Name:____ **Period:** __________

Accel. Pre-Calculus

Unit 6 Packet

Vectors

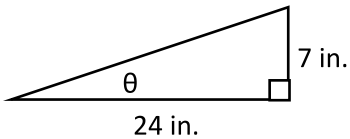
6.01: Review of Trigonometry at Any Angle **Date:** ____________________

Helpful Formulas:

Pythagorean Theorem: $a^2 + b^2 = c^2$ $\sin\theta = \frac{opp}{hyp}$ $\cos\theta = \frac{adj}{hyp}$ $\tan\theta = \frac{opp}{adj}$

Find the length of the hypotenuse and the measure of the angle of elevation for the given right triangle.

1.



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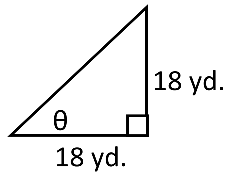

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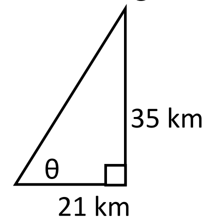
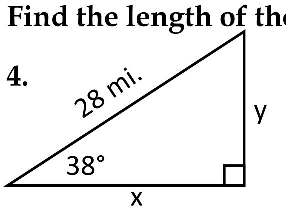


Find the length of the two legs for the given right triangle.

4.



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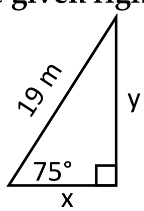

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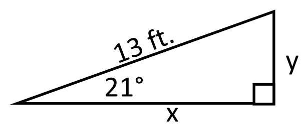


Sketch each angle in standard position and then state the quadrant where each angle has its terminal side.

9. θ = 140°

10. θ = 285°

11. The terminal side of θ passes through (-6, -5)

12. θ = -70°

13. θ = 220°

14. The terminal side of θ passes through (-12, 16)

More formulas: Pythagorean Theorem: $a^2 + b^2 = c^2$ $\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$

Sketch each angle described in standard position, where $0 \leq \theta < 360°$. Find the distance the point is from the origin and the measure of the angle.

15. The terminal side of θ passes through (2, 9)

16. The terminal side of θ passes through (-8, 6)

17. The terminal side of θ passes through (-10, -10)

18. The terminal side of θ passes through (13, -22)

<u>Self Check</u>**: Did your measures for the angles in #15 – 18 get larger each time? If not, you did something wrong.**

Sketch each point described with its given distance from the origin and angle measure in standard position. Determine the ordered pair coordinates described.

19. The point lies 15 inches from the origin on the terminal side of θ = 320°.

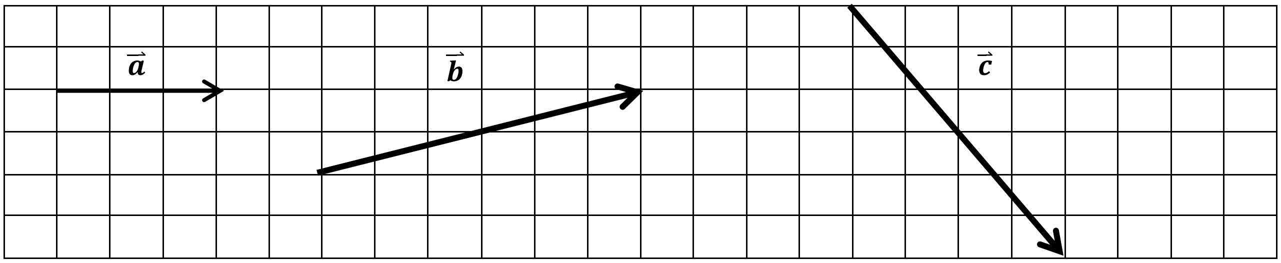
20. The point lies 29 mm from the origin on the terminal side of θ = 251°.

21. The point lies 7 yds. from the origin on the terminal side of θ = 120°.

22. The point lies 32 dm from the origin on the terminal side of θ = 38°.

APC 6.02 Geometric Vectors - Notes

Use the following to draw each vector diagram and resultant vector. Find the magnitude of the resultant.



1. $2\vec{b}$

2. $-3\vec{a}$

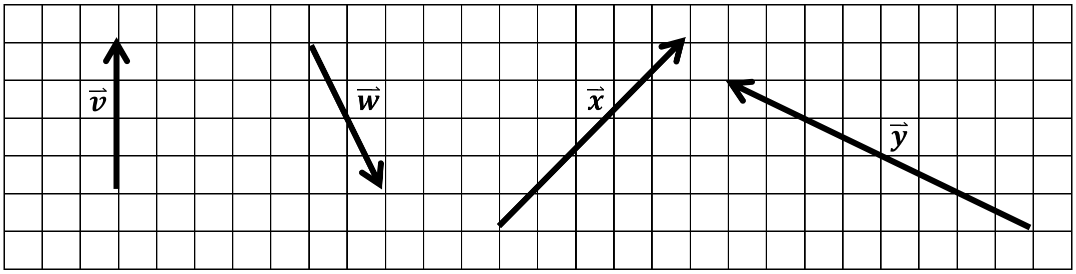
3. $2\vec{a} + \vec{c}$ (2 methods)

4. $\vec{b} - \frac{1}{2}\vec{c}$

State whether each quantity described is a *vector quantity* or a *scalar quantity*.

1. A box being pushed with a force of 125 newtons
2. Wind blowing 20 knots
3. A deer running 15 m/sec due west
4. A 15-pound tire hanging from a rope

6.02 Practice: Vectors Geometrically



Draw each and label the resultant. Then, find the magnitude of the resultant vector.

1. $\frac{1}{2}\vec{x}$

2. $-\frac{3}{4}\vec{y}$

3. $\vec{y} - \vec{w}$

4. $\vec{x} + 3\vec{w}$

5. $2\vec{v} - \vec{x}$

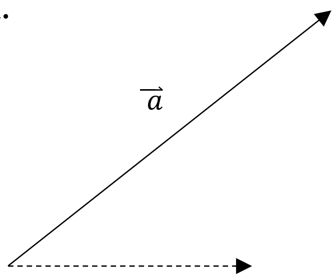
6. $-\vec{x} - \frac{1}{2}\vec{y}$

6.03 Notes: More Geometric Vectors

- **Use a ___________________________ to measure the magnitude of a vector.**
- **Use a ___________________________ to measure the direction of a vector.**

Examples: Measure the magnitude (in cm) and direction (in degrees) of each vector.

1.



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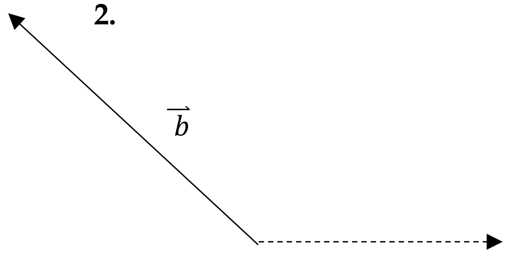

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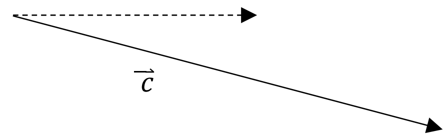


Mag = _______ Dir = _______ Mag = _______ Dir = _______ Mag = _______ Dir = _______

Now, draw each vector diagram and find the magnitude and direction of the resultant vector.

