Name ______________________________ Period ____________

Accelerated Pre-Calculus

Unit 1 Packet

Introduction to Trigonometry

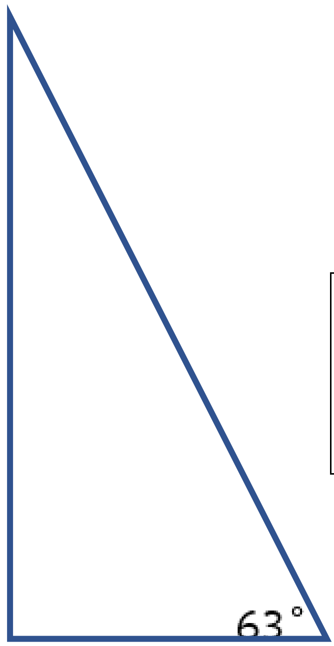
Name: ____________________

Accel Pre-Calculus Trig Ratio Investigation

Working with a partner, measure the side lengths of the following triangles (in cm) then find the sine, cosine, and tangent of theta for each triangle (remember SOHCAHTOA). Discuss the results with your partner.

1.

a.

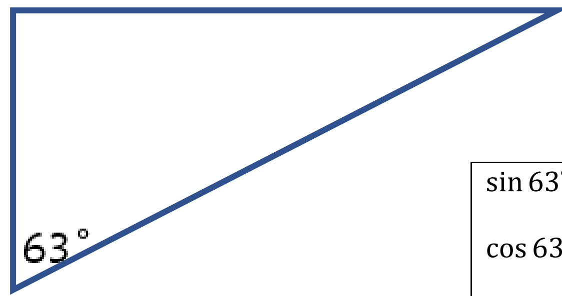


$\sin 63^\circ =$

$\cos 63^\circ =$

$\tan 63^\circ =$

b.

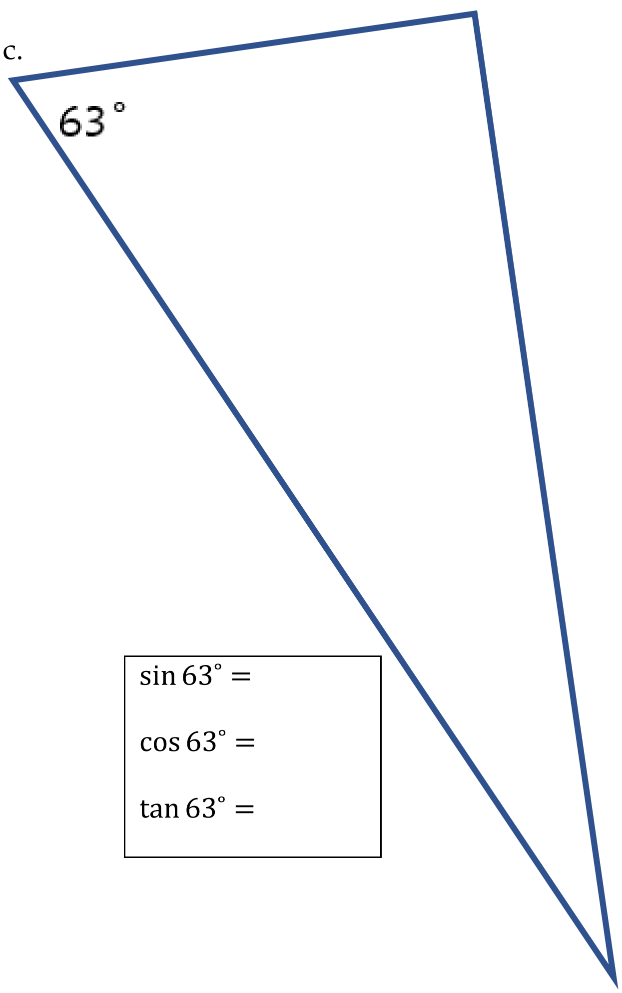


$\sin 63^\circ =$

$\cos 63^\circ =$

$\tan 63^\circ =$

c.

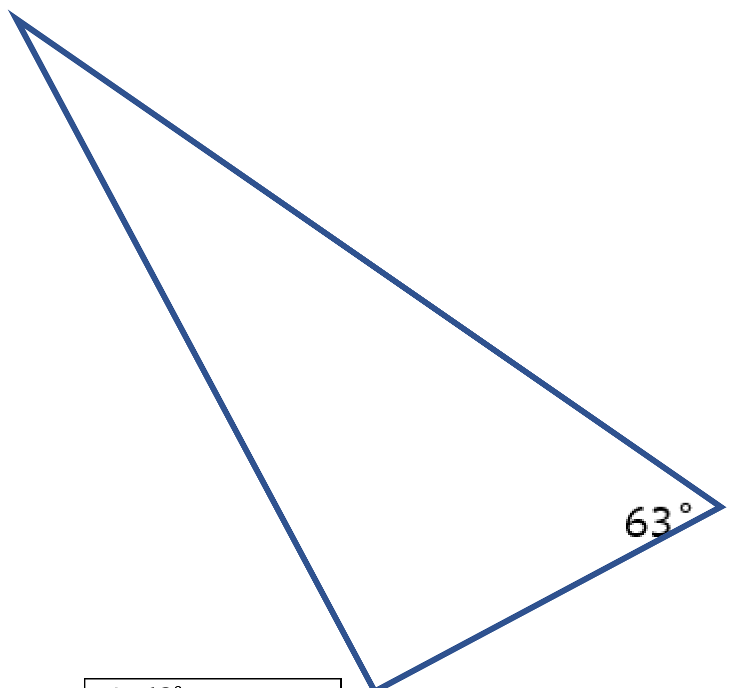


$\sin 63^\circ =$

$\cos 63^\circ =$

$\tan 63^\circ =$

d.



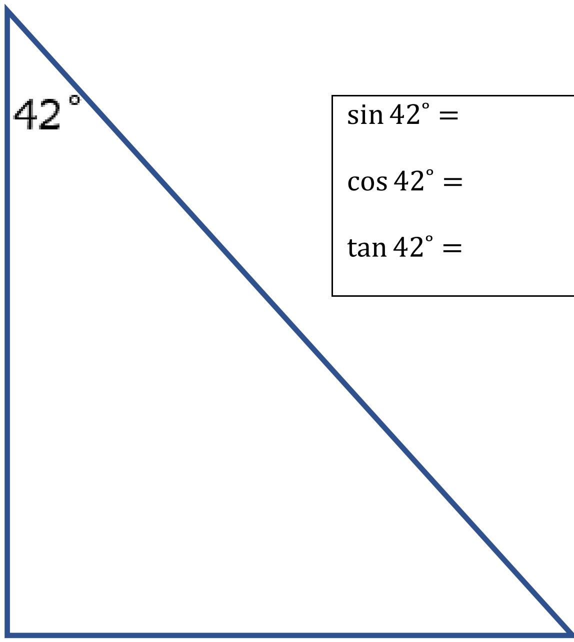
$\sin 63^\circ =$

$\cos 63^\circ =$

$\tan 63^\circ =$

2.

a.

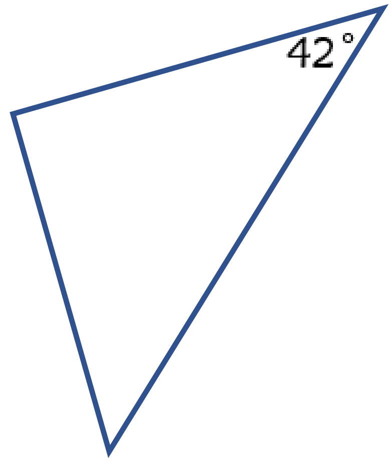


sin 42° =

cos 42° =

tan 42° =

b.

