Unit 9 Test Review WS #3 - Statistics

Key

6.10 Apartment rental rates. You want to rent an unfurnished one-bedroom apartment for next semester. The mean monthly rent for a random sample of 10 apartments advertised in the local newspaper is $540. Assume that the standard deviation is $80. Find a 95% confidence interval for the mean monthly rent for unfurnished one-bedroom apartments available for rent in this community.

$\bar{x} = 540$

$n = 10$

$\sigma = 80$

$\frac{1-c\%}{2}$

$\frac{1-0.95}{2}$

$0.025 \to z = -1.96$

$CI = 540 \pm 1.96\left(\frac{80}{\sqrt{10}}\right)$

$CI = 540 \pm 49.585$

$(490.41, 589.58)$

Confidence Intervals:

$\hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

$\bar{x} \pm z\frac{\sigma}{\sqrt{n}}$

Compare the margin of error for intervals with 90, 95, and 99% confidence:

90%

$\frac{1-0.9}{2} \to 0.05$

$z = -1.64$

$M.E. = -1.64\left(\frac{80}{\sqrt{10}}\right)$

$M.E. = 41.489$

95%

$M.E. = 49.585$

99%

$\frac{1-0.99}{2} = 0.005$

$z = -2.57$

$M.E = 2.57\left(\frac{80}{\sqrt{10}}\right) = 65.0164$

*Margin of Error increases with higher confidence percentage.

Suppose you desire a 90% confidence interval with a *width* of no more than $50. What sample size is needed?

$M.E. = 50$

$90\% \to 0.05 \to$

$z = -1.64$

$M.E. = z\left(\frac{\sigma}{\sqrt{n}}\right)$

$50 = 1.64\left(\frac{80}{\sqrt{n}}\right)$

$50\sqrt{n} = 131.2$

$\sqrt{n} = 2.624$

$n = 6.88$

$\boxed{n \geq 7}$

2)

A U.S. Coast Guard survey of 300 small boats in the Cape Cod area found 120 in violation of one or more safety regulations. Give a 99.8% confidence estimate for p, the proportion of all unsafe small boats.

$n = 300$

$\hat{p} = \frac{120}{300} = 0.4$

$\frac{1-0.998}{2} = 0.001$

$z = 3.09$

$CI = 0.4 \pm 3.09\sqrt{\frac{0.4(1-0.4)}{300}}$

$CI = 0.4 \pm 0.0874$

$\boxed{(0.3126, 0.4874)}$

3) The birth process of a newly discovered mammal is being studied, and the lengths of 18 observed pregnancies have been recorded. The mean gestation period was 97.3 days with s = 2.2 days. Find a 95% confidence interval for the mean time of pregnancy for this mammal.

$\bar{x} = 97.3$

$\sigma = 2.2$

$n = 18$

$\frac{1-c\%}{2} \to \frac{1-0.95}{2} \to 0.025$

$z = -1.96$

$CI = 97.3 \pm 1.96\left(\frac{2.2}{\sqrt{18}}\right)$

$CI = 97.3 \pm 1.0163$

$\boxed{(96.284, 98.316)}$

4) The *New York Times* and CBS News conducted a nationwide poll of 1048 randomly selected 13- to 17-year-olds. We can consider the sample to be a SRS. Of these 1048 teenagers, 692 had a television in their room. Give a 95% confidence interval for the proportion of all people in this age group who had a TV in their room at the time of the poll

$n = 1048$

$\hat{p} = \frac{692}{1048} = 0.6603$

$\frac{1-c\%}{2} \rightarrow \frac{1-0.95}{2} \rightarrow z = 1.96$

$\hat{p} - z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

$CI = 0.6603 \pm 1.96\sqrt{\frac{0.6603(1-0.6603)}{1048}}$

$CI = 0.6603 \pm 0.0287$

$\boxed{(0.6316, 0.689)}$

5)

Find n: A researcher wants to determine the 99% confidence interval for the mean number if hours per week that adults spend doing community service. How large of a sample should the researcher select so that the estimate will be within 1 hour of the population mean? Assume that the standard deviation for hours spent per week by adults doing community service is 3.

$\frac{1-c\%}{2} \rightarrow \frac{1-0.99}{2} \rightarrow 0.005 \rightarrow z = -2.57$

$n = \_?$

$M.E. = 1$

$\sigma = 3$

$M.E. = z\left(\frac{\sigma}{\sqrt{n}}\right)$

$1 = -2.57\left(\frac{3}{\sqrt{n}}\right)$

$\frac{1}{1} = \frac{-2.57(3)}{\sqrt{n}}$

$\sqrt{n} = -7.71$

$n = 59.4441$

$\boxed{n = 60}$