4. $\vec{a} + \vec{b}$

5. $2\vec{c} - \vec{a}$

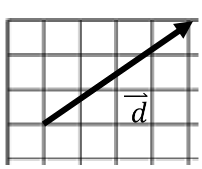
When a grid is used, first find the component form: <horizontal displacement, vertical displacement>. Then:

- **Use a ____ to calculate the magnitude of a vector.**
- **Use a _________________________ to calculate the direction of a vector.**

Caution: The calculator is not always right!

Examples: Find the component form, magnitude, and direction (using standard position) of $\vec{d}$ and $\vec{e}$.

6.



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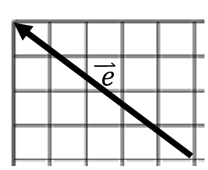
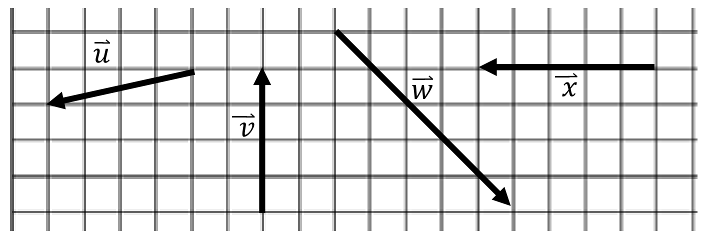


6.03 Practice: More Geometric Vectors

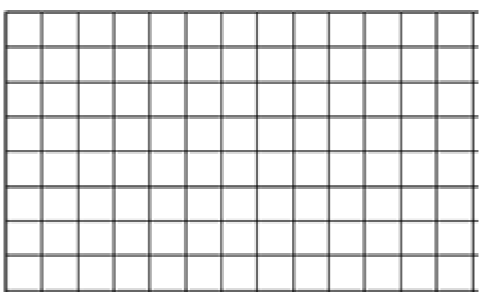
Use the vectors to the right to complete problems #1 – 6. Round answers to the nearest thousandth. Use standard position for the direction of a vector.

1. Find the component form, magnitude, and direction of $\vec{x}$.



2. Find the component form, magnitude, and direction of $\vec{u}$.

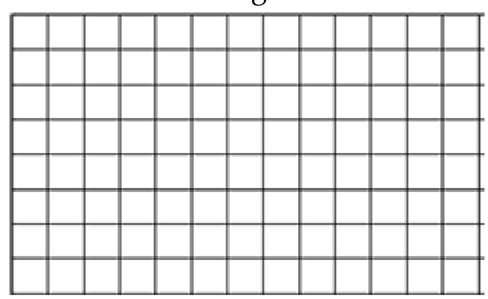
3. Draw the vector diagram of $\vec{a} = 2\vec{u} - \vec{w}$.



Component form of $\vec{a}$: ______________

Magnitude = ________ Direction = _________

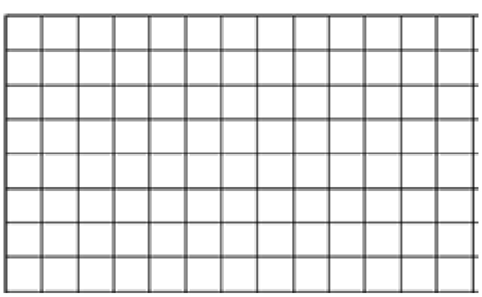
4. Draw the vector diagram of $\vec{b} = \vec{w} + 2\vec{v} + \vec{x}$



Component form of $\vec{b}$: ______________

Magnitude = ________ Direction = _________

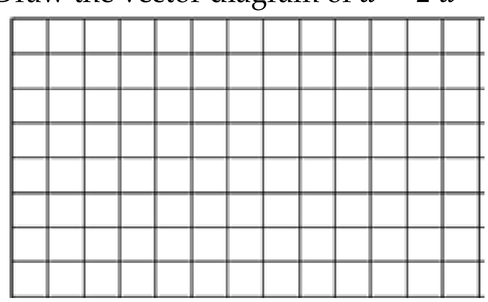
5. Draw the vector diagram of $\vec{c} = \vec{w} - \vec{u}$.



Component form of $\vec{c}$: ______________

Magnitude = __________ Direction = __________

6. Draw the vector diagram of $\vec{d} = 2\vec{u} - \vec{w} - \vec{x}$



Component form of $\vec{d}$: ______________

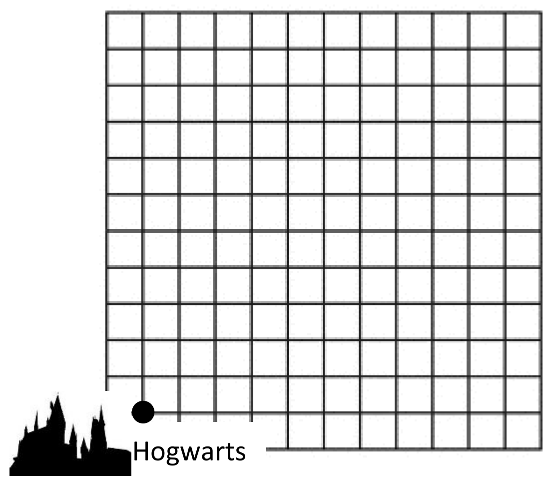
Magnitude = __________ Direction = __________

6.04 TASK: Walking & Flying Around Hogsmeade **Date:** ______________

Harry Potter needs to make a few stops around Hogsmeade. Harry's broom is broken, so he must walk between the buildings. The town is laid out in square blocks, which makes it easy to give directions. Here are the directions Harry must follow Monday:

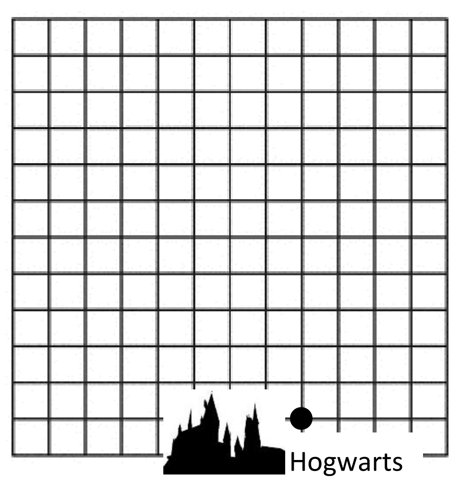
Monday – Start at Hogwarts		
	East/West	North/South
Stop 1 The 3 Broomsticks	3 blocks East	5 blocks North
Stop 2 Honeydukes	5 blocks East	2 blocks North
Stop 3 Gladrags	2 blocks East	1 block North

1. Draw the route of Harry's trip on the grid below.
2. Label each of Harry's stops.
3. How would Harry's trip through Hogsmeade change if he was able to ride his broom to his three stops? If this would make a different route for Harry, draw this new route in a *different color*.