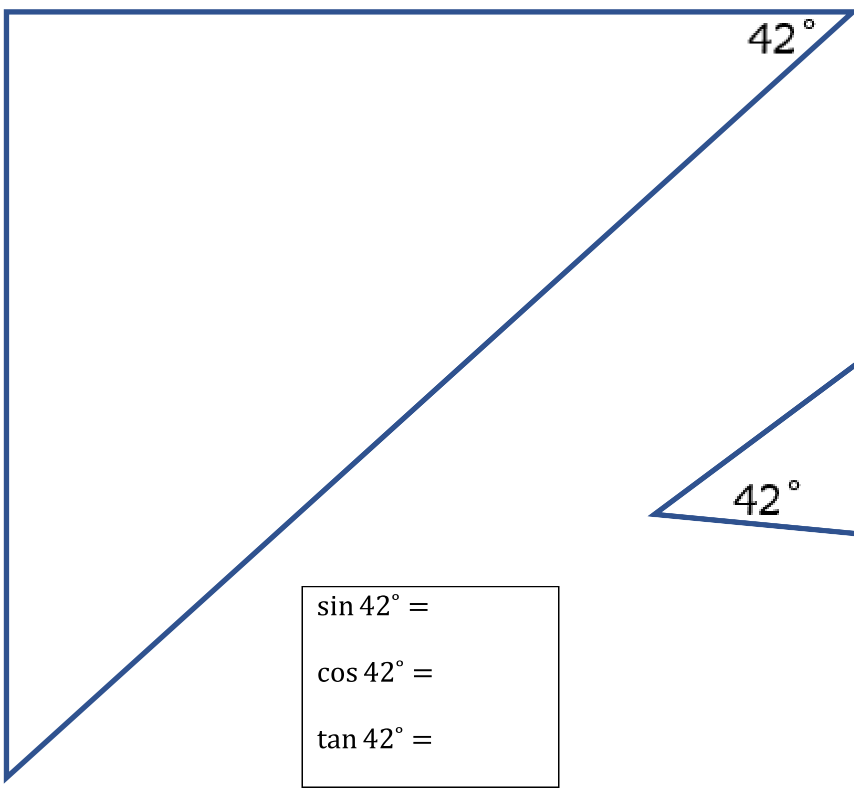


sin 42° =

cos 42° =

tan 42° =

c.

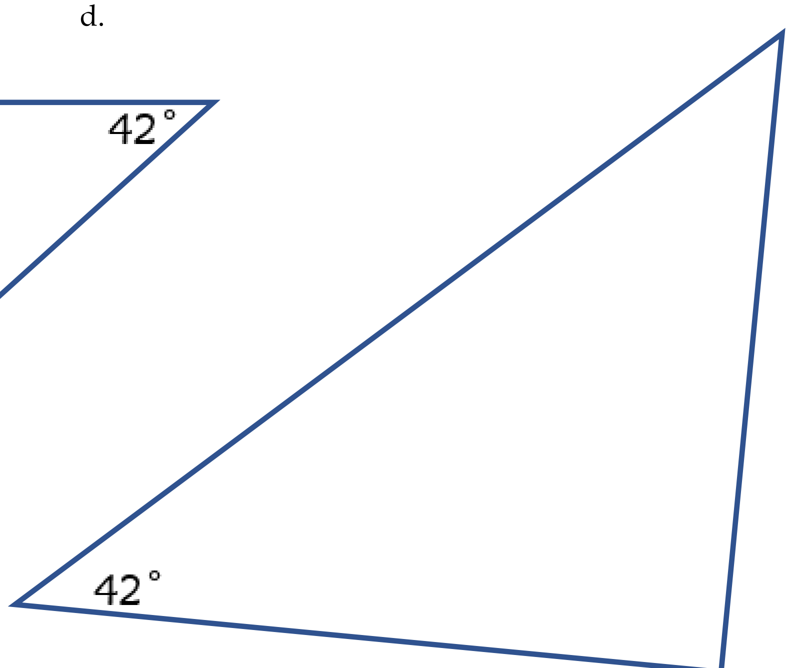


sin 42° =

cos 42° =

tan 42° =

d.



sin 42° =

cos 42° =

tan 42° =

Accel Pre-Calculus
1.02 Right Triangle Trig Notes

Name: ______________________________

Warm-up:

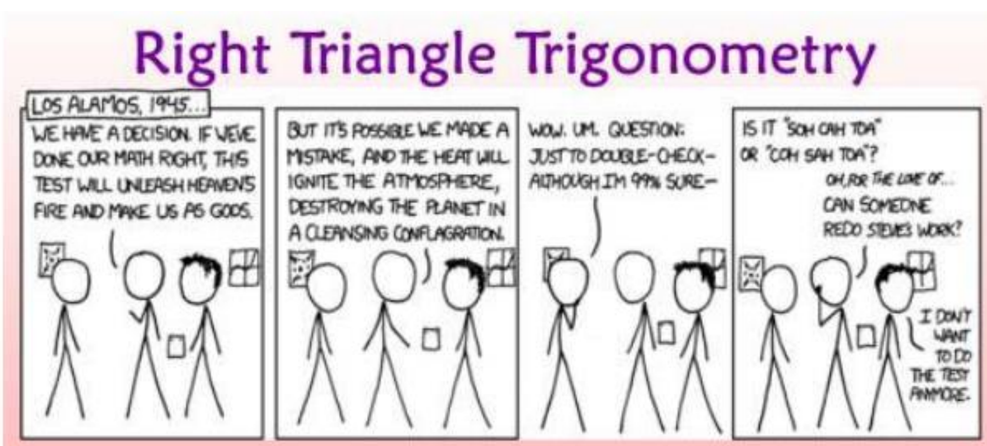
1. Rationalize $\dfrac{2}{\sqrt{5}}$

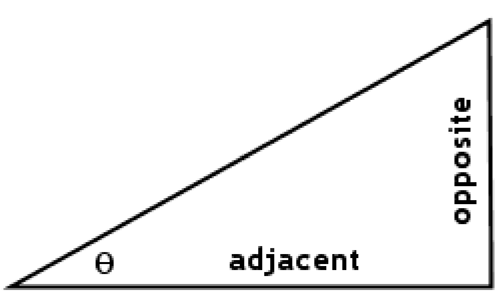
2. Solve for x.

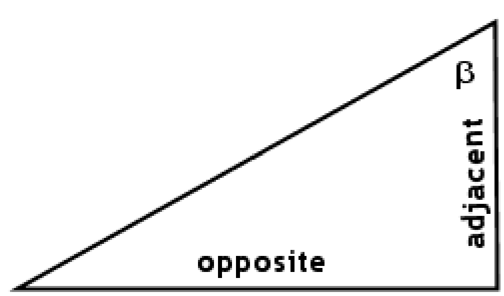
$$12 = \frac{x}{3} \qquad\qquad 20 = \frac{5}{x}$$

3. How do you solve for x?

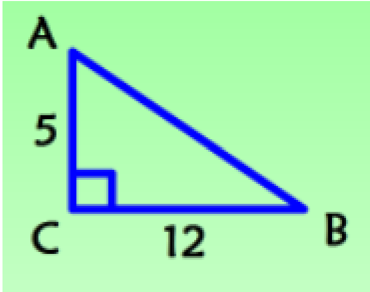
$$x + 7 = 19 \qquad\qquad x^2 = 30$$

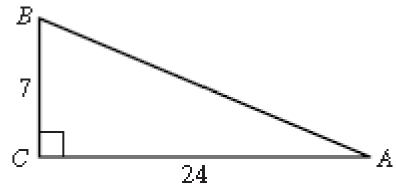




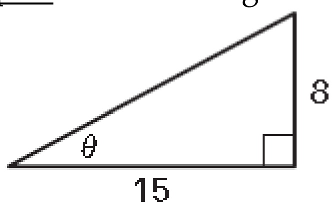


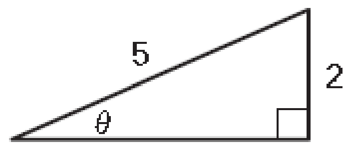
Examples: Find sine, cosine, and tangent of Angle A.

1. 

2. 

Examples: Find all 6 trig ratios from Angle A.

1. 

2. 

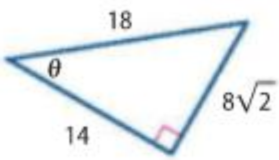
Example: Given $\cot\theta = \frac{13}{9}$, find the other 5 trig ratios from θ.

Accel Pre-Calculus Date: ______________________

1.02 Practice Right Triangle Trig Ratios

Find the exact values of the six trigonometric functions of θ. (Example 1)

1.



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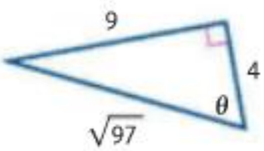

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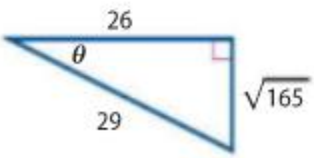

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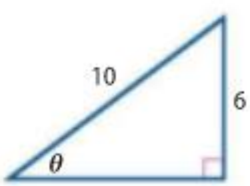


Use the given trigonometric function value of the acute angle θ to find the exact values of the five remaining trigonometric function values of θ. (Example 2)

9. $\sin\theta = \frac{4}{5}$

11. $\tan\theta = 3$

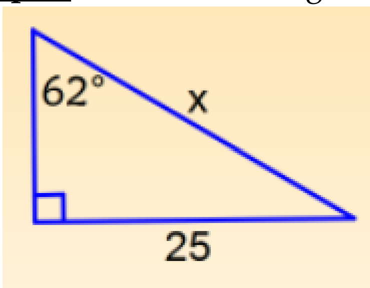
13. $\cos\theta = \frac{5}{9}$

Accel Pre-Calculus Date: ______________________________

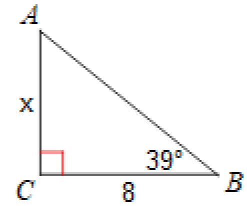
1.03 Solving Right Triangles Notes

Examples: Find the missing side length using trigonometry (solve for x).

