificates?

3. The table shows the number of individuals attending the movies over the weekend at a local theater. Determine the costs for a child, adult, and senior citizen to attend the movies.

| **Day** | **Child** | **Adult** | **Senior** | **Total Paid** |
|---|---|---|---|---|
| Fri | 80 | 110 | 25 | 1755 |
| Sat | 100 | 175 | 40 | 2685 |
| Sun | 45 | 85 | 30 | 1385 |

4. Mr. Wiley invested $5000 in three different accounts at the beginning of last year, yielding him a total of $182.50 of interest at the end of the year. The three accounts were a simple savings account earning 1%, a certificate of deposit earning 3.5%, and municipal bonds earning 4.3%. His municipal bond investment was 5 times the amount of money invested in the simple savings account. How much did he invest in each account?

5.10 More Practice with Matrix Applications Date: ___________________

**Identify the variables, create a system of equations, write the matrix equation, and solve. Show all work for solving 2x2 matrix equations. You may use technology to solve any 3x3 matrix equations.**

1. One group of people purchased 10 hot dogs and 5 soft drinks at a cost of \$8.75. A second group bought 7 hot dogs and 4 soft drinks at a cost of \$6.25. What is the cost of a single hot dog and a single soft drink?

2. Four large cheeseburgers and two chocolate milkshakes cost a total of \$7.90. Two milkshakes cost \$0.15 more than one cheeseburger. What is the cost of a cheeseburger and what is the cost of a milkshake?

3. Thompson's Furniture Store borrowed \$650,000 to expand its facilities and extend its product line. Some of the money was borrowed at 4%, some at 6.5%, and the rest at 9%. How much was borrowed at each rate if the annual interest was \$46,250 and the amount borrowed at 9% was twice the amount borrowed at 4%?

4. At the Pittsburgh zoo, children ride a train for 25 cents, adults pay \$1.00, and senior citizens 75 cents. On a given day, 1400 passengers paid a total of \$740 for the rides. There were 250 more children riders than all other riders. Find the number of children, adult, and senior riders.

5. Grace receives an $80,000 inheritance. She invests part of it in CDs (certificates of deposit) earning 6.7% APY (annual percentage yield), part in bonds earning 9.3% APY, and the remainder in a growth fund earning 15.6% APY. She invests three times as much in the growth fund as in the other two combined. How much does she have in each investment if she receives $10,843 interest the first year?

6. Sophia has 74 coins consisting of nickels, dimes, and quarters in her coin box. The total value of the coins is $8.85. If the number of nickels and quarters is four more than the number of dimes, find how many of each coin Sophia has.

7. A company manufactures tables, chairs, and stools. Last week it built a total of 275 items. The number of chairs built was four times the total number of tables and stools built. The total value of these items is $42,125 with a chair selling for $150, a table for $200, and a stool for $75. Determine the number of each item built last week.

8. A sports equipment company took out three different loans totaling $350,000 from a bank to buy treadmills. The interest rates for each are: Loan 1 = 6.5%, Loan 2 = 7%, and Loan 3 = 9%. After one year the interest paid was $24,950.00. The amount borrowed in Loan 1 was $50,000 less than the amounts borrowed in the other two Loans combined. How much was borrowed in each loan?

5.11 Matrices Test Review Date: ____________________

**Solve for X, given the following matrices. If not possible, state the reason why. *Show work!!***

$A = \begin{bmatrix} -2 & 0 \\ 4 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 4 & a & -5 \\ 2 & 0 & -3 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 0 & 2 \\ -3 & 6 & 1 \end{bmatrix}$ $D = \begin{bmatrix} 12 \\ 6 \end{bmatrix}$

1. X – A = D

2. X = C – B

3. -2X = A

4. 3C = X – 2B

5. X = AB

6. X = 2CD

7. X = $A^{-1}$

8. AX = D

9. The **determinant** of $\begin{bmatrix} 1 & 1 & 0 \\ 3 & 4 & -2 \\ -5 & -6 & x \end{bmatrix}$ is 12. **Solve for x**.

**Write each matrix equation, then solve the system of equations using an Inverse Matrix.** ***Show work!!***

10. $2x + y = 3$
$5x + 6y = 4$

11. $4x - 18 = 3y$
$8x - 7y = 34$

**Write each as a matrix equation, then solve the 3 variable system of equations using a calculator.**

12. $-x + 2y + 7z = 13$
$2x - y - 2z = -2$
$3x + 5y + 2z = -14$

13. A doctor's prescription calls for a daily intake containing 40 mg of vitamin C and 30 mg of vitamin D. Your pharmacy stocks 2 liquids that can be used: one contains 20% vitamin C and 30% vitamin D, the other contains 40% vitamin C and 20% vitamin D. How many milligrams of each compound should be mixed to fill the prescription?

14. John has $20,000 to invest. As his financial consultant, you recommend that he invest in Treasury bills that yield 5%, Treasury bonds that yield 7%, and corporate bonds that yield 9%. John wants to have an annual income of $1280, and the amount invested in Treasury bills must be two times the amount invested in corporate bonds. Find the amount in each investment.