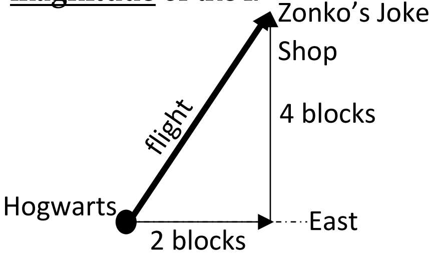
On Tuesday, Harry has more errands to run. When he is done, he will meet Ron at the Shrieking Shack, found 3 blocks West and 3 blocks North of Hogwarts. His directions for the first 3 stops are listed in the chart below. Draw the route of Harry's trip on the grid. Use the graph to determine his path to the Shrieking Shack.

Tuesday – Start at Hogwarts		
	East/West	North/South
Stop 1 Zonko's Joke Shop	2 blocks East	4 blocks North
Stop 2 Scrivenshaft's	4 blocks West (or -4 blocks East)	3 blocks North
Stop 3 Dervish and Bangs	5 blocks West (or ____ blocks East)	3 blocks South (or ____ blocks North)
Stop 4 Shrieking Shack	_____ blocks _____	_____ blocks _____



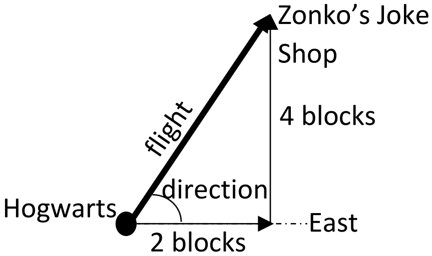
Harry's trusted owl, Hedwig, can fly over buildings, so she travels in a straight line from each stop to the next and waits for Harry to arrive. On Tuesday's graph, use a different color to draw arrows representing Hedwig's path.

How far did Hedwig fly to get to Stop 1 on Tuesday? This is known as the **magnitude** of the fl



There are several ways to describe Hedwig's direction during this leg of the trip. We could simply say she traveled "north-east", but this would not be precisely accurate or an exact description. Why not?

For **vectors**, we include an angle, often measured in **standard position**, to indicate the **direction**. Use inverse trigonometry to find the direction of Hedwig's flight from Hogwarts to Tuesday's first stop.



Next stop: Scrivenshaft's. We might say that Hedwig flew north-west, but it would not be an exact description. What is the possible range of values for all angles measured in **standard position** that are generally pointing to the north-west? What is *the* value for an angle pointing precisely to north-west?

Find the **magnitude** (distance) and **direction** of Hedwig's path from Stop 1 to Stop 2 on Tuesday. Show work. CAUTION: The angle must be measured in **standard position**, meaning it *opens counter-clockwise and is measured from the positive x-axis*. **The direction is NOT an acute angle measure!**

Find the **<u>magnitude</u>** (distance) and **<u>direction</u>** of Hedwig's path from Stop 2 to Stop 3 on Tuesday. Show work.

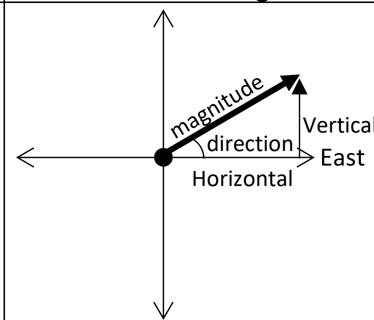
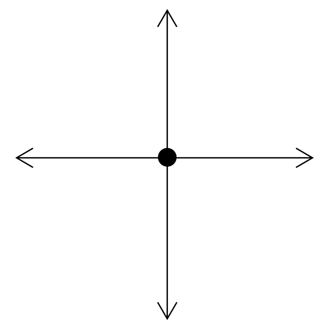
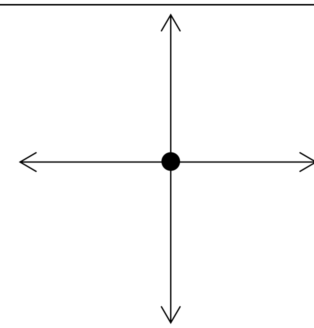
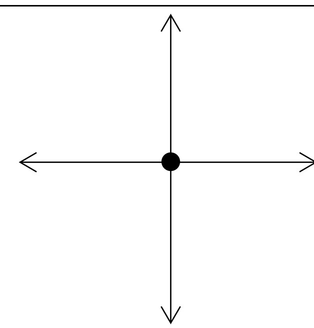
Find the **<u>magnitude</u>** (distance) and **<u>direction</u>** of Hedwig's path from Stop 3 to Stop 4 on Tuesday. Show work.

Ron sets out from Hogwarts to the Shrieking Shack to meet Harry. What directions does he take for the shortest path that follows the town's square blocks?

How does this route compare to the <u>overall displacement</u> of Harry during his errands on Tuesday?

The way we have expressed Harry's routes is known as **component form**, since it is made up of two parts, or components, that describe the changes in the horizontal direction and in the vertical direction. The way we have expressed Hedwig's path is known as **magnitude-direction form**, since it gives the distance directly to each stop and the angle measurement for the flight path.

It is important to be able to convert from one form to another. Practice this skill by filling in the table for Harry's route on Wednesday. Use the Pythagorean Theorem, trigonometry, and inverse trigonometry. Leaving the point (Hogwarts), draw Harry's path as horizontal and vertical components and Hedwig's path as a direct flight to the destination. Remember to include units. (Hint: think reference triangles)

Wednesday – Start at Hogwarts					
	Harry's description		**Hedwig's description**		
	Horizontal	**Vertical**	**Magnitude**	**Direction**	**Drawing**
a.	6 blocks East	3 blocks North			
b.			16 blocks	113°	
c.	2 blocks West	6 blocks South			
d.			10 blocks	315°	

Challenge: Thursday, Harry followed these directions through town: 2 blocks East, 5 blocks South, 1 block East, 3 blocks North, 4 blocks West, 2 blocks North, 3 blocks East, 1 block North, 3 blocks West, and 5 blocks South. If Ron wants to walk to Harry's final destination, what route should he take for the shortest trip?

Hedwig flies from Hogwarts directly to Harry's final destination. What is the **magnitude** of her flight? What **direction**, measured in **standard position**, does she fly?

6.05 Algebraic Vectors Notes

Component form: <a, b> a = ________________________ and b = _______________________

Ex: Draw the following vectors.

<2, 4> **<-3, 1>** **<-5, -5>** **<2, -10>**

Find component form of $\overrightarrow{CD}$ given initial and terminal points: C (7,-3) and D (9,1)

Operations with vectors:

If $\vec{u}$ = <a_1, b_1> and $\vec{v}$ = <a_2, b_2>

$\vec{u}+\vec{v}$ = $\vec{u}-\vec{v}$ = $k\vec{u}$ =

Ex: Perform the indicated operations given $\vec{u}$ = <-4, 1> and $\vec{v}$ = <2, 5>.

$\vec{u}+\vec{v}$ = $\vec{u}-\vec{v}$ =

$2\vec{u}-\vec{v}$ = $-3\vec{u}-4\vec{v}$ =

Unit Vector:

i =

j =

Linear combination of unit vectors (or "sum of unit vectors"):

Ex: Write <-3, 8> as a sum of unit vectors. ____________________

Ex: Write <0, -5> as a linear combination of unit vectors. ____________________

Ex: Find the vector with initial point X (5,5) and terminal point Y (-2,6) as a sum of unit vectors.