1.

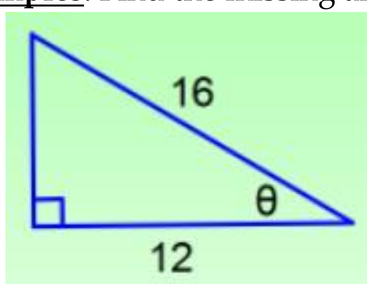


2.

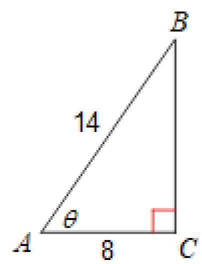


Examples: Find the missing angle measure using trigonometry (solve for θ).

3.

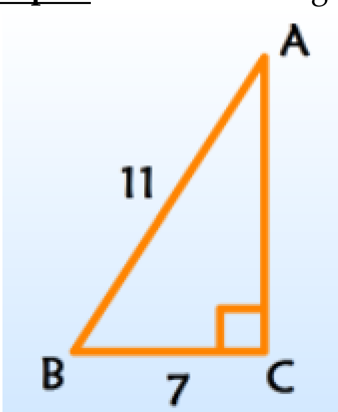


4.



Examples: Solve the triangle (find all side lengths and all angle measures).

5.



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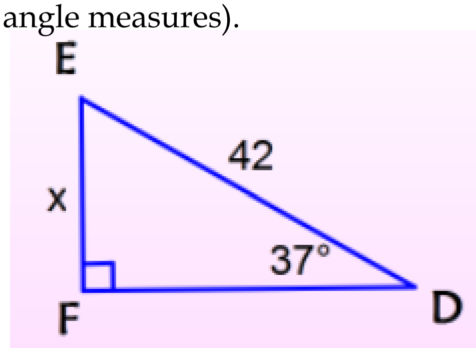
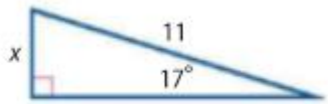


Find the value of x. Round to the nearest tenth, if necessary. (Example 3)

19.



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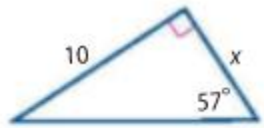

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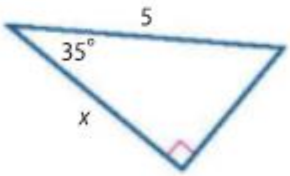

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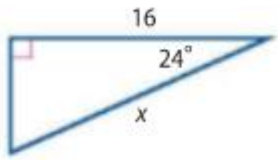

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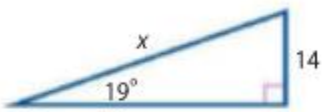

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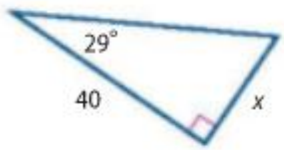

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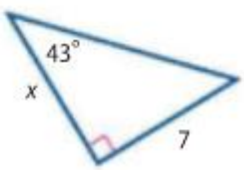

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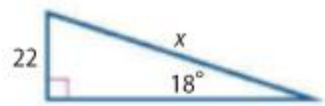
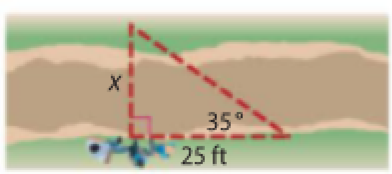


27 **MOUNTAIN CLIMBING** A team of climbers must determine the width of a ravine in order to set up equipment to cross it. If the climbers walk 25 feet along the ravine from their crossing point, and sight the crossing point on the far side of the ravine to be at a 35° angle, how wide is the ravine? (Example 4)



28. **SNOWBOARDING** Brad built a snowboarding ramp with a height of 3.5 feet and an 18° incline. (Example 4)

a. Draw a diagram to represent the situation.

b. Determine the length of the ramp.

29. **DETOUR** Traffic is detoured from Elwood Ave., left 0.8 mile on Maple St., and then right on Oak St., which intersects Elwood Ave. at a 32° angle. (Example 4)

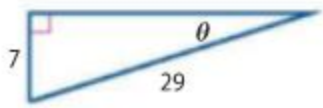
a. Draw a diagram to represent the situation.

b. Determine the length of Elwood Ave. that is detoured.

30. **PARACHUTING** A paratrooper encounters stronger winds than anticipated while parachuting from 1350 feet, causing him to drift at an 8° angle. How far from the drop zone will the paratrooper land? (Example 4)

Find the measure of angle θ. Round to the nearest degree, if necessary. (Example 5)

31.



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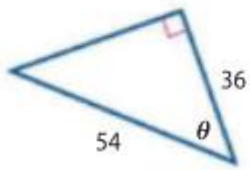

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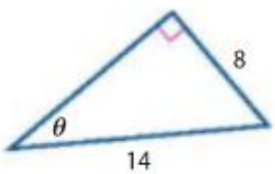

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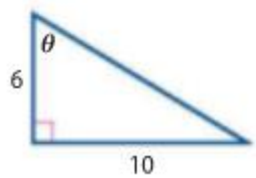
35.



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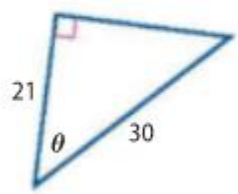

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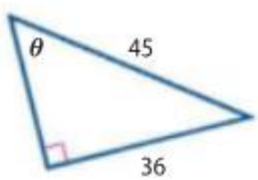

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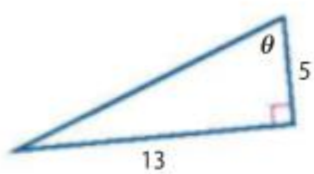
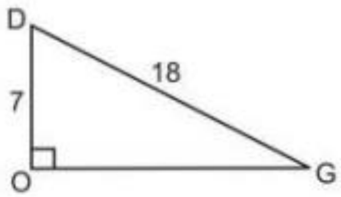
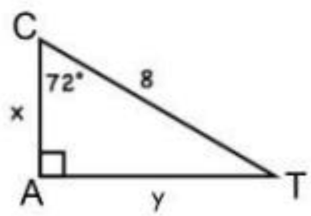


A Quick Rewind:

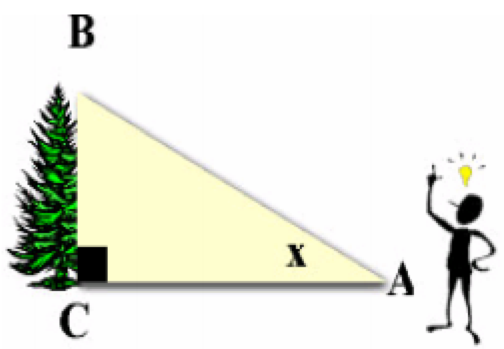
1. Solve the triangle.



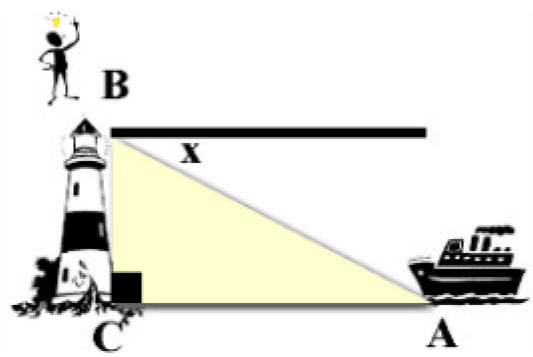
2. Solve the triangle.



Trigonometric ratios have many practical real-world examples. Angles of elevation and depression are formed by the horizontal lines that a person's lines of sight to an object form. If a person is looking up, the angle is an elevation angle. If a person is looking down, the angle is a depression angle.



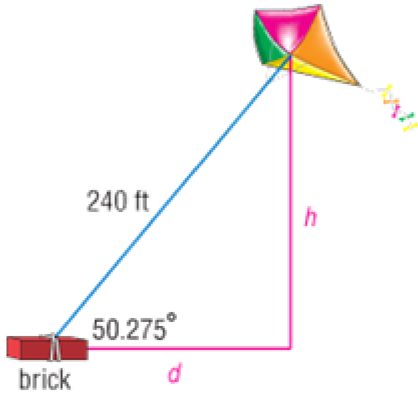
x = angle of elevation from ground to top of tree



x = angle of depression from lighthouse to boat

Example #3: A plane is coming in for a landing with an angle of depression of 32°. The plane is currently 3200 feet in the air. How far does the plane have to travel before it hits the runway?

Example #4: A child holding on to the string of a kite gets tired and decides to put the string on the ground and secure it with a brick. The length of the string from the brick to the kite is 240 feet.



a. If the angle formed by the string and the ground is $50.275°$, how high is the kite?

b. What is the horizontal distance between the kite and the brick?

Example #5: A submersible traveling at a depth of 250 feet dives at an angle of $15°$ with respect to a line parallel to the water's surface. It travels a horizontal distance of 1500 feet during the dive. What is the depth of the submersible after the dive?

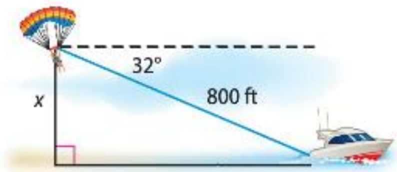
Example #6: The steepest railway in the world is the Katoomba Scenic Railway in Australia. The passenger car is pulled up the mountain by twin steel cables. It travels along the tract 1020 feet to obtain a change in altitude of 647 feet.

a. Find the angle of elevation of the railway.

b. How far does the car travel in a horizontal direction?

1.04 Practice Trig Applications Date: _______________

39. PARASAILING Kayla decided to try parasailing. She was strapped into a parachute towed by a boat. An 800-foot line connected her parachute to the boat, which was at a 32° angle of depression below her. How high above the water was Kayla? (Example 6)

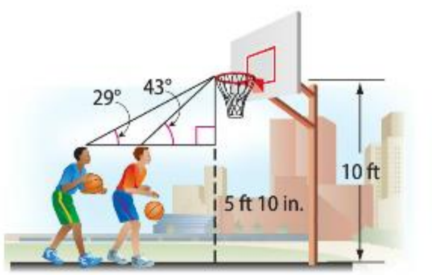


41. ROLLER COASTER On a roller coaster, 375 feet of track ascend at a 55° angle of elevation to the top before the first and highest drop. (Example 6)

a. Draw a diagram to represent the situation.

b. Determine the height of the roller coaster.

43. BASKETBALL Both Derek and Sam are 5 feet 10 inches tall. Derek looks at a 10-foot basketball goal with an angle of elevation of 29°, and Sam looks at the goal with an angle of elevation of 43°. If Sam is directly in front of Derek, how far apart are the boys standing? (Example 7)



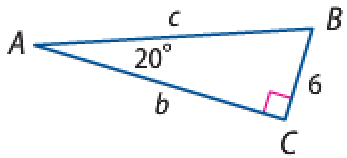
45 LIGHTHOUSE Two ships are spotted from the top of a 156-foot lighthouse. The first ship is at a 27° angle of depression, and the second ship is directly behind the first at a 7° angle of depression. (Example 7)

a. Draw a diagram to represent the situation.

b. Determine the distance between the two ships.

Solve each triangle. Round side lengths to the nearest tenth and angle measures to the nearest degree. (Example 8)

47.



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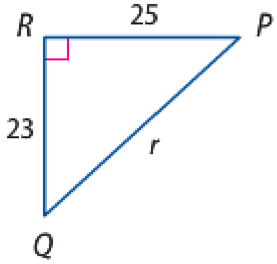


55. BASEBALL Michael's seat at a game is 65 feet behind home plate. His line of vision is 10 feet above the field.

a. Draw a diagram to represent the situation.

b. What is the angle of depression to home plate?

56. HIKING Jessica is standing 2 miles from the center of the base of Pikes Peak and looking at the summit of the mountain, which is 1.4 miles from the base.

a. Draw a diagram to represent the situation.

b. With what angle of elevation is Jessica looking at the summit of the mountain?

71. SCUBA DIVING A scuba diver located 20 feet below the surface of the water spots a shipwreck at a 70° angle of depression. After descending to a point 45 feet above the ocean floor, the diver sees the shipwreck at a 57° angle of depression. Draw a diagram to represent the situation, and determine the depth of the shipwreck.

1.05 Radian Investigation **Date: ___________________**

Materials: Circles Handout, Protractor, Ruler, 5 Twizzler strands

Part 1: Defining a Radian:

Measuring the Radii

Work on one circle at a time

Step 1: Use a Twizzler strand to measure the radius of the circle. Cut your Twizzler to that length.

Step 2: Wrap your radius Twizzler along the circle, starting at the line and in either direction.

Step 3: Make a mark on your paper where the Twizzler ends.

Repeat for the other circles. In Step 2, make sure to wrap the Twizzler strand around in the same direction as you did for the first circle.

Measuring the Angle Formed

Step 1: From the center, draw a line of best fit passing between your three points.

Step 2: Using a protractor, measure the angle that is created: _________

Step 3: Share your measurement with the class.

Definition of Radian

The angle that you submitted is measured in degrees. **Radian** is another unit that we can use to express angle measurement. More specifically, a radian is defined as

____

____

____

Part 2: Converting Between Degrees and Radians:

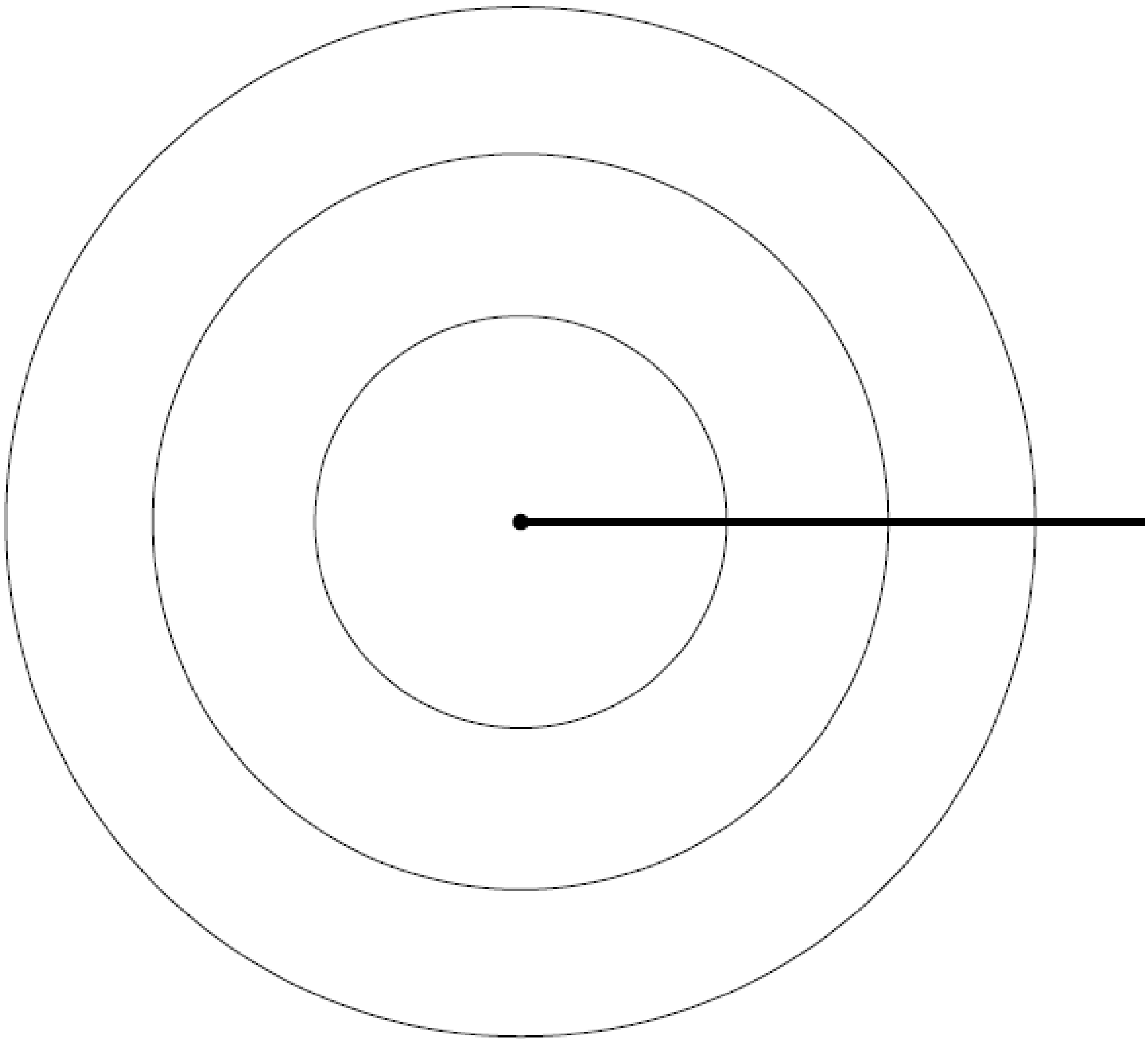
Degrees vs. Radians

So we know:

Therefore, each radian is how many degrees?