6.05 Practice: ODDS #1 – 17, 29−35

Find component form of $\overrightarrow{AB}$ with the given initial and terminal points.

1. $A(-3, 1), B(4, 5)$

2. $A(2, -7), B(-6, 9)$

3. $A(10, -2), B(3, -5)$

4. $A(-2, 7), B(-9, -1)$

5. $A(-5, -4), B(8, -2)$

6. $A(-2, 6), B(1, 10)$

7. $A(2.5, -3), B(-4, 1.5)$

8. $A(-4.3, 1.8), B(9.4, -6.2)$

9. $A\left(\frac{1}{2}, -9\right), B\left(6, \frac{5}{2}\right)$

10. $A\left(\frac{3}{5}, -\frac{2}{5}\right), B(-1, 7)$

Find each of the following for $\mathbf{f} = \langle 8, 0\rangle$, $\mathbf{g} = \langle -3, -5\rangle$, and $\mathbf{h} = \langle -6, 2\rangle$. (Example 3)

11. $4\mathbf{h} - \mathbf{g}$

12. $\mathbf{f} + 2\mathbf{h}$

13. $3\mathbf{g} - 5\mathbf{f} + \mathbf{h}$

14. $2\mathbf{f} + \mathbf{g} - 3\mathbf{h}$

15. $\mathbf{f} - 2\mathbf{g} - 2\mathbf{h}$

16. $\mathbf{h} - 4\mathbf{f} + 5\mathbf{g}$

17. $4\mathbf{g} - 3\mathbf{f} + \mathbf{h}$

18. $6\mathbf{h} + 5\mathbf{f} - 10\mathbf{g}$

Let $\overrightarrow{DE}$ be the vector with the given initial and terminal points. Write $\overrightarrow{DE}$ as a linear combination of the vectors $\mathbf{i}$ and $\mathbf{j}$. (Example 5)

28. $D(4, -1), E(5, -7)$

29. $D(9, -6), E(-7, 2)$

30. $D(3, 11), E(-2, -8)$

31. $D(9.5, 1), E(0, -7.3)$

32. $D(-3, -5.7), E(6, -8.1)$

33. $D(-4, -6), E(9, 5)$

34. $D\left(\frac{1}{8}, 3\right), E\left(-4, \frac{2}{7}\right)$

35. $D(-3, 1.5), E(-3, 1.5)$

6.06 Notes: More Algebraic Vectors

Magnitude:

Ex: Find $|\vec{v}|$ if $\vec{v}$ = <-3, 8>.

Ex: Find the magnitude of a vector with initial point A (-2,3) and terminal point B (4,5).

Direction:

Ex: Find the direction of $\vec{e} = \langle 8, 5 \rangle$

Ex: Find the direction of $\vec{f} = -6i - 7j$

<u>Think about it</u>: If the direction of $\vec{x}$ is 50°, how would direction change for each of the following:

- **$5\vec{x}$**
- **$-3\vec{x}$**
- **$\frac{1}{2}\vec{x}$**

Components from Magnitude and Direction:

Ex: Find the components of $\vec{g}$ if it has magnitude of 5 and direction of 25°.

6.06 Practice: ODDS #1−9, 39−51

Find the magnitude of $\overrightarrow{AB}$ with the given initial and terminal points (same 1-9 as yesterday).

1. $A(-3, 1), B(4, 5)$

2. $A(2, -7), B(-6, 9)$

3. $A(10, -2), B(3, -5)$

4. $A(-2, 7), B(-9, -1)$

5. $A(-5, -4), B(8, -2)$

6. $A(-2, 6), B(1, 10)$

7. $A(2.5, -3), B(-4, 1.5)$

8. $A(-4.3, 1.8), B(9.4, -6.2)$

9. $A\left(\frac{1}{2}, -9\right), B\left(6, \frac{5}{2}\right)$

10. $A\left(\frac{3}{5}, -\frac{2}{5}\right), B(-1, 7)$

Find the component form of *v* with the given magnitude and direction angle. (Example 6)

38. $|\mathbf{v}| = 12, \theta = 60°$

39. $|\mathbf{v}| = 4, \theta = 135°$

40. $|\mathbf{v}| = 6, \theta = 240°$

41. $|\mathbf{v}| = 16, \theta = 330°$

42. $|\mathbf{v}| = 28, \theta = 273°$

43. $|\mathbf{v}| = 15, \theta = 125°$

Find the direction angle of each vector to the nearest tenth of a degree. (Example 7)

44. $3\mathbf{i} + 6\mathbf{j}$

45. $-2\mathbf{i} + 5\mathbf{j}$

46. $8\mathbf{i} - 2\mathbf{j}$

47. $-4\mathbf{i} - 3\mathbf{j}$

48. $\langle -5, 9 \rangle$

49. $\langle 7, 7 \rangle$

50. $\langle -6, -4 \rangle$

51. $\langle 3, -8 \rangle$

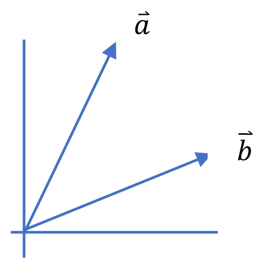
6.07 Angle Between Vectors Notes

Dot Product: if u = <a$_1$, b$_1$> and v = <a$_2$, b$_2$> then $\vec{u} \cdot \vec{v} =$

Ex: Find the dot product between the following pairs of vectors.

1. u = <3, 6>, v = <-4, 2> **2. a = 2i + 5j, b = 8i – 4j**

Angle Between 2 Vectors:



$\vec{a} =$

$\vec{b} =$

$\vec{a} \cdot \vec{b} =$

Ex: Find the angle between vectors $\vec{c} = \langle -5, -2 \rangle$ and $\vec{d} = \langle 4, 8 \rangle$.

Orthogonal Vectors:

Ex: Create a non-zero vector $\vec{n}$ that is orthogonal to $\vec{m} = \langle -4, 2 \rangle$.

Parallel Vectors:

Ex: Determine the angle between the following pairs of vectors.

$< -2, 5 > \text{ and } < -6, 15 >$ $< 8, 12 > \text{ and } < 6, 9 >$ $< 10, 11 > \text{ and } < -5, -5.5 >$

6.07 Practice: #1-9, 19-24

Determine if the following vectors are parallel, orthogonal, or neither.

1. $\boldsymbol{u} = < 3, -5 >, \boldsymbol{v} = < 6, 2 >$

2. $\boldsymbol{u} = < -10, -16 >, \boldsymbol{v} = < -8, 5 >$

3. $\boldsymbol{u} = < 9, -3 >, \boldsymbol{v} = < 1, 3 >$

4. $\boldsymbol{u} = < 4, -4 >, \boldsymbol{v} = < 7, 5 >$

5. $\boldsymbol{u} = < 1, -4 >, \boldsymbol{v} = < 2, -8 >$

6. $\boldsymbol{u} = 11\boldsymbol{i} + 7\boldsymbol{j};\ \boldsymbol{v} = -7\boldsymbol{i} + 11\boldsymbol{j}$

7. $\boldsymbol{u} = < -4, 6 >, \boldsymbol{v} = < -5, -2 >$

8. $\boldsymbol{u} = 8\boldsymbol{i} + 6\boldsymbol{j}; \boldsymbol{v} = -\boldsymbol{i} + 2\boldsymbol{j}$

9. **SPORTING GOODS** The vector $\mathbf{u} = \langle 406, 297 \rangle$ gives the numbers of men's basketballs and women's basketballs, respectively, in stock at a sporting goods store. The vector $\mathbf{v} = \langle 27.5, 15 \rangle$ gives the prices in dollars of the two types of basketballs, respectively. (Example 1)

a. Find the dot product $\mathbf{u} \bullet \mathbf{v}$.

b. Interpret the result in the context of the problem.

Find the angle θ between u and v to the nearest tenth of a degree. (Example 3)

16. $\mathbf{u} = \langle 0, -5 \rangle$, $\mathbf{v} = \langle 1, -4 \rangle$

17. $\mathbf{u} = \langle 7, 10 \rangle$, $\mathbf{v} = \langle 4, -4 \rangle$

18. $\mathbf{u} = \langle -2, 4 \rangle$, $\mathbf{v} = \langle 2, -10 \rangle$

19. $\mathbf{u} = -2\mathbf{i} + 3\mathbf{j}$, $\mathbf{v} = -4\mathbf{i} - 2\mathbf{j}$

20. $\mathbf{u} = \langle -9, 0 \rangle$, $\mathbf{v} = \langle -1, -1 \rangle$

21. $\mathbf{u} = -\mathbf{i} - 3\mathbf{j}$, $\mathbf{v} = -7\mathbf{i} - 3\mathbf{j}$

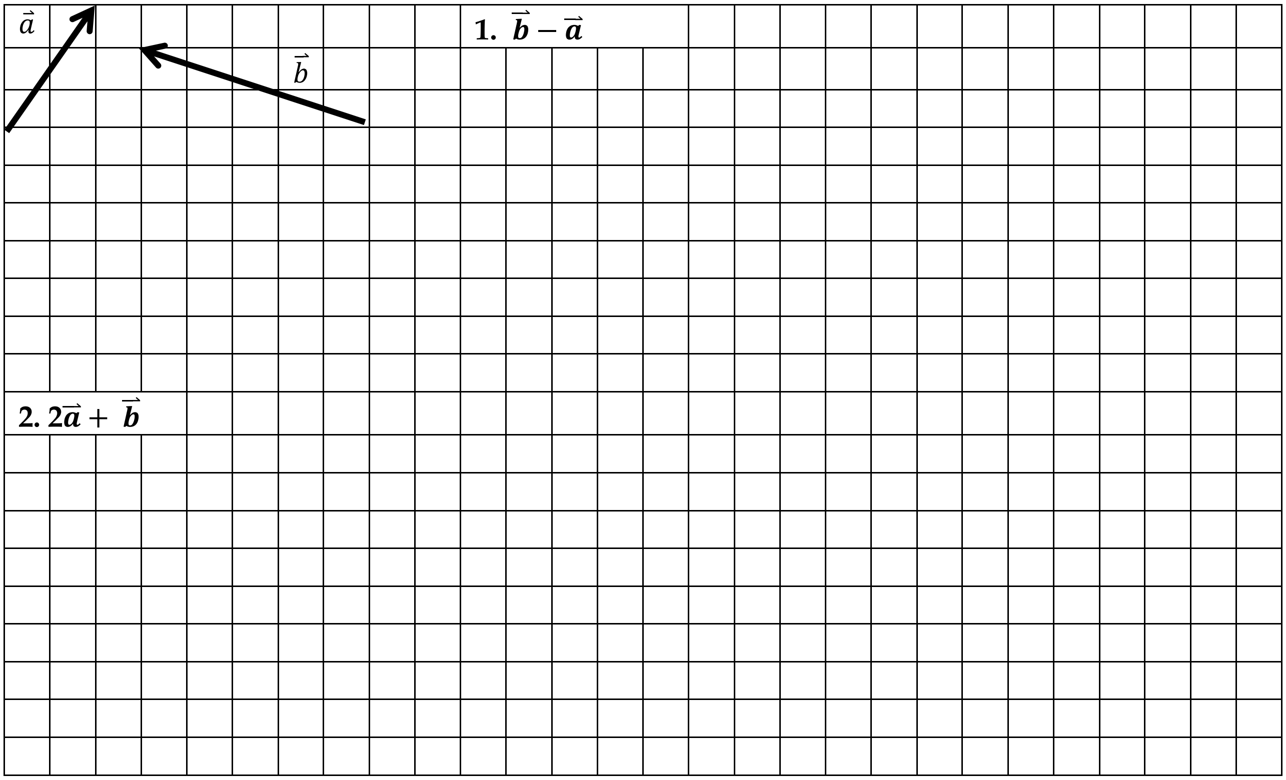
22. $\mathbf{u} = \langle 6, 0 \rangle$, $\mathbf{v} = \langle -10, 8 \rangle$

23. $\mathbf{u} = -10\mathbf{i} + \mathbf{j}$, $\mathbf{v} = 10\mathbf{i} - 5\mathbf{j}$

24. **CAMPING** Regina and Luis set off from their campsite to search for firewood. The path that Regina takes can be represented by $\mathbf{u} = \langle 3, -5 \rangle$. The path that Luis takes can be represented by $\mathbf{v} = \langle -7, 6 \rangle$. Find the angle between the pair of vectors. (Example 3)

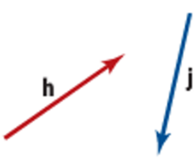
6.08 2D Vector Quiz Review

Draw each and label the resultant. Then find the component form and magnitude of each resultant.

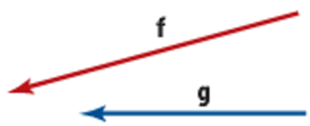


Draw the vector diagram and the resultant vector. Use either the triangle or parallelogram method.

3. $\vec{h} + \vec{j}$



4. $\vec{f} - 2\vec{g}$



Use each set of points to create a vector with initial point & terminal point in *alphabetical order*. For each, find the component form, write as a sum of unit vectors, and find the magnitude and direction.

5. A $(-3, 5)$ and B $(5, -1)$

6. $C(10, -6)$ and $D(-8, 2)$

7. $S(1,12)$ and $T(-2, -9)$

Given: $\vec{u} = <5, -12>$, $\vec{v} = i - 2j$ and $\vec{w} = <9, 3>$

8. a) Find: $-\vec{u} - \frac{1}{3}\vec{w}$ b) $2\vec{v} - \vec{u}$

c) Are any of the vectors orthogonal? Show work.

d) If the vectors are not orthogonal, use $\cos\theta = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|}$ to find the angle between each pair of vectors.

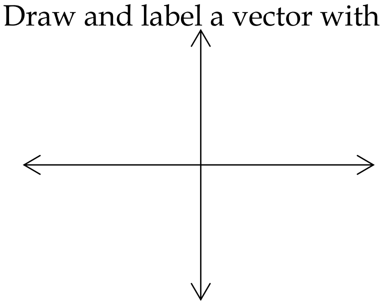
Find the component form of $\vec{v}$ given the following.