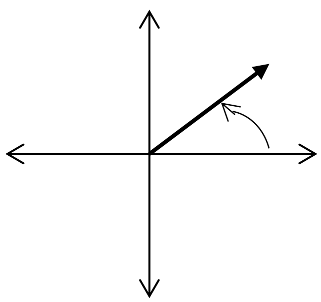
Convert from Degrees to Radians:

Convert from Radians to Degrees:

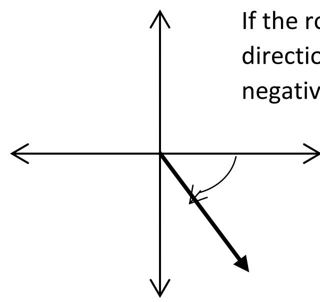


Accel Pre-Calculus
1.06 Angle Measures in Degrees and Radians

An **angle** is formed by two rays that share a fixed endpoint known as the **vertex**. One of the rays is fixed to form the **initial side** of the angle and the other ray rotates to form its **terminal side**. An angle with its vertex at the origin and its initial side along the positive x-axis is in **standard position.**

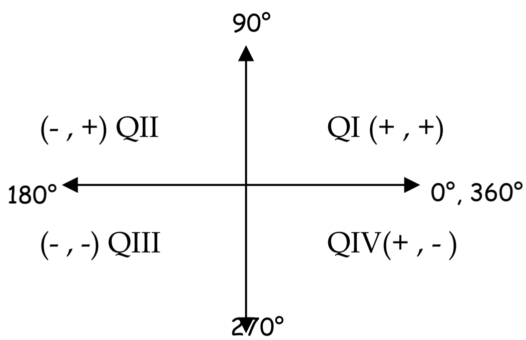


If the rotation is in a counter-clockwise direction, the angle formed is positive.



If the rotation is in a clockwise direction, the angle formed is negative.

Know the quadrants and the signs for x and y in each quadrant! That is very important in trigonometry. Degree or angle measures are read with respect to the quadrants starting at the positive x-axis (standard position) and moving in a counter-clockwise direction.



If the terminal side of an angle falls on one of the axes, the angle is a **quadrantal** angle.

There are four (4) quadrantal angles on a unit circle. They are:
0° (360°) on the x-axis, 90° on the y-axis, 180° on the –x-axis, and 270° on the –y-axis.

More Vocabulary from Geometry:

Right Angle: an angle whose measure is exactly 90°

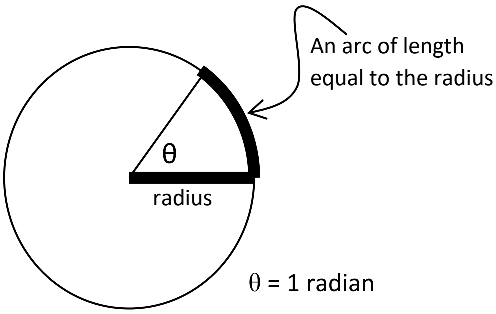
Acute Angle: an angle whose measure is less than 90°

Obtuse Angle: an angle whose measure is greater than 90°

Complementary Angles: two angles the sum of whose measure is 90°

Supplementary Angles: two angles the sum of whose measure is 180°

A new *unit of measurement* for angles: a **radian** is the measure of an angle in a circle whose intercepted arc has a length equal to the radius.
Remember the formula relating the length of a radius (r) of a circle to the circumference of the circle (C):
$\boldsymbol{C = 2\pi r}$



About the center of a circle is 360° (degrees).

Also about the center of a circle is 2π radians.

Therefore: 360° is equivalent to 2π radians

So 360° = 2π radians

Or simplified, **180° = π radians**

(This is our conversion unit!)

Degrees ➔ Radians

Convert from degrees to radians.
State the quadrant in which the angle lies.

a. 120˚ Multiply by $\frac{\pi}{180°}$ to

cancel the degrees and get radians.

Radians ➔ Degrees

Convert from radians to degrees.
State the quadrant in which the angle lies.

b. $\frac{5\pi}{6}$ radians Multiply by $\frac{180°}{\pi}$ to

cancel the radians and get degrees.

1.06 Degree & Radian Conversion Practice

Directions: Complete #10 – 17 all

Write each degree measure in radians as a multiple of π and each radian measure in degrees. (Example 2)

10. 30° **11.** 225°

12. −165° **13.** −45°

14. $\frac{2\pi}{3}$ **15.** $\frac{5\pi}{2}$

16. $-\frac{\pi}{4}$ **17.** $-\frac{7\pi}{6}$

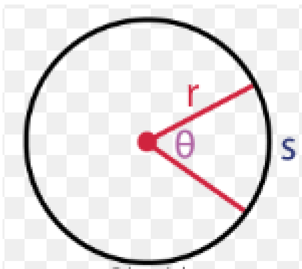
 Date: ________________________

Review:

a) Convert $315°$ to radians.

b) Convert $\frac{11\pi}{6}$ to degrees.

If θ is the measure in **radians** of a central angel in a circle with radius *r*, then the length, *s*, of the arc intercepted by θ is given by $s = r\theta$. NOTE: θ must be in radians.

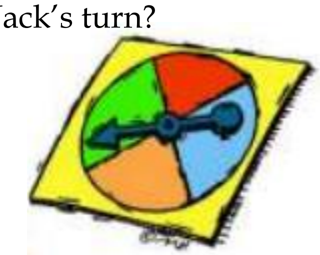


<u>Example 1</u>: Find the arc length if the radius is 8 cm and the central angle measures $\frac{3\pi}{4}$ radians.

<u>Example 2</u>: Find the arc length if the radius is 12 cm and the central angle measures $\frac{5\pi}{6}$ radians.

<u>Example 3</u>: Find the arc length if the radius is 2.5 mi and the central angle measures $300°$.

<u>Example 4</u>: While playing a game of chance, Jack flicks a spinner with a radius of 2 inches. If the spinner swings through $2665°$, how far did the arrowhead travel during Jack's turn?



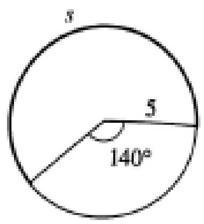
1.07 Practice- Applications with Arc Length **Date: _______________**

Directions: Complete #1,2 51-60, 63, 33

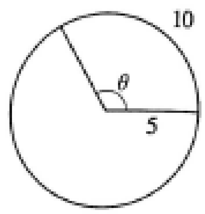
1. Find the length of an arc that subtends a central angle of 3 radians in a circle with radius 2 in.

2. Find the length of a radius of a circle if an arc of length 7 cm is subtended by an angle of 2 rad.

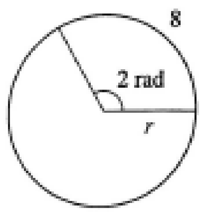
51. Find the length of the arc s in the figure.



52. Find the angle θ in the figure.



53. Find the radius r of the circle in the figure.



54. Find the length of an arc that subtends a central angle of 45° in a circle of radius 10 m.

55. Find the length of an arc that subtends a central angle of 2 rad in a circle of radius 2 mi.

56. A central angle θ in a circle of radius 5 m is subtended by an arc of length 6 m. Find the measure of θ in degrees and in radians.

57. An arc of length 100 m subtends a central angle θ in a circle of radius 50 m. Find the measure of θ in degrees and in radians.

58. A circular arc of length 3 ft subtends a central angle of 25°. Find the radius of the circle.

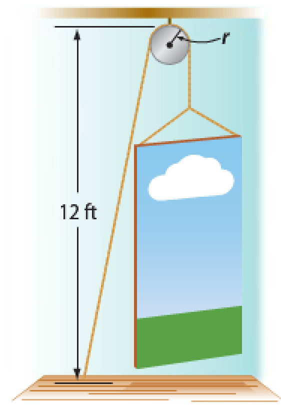
59. Find the radius of the circle if an arc of length 6 m on the circle subtends a central angle of $\pi/6$ rad.

60. Find the radius of the circle if an arc of length 4 ft on the circle subtends a central angle of 135°.

63. DRAMA A pulley with radius r is being used to remove part of the set of a play during intermission. The height of the pulley is 12 feet.

a. If the radius of the pulley is 6 inches and it rotates 180°, how high will the object be lifted?

b. If the radius of the pulley is 4 inches and it rotates 900°, how high will the object be lifted?



33 AMUSEMENT PARK A carousel at an amusement park rotates 3024° per ride. (Example 4)

a. How far would a rider seated 13 feet from the center of the carousel travel during the ride?

b. How much farther would a second rider seated 18 feet from the center of the carousel travel during the ride than the rider in part **a**?

1.08 Co-terminal Angles and Reference Angles

Co-terminal angles are angles that have the same terminal side. Not only are co-terminal angles created by measuring an angle both in the negative and in the positive directions, but they can be created by doing more than one revolution (360˚). Yes, angles can measure more than 360˚!