9. $|\vec{v}| = 4, \theta = 135°$ 10. $|\vec{v}| = 6, \theta = 240°$ 11. $|\vec{v}| = 15, \theta = 330°$

12. A boat is traveling west at 25 mph. The current is moving south at 3 mph. What is the boat's resultant speed? What is the direction of the boat's movement?

13. Alvin pulls a sled through the snow with a force of 50 newtons at an angle of 35° with the horizontal. Find the magnitude of the force.

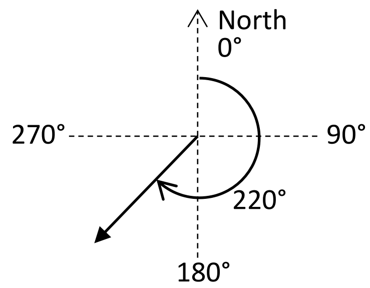
14. Draw and label a vector with magnitude of 15 meters per second at a direction of 230°.



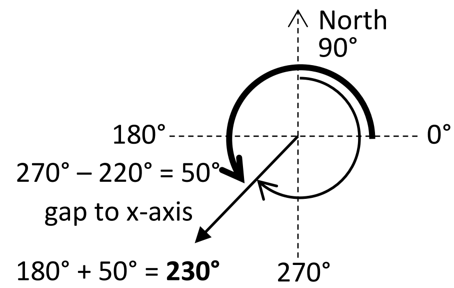
6.10 Bearing and Directional Bearing

<u>Bearing</u> directions are angles given as rotations measured clockwise from North.

Example: bearing of 220°

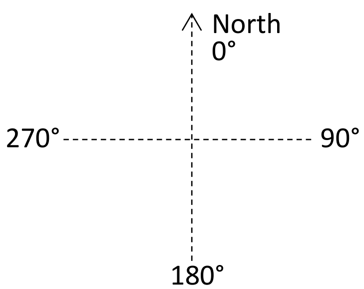


To convert to a measurement in standard position, determine the size of the angle measured from East (remember, quadrants are 90°)



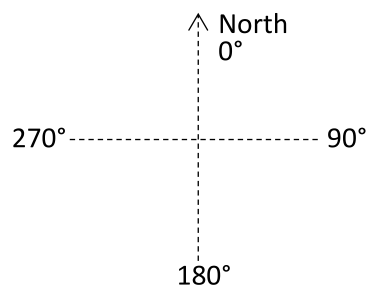
Directions: Sketch the angle given in bearing. Then, determine the angle measured in standard position that is coterminal with the given angle.

1. bearing of 160°



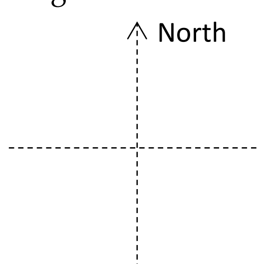
Standard Position: ____________

2. Bearing of 335°



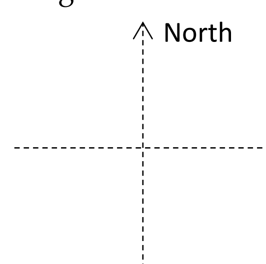
Standard Position: ____________

3. bearing of 212°



Standard Position: ____________

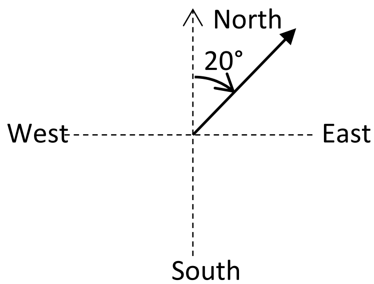
4. Bearing of 241°



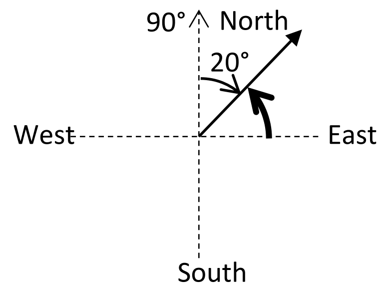
Standard Position: ____________

<u>Directional bearing</u> (also called Quadrant Bearing) directions are angles given as a starting direction (either North or South) and then an acute angle of rotation measured toward the second direction (either East or West).

Example: bearing of N 20° E

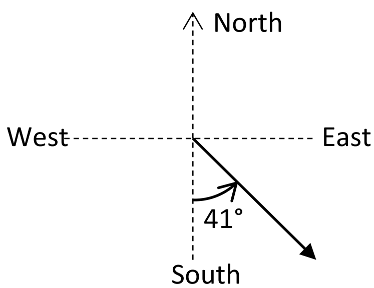


To convert to a measurement in standard position, determine the size of the angle measured from East (remember, quadrants are 90°)

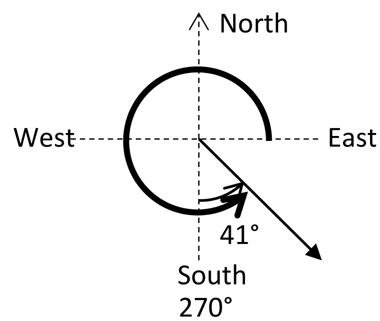


Direction occurs 20° *before* 90° quadrantal
90° – 20° = **70°**

Example: bearing of S 41° E



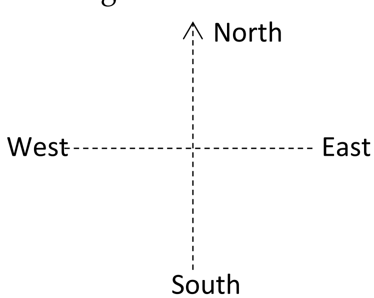
To convert to a measurement in standard position, determine the size of the angle measured from East (remember, quadrants are 90°)



Direction occurs 41° *after* 270° quadrantal
270° + 41° = **311°**

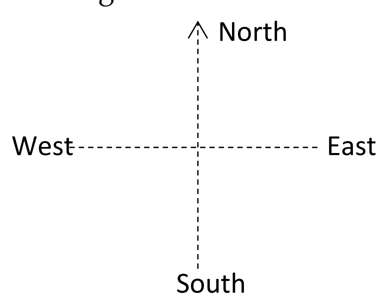
Directions: Sketch the angle given in bearing. Then, determine the angle measured in standard position that is coterminal with the given angle.

5. bearing of N 60° W



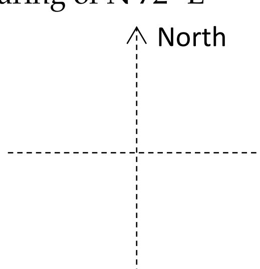
Standard Position: ____________

6. bearing of S 35° E



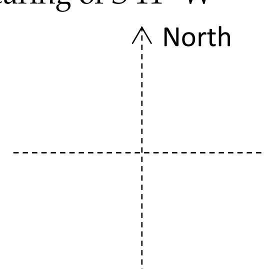
Standard Position: ____________

7. bearing of N 72° E



Standard Position: ____________

8. bearing of S 11° W



Standard Position: ____________

6.11 Applications of Vectors: Notes **Date:** ____________

Formulas:

Ex 1: Train A and Train B depart from the same station. The path that train A takes can be represented by $\langle 33, 12 \rangle$. If the path that train B takes can be represented by $\langle 55, 4 \rangle$, find the angle between the pair of vectors.