Co-terminal angles can be found at the same location just another revolution more or less. So, to find co-terminal angles, we must add or subtract 360˚ and we will end in the same location for the terminal side.

Example 1: Find one positive and one negative co-terminal angle for the given angle.

a. 30˚ b. -34˚ c. $\frac{7\pi}{6}$

Can we ever identify *all* co-terminal angles? **No, there are infinitely many!**

We can use this process for angles larger than 360˚ by subtracting 360 from the larger angle measure until we find a positive and a negative co-terminal angle.

Example 2: Find one positive and one negative co-terminal angle for the given angle. State the quadrant in which the terminal side lies.

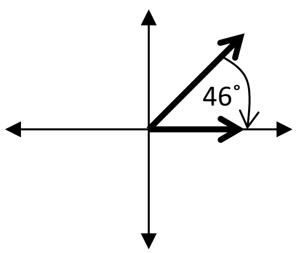
a. 800˚ b. -3732˚ c. 3945˚

Reference Angles

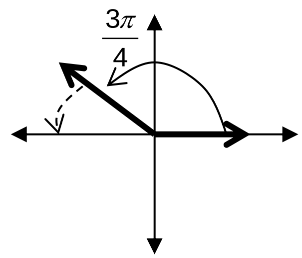
A Reference angle (θ') is the angle formed by the *terminal side* of the angle and the closest part of the *x-axis.*

Examples: Find the measure of the reference angle (θ') for each angle.

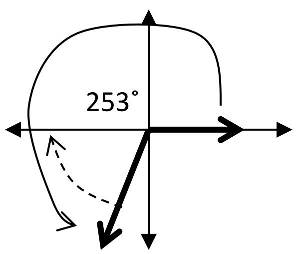
a. 46°



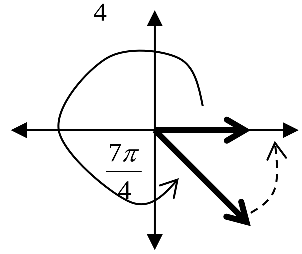
b. $\frac{3\pi}{4}$



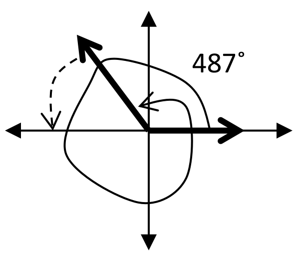
c. 253°



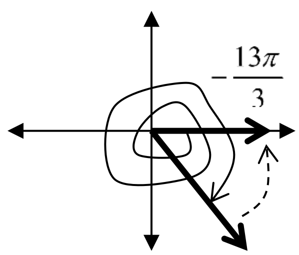
d. $\frac{7\pi}{4}$



e. 487° (first find a positive co-terminal angle between 0° and 360°)



f. $-\frac{13\pi}{3}$ (first find a positive co-terminal angle between 0 and 2π)



How large can the *reference angle* be? **Up to 90˚ - reference angles are <u>always</u> <u>acute</u> and <u>positive!</u>**

In summary, to find the reference angle (θ') based on the quadrant in which the terminal side of θ lies:

QII $\theta' =$ $\theta' =$	QI $\theta' =$
QIII $\theta' =$ $\theta' =$	QIV $\theta' =$ $\theta' =$

Reminder: to use the rules in this table, the angle θ must be between 0° and 360° (or between 0 and 2π).

If this is not the case, then find a positive, co-terminal angle for θ between 0° and 360° to use the table.

1.08 Coterminal & Reference Angles **Date: _______________**

Directions: Complete #18 – 26 all

Identify all angles that are coterminal with the given angle. Then find and draw one positive and one negative angle coterminal with the given angle. (Example 3)

18. $120°$

19. $-75°$

20. $225°$

21. $-150°$

22. $\frac{\pi}{3}$

23. $-\frac{3\pi}{4}$

24. $-\frac{\pi}{12}$

25. $\frac{3\pi}{2}$

26. **GAME SHOW** Sofia is spinning a wheel on a game show. There are 20 values in equal-sized spaces around the circumference of the wheel. The value that Sofia needs to win is two spaces above the space where she starts her spin, and the wheel must make at least one full rotation for the spin to count. Describe a spin rotation in degrees that will give Sofia a winning result. (Example 3)

Directions: Complete #17 – 24 all

Sketch each angle. Then find its reference angle. (Example 3)

17. $135°$ **18.** $210°$

19. $\frac{7\pi}{12}$ **20.** $\frac{11\pi}{3}$

21. $-405°$ **22.** $-75°$

23. $\frac{5\pi}{6}$ **24.** $\frac{13\pi}{6}$

1.09 Coterminal and Reference Angles **Date: ______________**

Find a coterminal angle between 0° and 360°.

1) 885° 2) -435°

Find a coterminal angle between 0 and 2p for each given angle.

3) $\frac{17\pi}{6}$ 4) $-\frac{\pi}{4}$

Find a positive and a negative coterminal angle for each given angle.

5) 240° 6) -166°

7) $-\frac{5\pi}{2}$ 8) $\frac{17\pi}{12}$

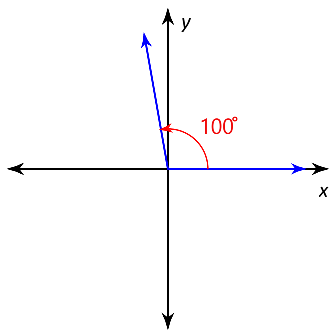
State if the given angles are coterminal.

9) $115°, 475°$

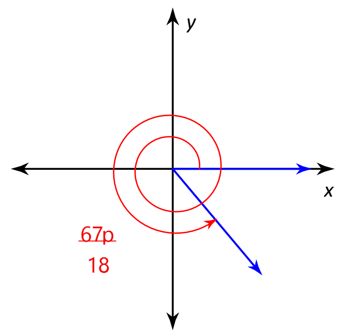
10) $\frac{5\pi}{6}, \frac{23\pi}{6}$

Find the reference angle.

11)



12)



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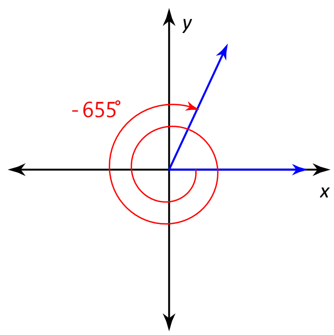
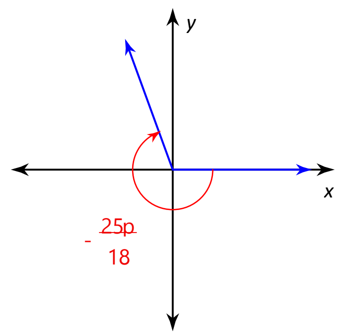


14)



15) -510°

16) $\frac{19\pi}{9}$

17) 320°

18) -595°

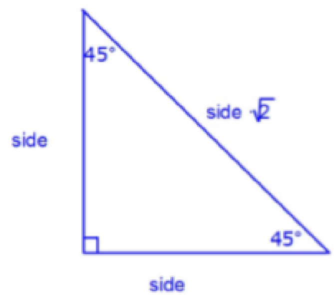
19) $-\frac{4\pi}{3}$

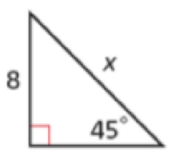
20) 345°

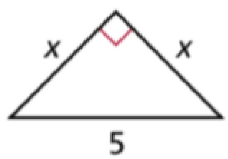
1.11 Special Right Triangles Review Name: _______________

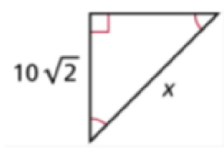
You may remember special right triangles from Geometry. Here's a refresher in case you don't. 😊