Ex 2: An airplane is flying at a direction of 115° at 530 mph. Find the component form of the velocity of the airplane.

Ex 3: A captain sails a boat for 200 kilometers at a bearing of 150°. Find the component form of the velocity of the boat.

Ex 4: Jordan is riding the bus to school. The bus travels north for 4.5 miles, east for 2 miles, and then N60°E for 1.5 miles. Find the component form of the resultant.

Ex 5: An airplane is flying with an airspeed of 500 miles per hour on a heading due north. If a 50-mile per hour wind is blowing at a bearing of 280°, determine the velocity and direction of the plane relative to the ground.

6.11 Applications of Vectors Practice Day 1

1. Two clay pigeons are thrown at the same time. If the path of the clay pigeons can be represented by the vectors $\mathbf{p} = \langle 42, 58 \rangle$ and $\mathbf{c} = \langle 59, 73 \rangle$, what is the measure of the angle between the clay pigeons?

2. A hiker is walking a trail at 2.5 miles per hour at a bearing of N50°W. Find the component form of the velocity of the hiker.

3. An airplane is traveling 300 kilometers per hour due east. A wind is blowing 35 kilometers per hour at an angle of 255°. A) What is the resulting speed of the airplane? B) What is the direction of the plane?

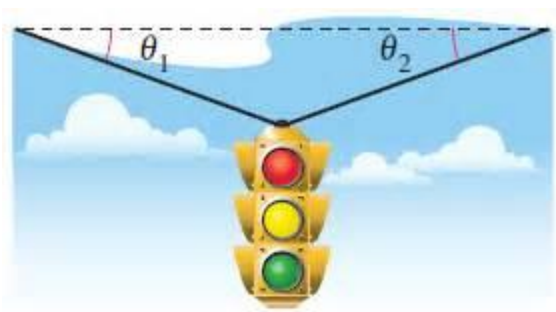
4. A helicopter is moving at a bearing of 105° with a velocity of 52 km/h. If a 30-kilometer per hour wind is blowing at S25°E, find the helicopter's resulting velocity and direction.

5. Meredith is skateboarding along a path at a bearing of 70° for 35 meters. She then changes paths and travels for 45 meters along path at a bearing of 60°. A) Find the resulting distance, and B) the direction (bearing) of her path.

6.12 Apps of Vectors Day 2: Notes **Date:** ____________

Ex 1: To reach a destination, a pilot is plotting a course that will result in a velocity of 450 miles per hour at an angle of N60°W. The wind is blowing 50 miles per hour to the north. Find the direction and speed the pilot should set to achieve the desired resultant.

Ex 2: A traffic light at an intersection is hanging from two wires of equal length at 15° below the horizontal as shown. If the traffic light weighs 560 pounds, what is the tension in each wire keeping the light at equilibrium?



6.12 Applications of Vectors Practice Day 2

1. Anne and Mike are lifting a stone statue and moving it to a new location in their garden. Anne is pushing the statue with a force of 120 newtons at a 60° angle while Henry is pulling the statue with a force of 180 newtons at a 40° angle. What is the magnitude of the combined force they exert on the statue?

2. Dr. Smith is hanging a sign for her medical practice that will be held by two support bars. If the bars make a 60° angle with each other and the sign weighs 100 pounds, what are the magnitudes of the forces exerted by the sign of each support bar?

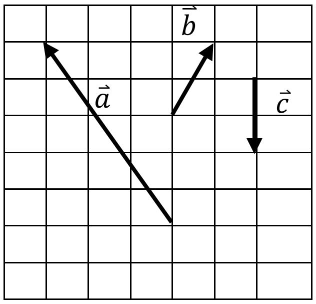
3. A person in a canoe wants to cross a 65-foot-wide river. He begins to paddle straight across the river at 1.2 m/s while a current is flowing perpendicular to the canoe. If the resulting velocity of the canoe is 3.2 m/s, what is the speed of the current to the nearest tenth?

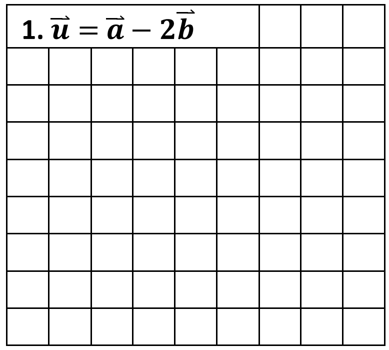
4. Kristin walks N70^{o}W for 200 meters. She then walks due east for 90 meters. How far and at what bearing is Kristin from her starting point?

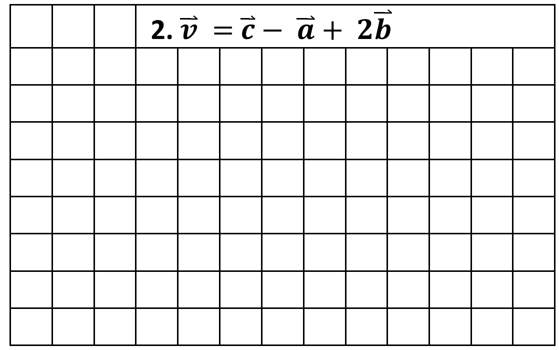
5. A pilot needs to plot a course that will result in a velocity of 500 miles per hour in a direction of due west. If the wind is blowing 100 miles per hour from 192^{o}, find the direction and the speed the pilot should set to achieve this resultant.

6.13 Test Review - Vectors **Date:** ___________________

Given the vectors below, draw and label the given resultant vectors.







3. a) Give the **<u>component</u>** form of $\vec{u}$ **:** _____________________ b) Find $|\vec{v}|$ **:** __________________________

4. Given points A(4,-5) and B(1,-3):

a) write $\overrightarrow{AB}$ in component form a) _________________

b) calculate $|\overrightarrow{AB}|$ b) _________________

c) write as the sum of unit vectors c) _________________

5. Give an example of 2 vectors, < ___, ___ > and < ___, ___ > that are *not* perpendicular. Show why they are not.

6. Given: $\vec{a} = < -3, 6 >, \vec{b} = 5\vec{i} - 2\vec{j}, \vec{c} = < 2, 3 >$, write $\vec{v} = 2\vec{b} - \vec{c} + 3\vec{a}$ as the sum of unit vectors.

6. __________________

7. A ship leaves port and sails for 40 miles in a direction N55°E. Find the component form (ordered pair) of the distance the ship sails.

7. __________________

8. Two hot air balloons take off at a spring festival. After about twenty minutes the path of the first balloon can be represented by $\langle 55, 81 \rangle$. If the path of the second balloon can be represented by $\langle 62, 77 \rangle$, find the angle between the vectors. Use: $\cos\theta = \frac{\vec{v} \cdot \vec{w}}{|\vec{v}|\,|\vec{w}|}$

9. A batter on the opposing softball team hits a ground ball that rolls out to left field. The left fielder runs toward the ball at a velocity of 3 meters per second, scoops it, and proceeds to throw it to the catcher at a speed of 30 meters per second and at an angle of 25° with the horizontal in an effort to throw out the runner. What is the resultant speed and direction of the throw?