45°- 45°- 90° Triangle

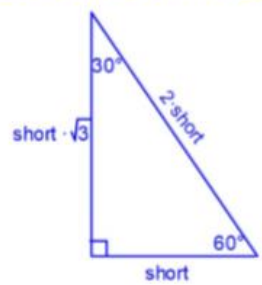


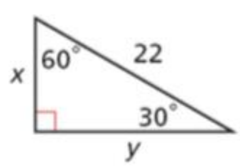


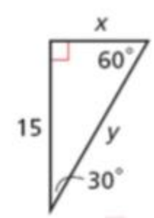


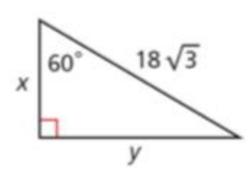


30°- 60°- 90° Triangle





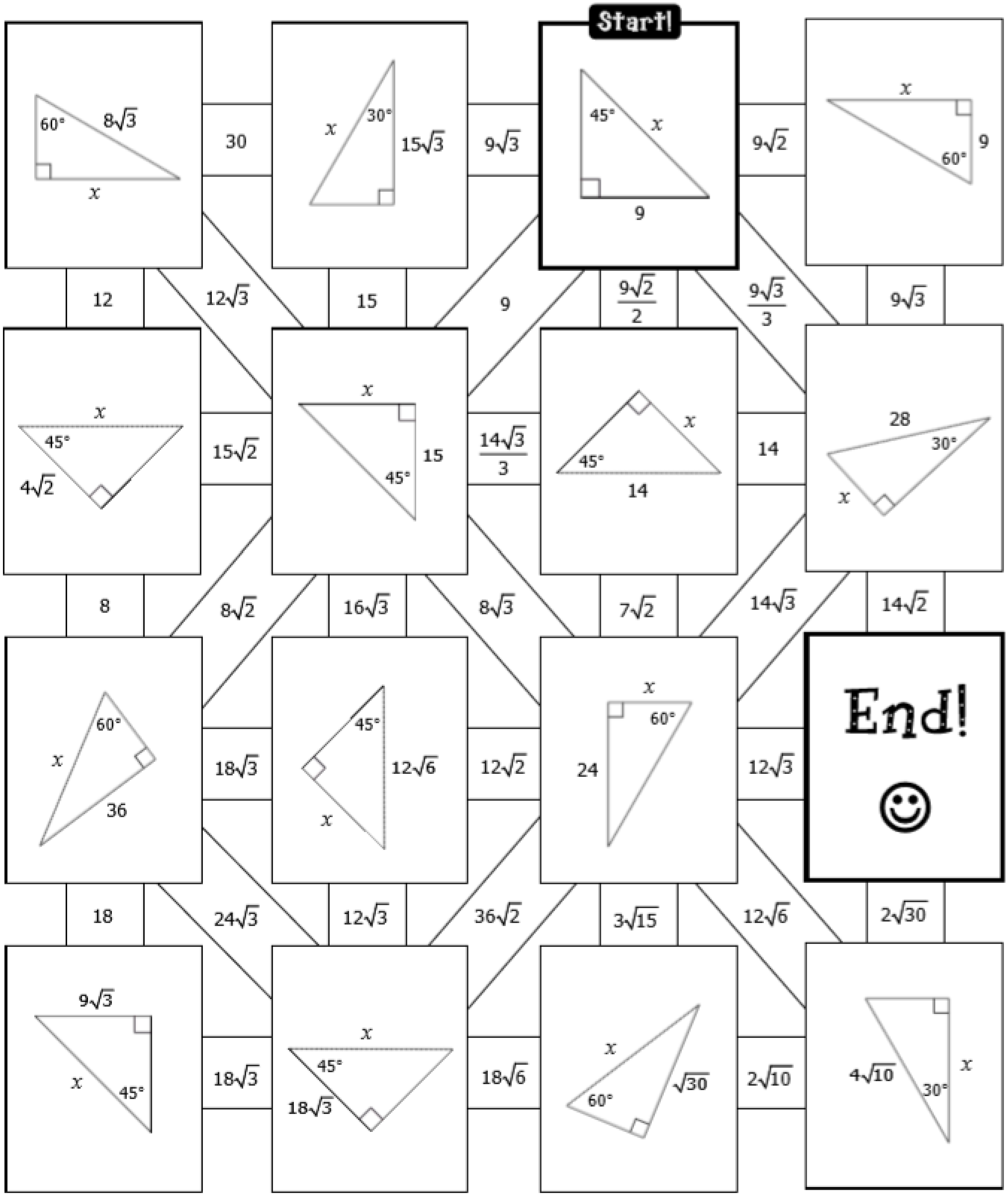




A manufacturer wants to make a larger clock with a height of 30 centimeters. What is the length of each side of the frame? Round to the nearest tenth.

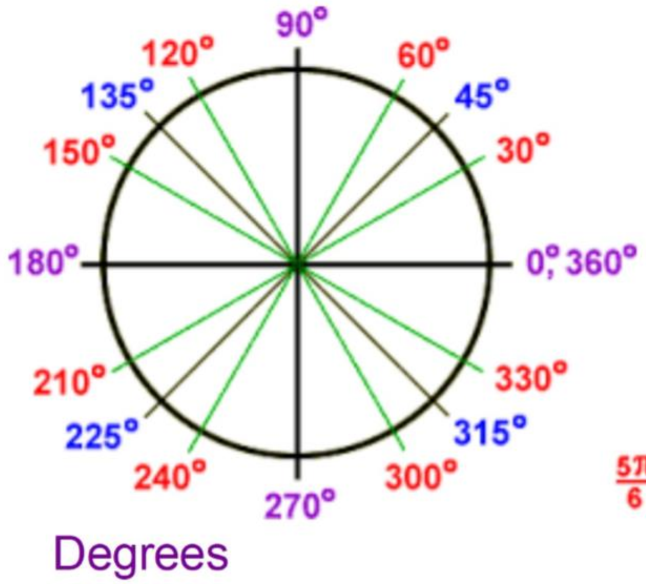
Special Right Triangles !

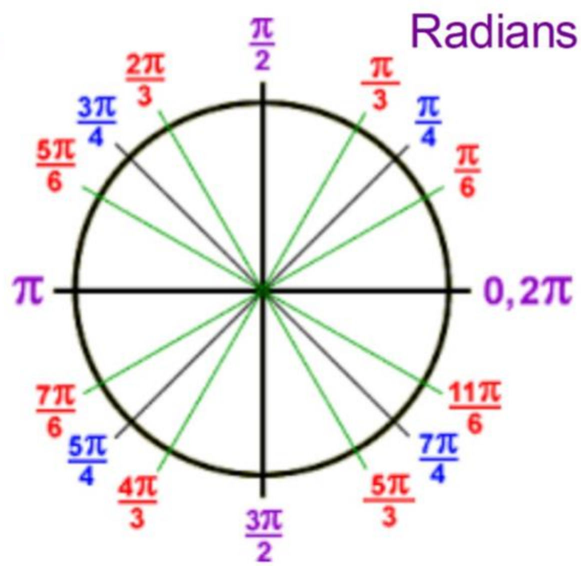
Directions: Find each missing side. Write all answers in simplest radical form. Use your solutions to navigate through the maze. **Staple all work to this paper!**



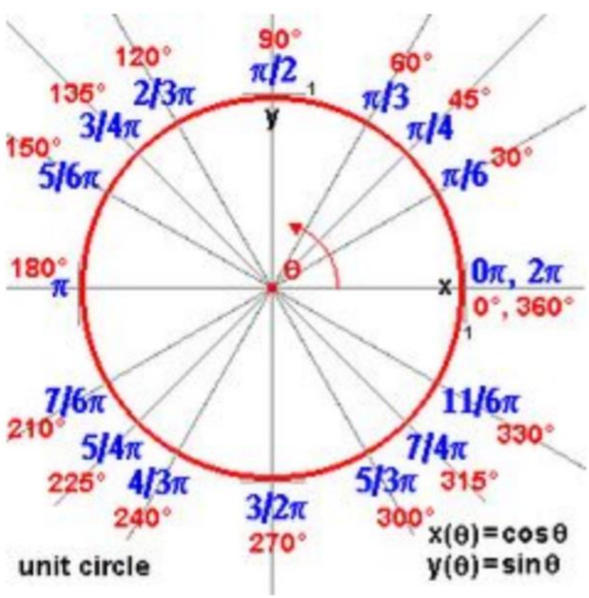
Version 1: Simplest Radical Form

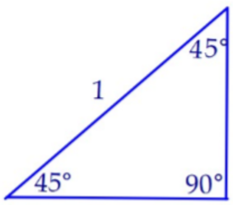
1.12 Intro to Unit Circle Date: ______________________