10. A corporate jet is flying at a bearing of 320° at 425 mph. If a 40 mph wind blows at a bearing of 290°, find the bearing and speed that the pilot must use to maintain the jet's former course.

11. The lighting system for Milton theater is supported equally by two cables suspended from the ceiling of the auditorium. The cables form a 140° angle with each other. If the lighting system weighs 950 pounds, what is the force exerted by each of the cables on the lighting system?

6.14 More Vector Review

Date: ___________________

Given: X (-2, 8) and Y (-5, 12)

1. Find the component form of $\overrightarrow{XY}$.

2. Find the direction, in standard position, of $\overrightarrow{XY}$.

3. Write $\overrightarrow{YX}$ as the sum of unit vectors.

4. Find the magnitude of $\overrightarrow{YX}$.

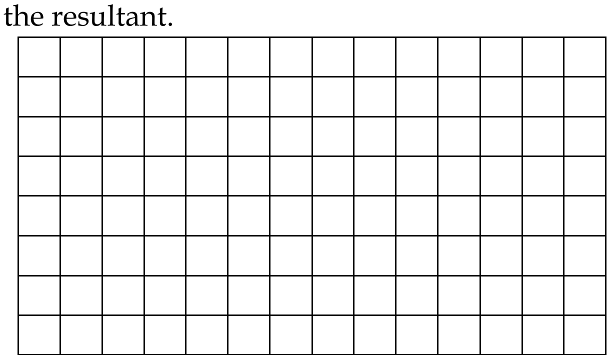
Given: $\vec{u}$ = <-5, -1>, $\vec{v}$ = <4, -2>

5. Find the angle between $\vec{u}$ and $\vec{v}$.

6. Find the magnitude and direction of $\vec{u}$.

7. Find $\vec{m} = \vec{u} - \frac{1}{2}\vec{v}$. Show work algebraically.

8. Draw the vector diagram for $\vec{m} = \vec{u} - \frac{1}{2}\vec{v}$ and label the resultant.



9. Find the magnitude and direction of $\vec{m} = \vec{u} - \frac{1}{2}\vec{v}$.

10. Write $\vec{m}$ as the sum of unit vectors.

11. a) A pilot directs his plane to fly on a bearing of 210° at 460 mph. State the measurement of the angle for the direction in standard position. Find the component form of the velocity of the airplane.

b) An 80 mph wind blowing in the direction of S15°E is pushing the plane off course. Find the ground speed and direction of the airplane (resultant path).

12. A new score board is being place in the gym. It will be supported from the ceiling by 2 cables. Each cable can withstand 150 pounds of tension. If the score board weighs 250 pounds, what is the largest possible measurement for the angle that the two cables make with each other?

13. A pilot needs to plot a course that will result in a velocity of 520 miles per hour in a direction of S32°W. If the wind is blowing 65 miles per hour in the direction of 12°, find the direction and the speed the pilot should set to achieve this resultant.

Accelerated Pre-Calculus
January & February 2022
Unit 6 – Vectors

Monday	Tuesday	Wednesday	Thursday	Friday
				28 6.01 Review Right Triangle Trig HW: 6.01 Practice
31 6.02 Introduction to Geometric Vectors • Geometric Vectors HW: 6.02 Practice	Feb 1 6.03 More Geometric Vectors • Vector Game • Direction HW: 6.03 Practice	2 6.04 Geometric and Algebraic Vectors Harry Potter Task HW: Finish 6.04 Task	3 6.05 Algebraic Vectors • Component Form • Operations • Unit Vectors HW: 6.05 Practice	4 6.06 More Algebraic Vectors • Magnitude • Direction HW 6.06 Practice
7 6.07 Angle with Vectors • Angle Between Vectors • Orthogonal Vectors • Parallel Vectors HW: 6.07 Practice	8 6.08 Quiz Review HW: Study for Quiz	9 **6.09 Quiz** **Vector Operations**	10 6.10 Bearings HW: Finish 6.10	11 6.11 Vector Applications Day 1 HW: 6.11 Application Practice Worksheet
14 6.12 Vector Applications Day 2 HW: 6.12 Application Day 2 Practice Wkst and Test Review	15 6.13 Test Review HW: 6.13 Test Review Worksheet	16 Test Review	17 **TEST: Vectors in 2 dimensions**	18

Vectors

2D Vectors: $\vec{u} = \langle a_1, b_1 \rangle$ and $\vec{v} = \langle a_2, b_2 \rangle$

1. **Component form** shows the vector from the *initial point* to the *terminal point* based on the displacement of its dimensional values:
   - 2D vector, from (x_1, y_1) to (x_2, y_2): $\vec{v} = \langle x_2 - x_1, y_2 - y_1 \rangle$

2. **Unit vector** is a vector of length 1. The standard unit vectors are $\vec{\imath} = \langle 1,0 \rangle$ and $\vec{\jmath} = \langle 0,1 \rangle$. A vector can be written as the *sum of unit vectors* by using its components as scalars of standard unit vectors:
   - 2D vector: $\vec{v} = a\vec{\imath} + b\vec{\jmath}$

3. **Magnitude** (length) of a vector:
   - 2D vector: $|\vec{v}| = \sqrt{a^2 + b^2}$

4. **Direction** of a vector:
   - 2D vector: $\theta = \tan^{-1}\left(\frac{b}{a}\right)$, add 180° if in quadrants 2 or 3.

5. Given the **magnitude** and the **direction** of a vector, it is possible to determine its components:
   - 2D vector with magnitude $|\vec{v}|$ and direction θ, $\vec{v} = |\vec{v}| \langle \cos\theta, \sin\theta \rangle = \langle |\vec{v}| \cos\theta, |\vec{v}| \sin\theta \rangle$

6. **Resultant vector** is the sum of two or more vectors.
   - Geometrically, this is shown with the *tip-to-tail* method, also known as the *triangle* method. The *parallelogram* method also can determine the resultant vector.
   - Algebraically, this is calculated by finding the sum of the corresponding components.
   - 2D vectors: $\vec{u} + \vec{v} = \langle a_1 + a_2, \ b_1 + b_2 \rangle$

7. **Scalar multiplication**:
   - 2D vector: $k\vec{v} = \langle ka, kb \rangle$

8. **Dot product** (inner product) is used to determine if two vectors are perpendicular:
   - 2D vectors: $\vec{u} \cdot \vec{v} = a_1 a_2 + b_1 b_2$
   - For magnitude: $|\vec{v}| = \sqrt{\vec{v} \cdot \vec{v}}$
   - 2 vectors are orthogonal (perpendicular) if their dot product equals 0.

9. **Angle between two vectors** can be found with a dot product:
   - 2D vectors: $\cos\theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}|\,|\vec{v}|}$

10. **Angles** have different ways of being measured:
    - **Standard Position** is measured from the positive x-axis, with positive angles opening counter-clockwise.
    - **True Bearing** or **Compass Bearing** is measured from North, with positive angles opening clockwise. True bearing measurement = 450° − Standard position measurement
      Standard position measurement = 450° − True bearing measurement
    - **Quadrant Bearing** is measured either from North or from South, opening toward East or toward West in such a way that the angle value is always acute.