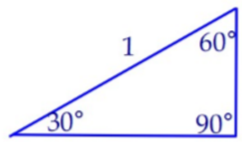


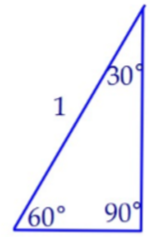


Degrees and Radians Together









sin45° =

cos45° =

tan45° =

sin30° =

cos30° =

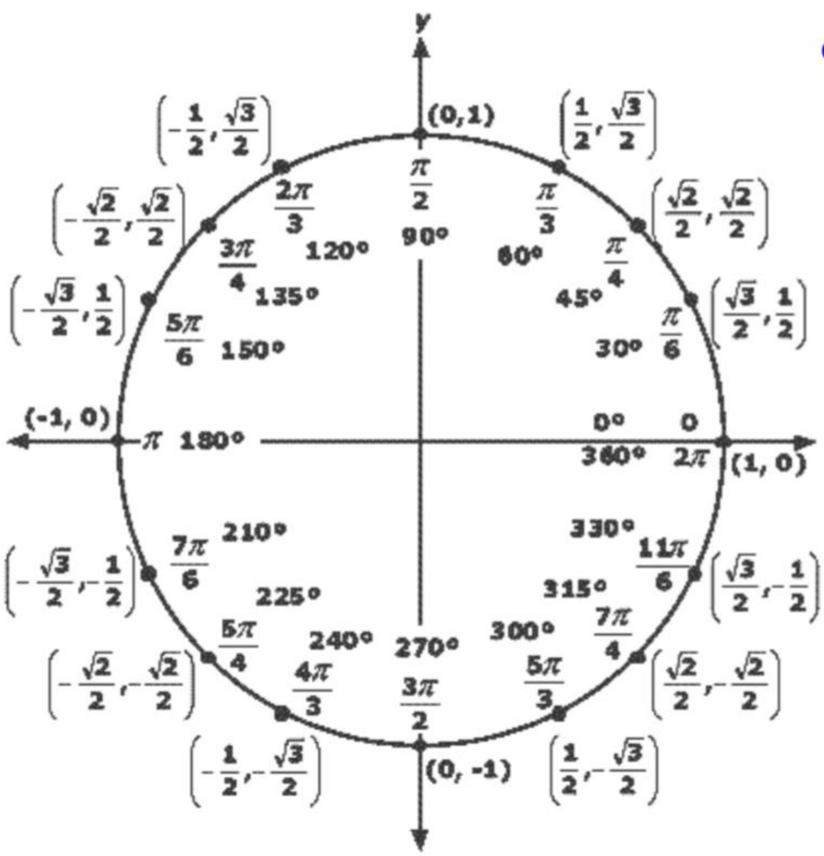
tan30° =

sin60° =

cos60° =

tan60° =

Unit Circle



coordinates = (x,y)

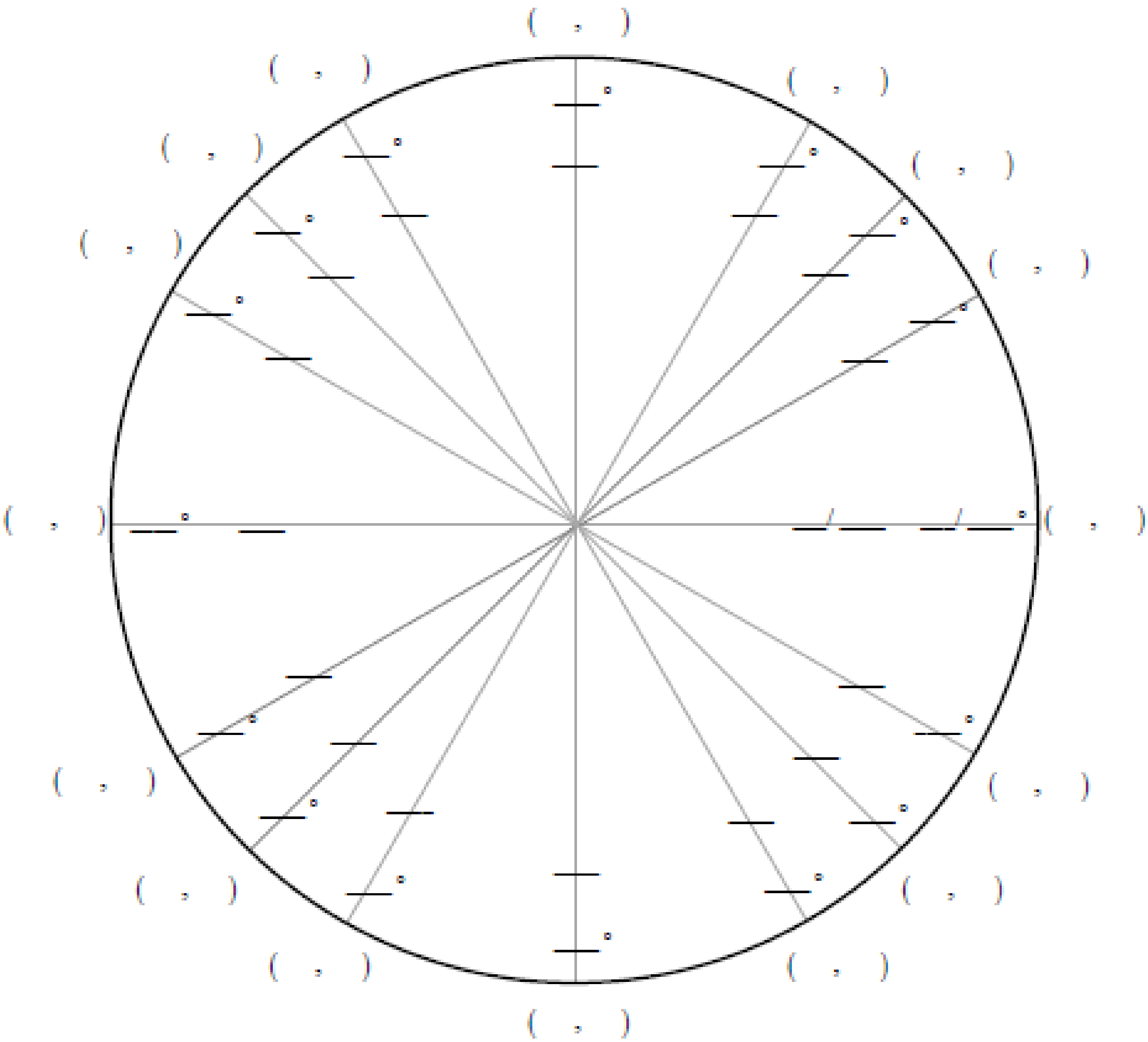
cosθ = x
sinθ = y
tanθ = y/x

secθ = 1/x
cscθ = 1/y
cotθ = x/y

<u>**Reciprocals**</u>

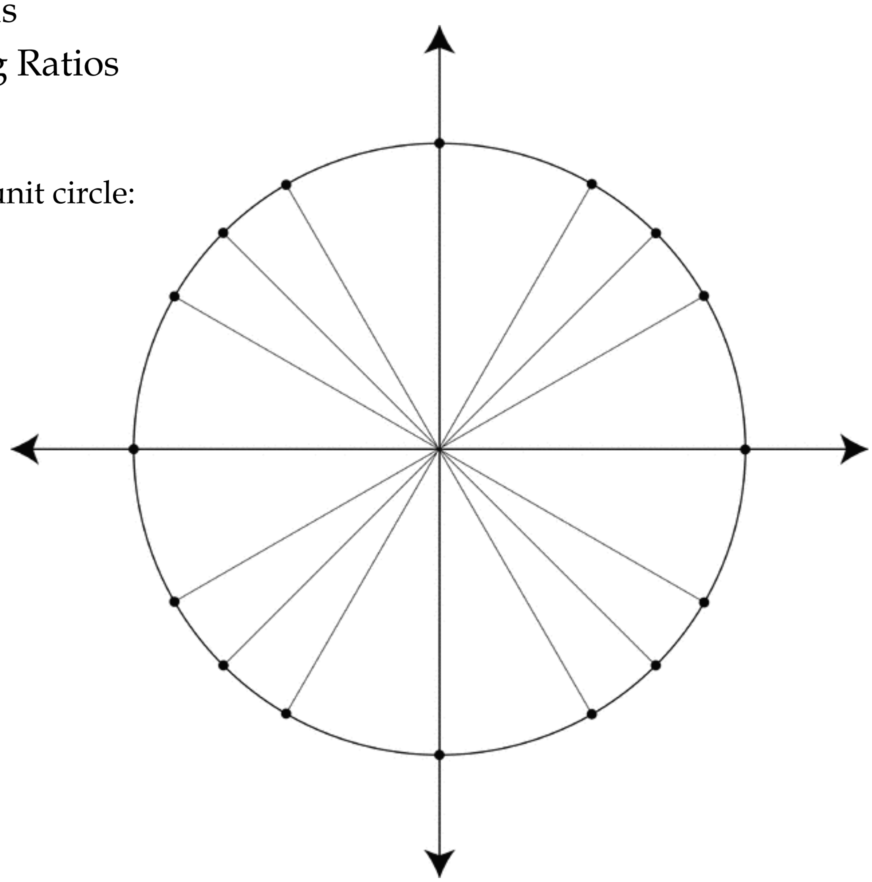
<u>co</u>sine	secant
sine	<u>co</u>secant
tangent	<u>co</u>tangent

1.12 HW: Try filling out the unit circle



Accelerated Pre-Calculus

1.13 Unit Circle and Trig Ratios



To find trig ratios from the unit circle:

Remember
x = cos
y = sin
y/x = tan

Using the Unit Circle, find the following.

1. $sin 30° =$
2. $cos 45° =$
3. $tan \frac{\pi}{6} =$
4. $csc \frac{\pi}{3} =$
5. $cot 90° =$
6. $sec 270° =$

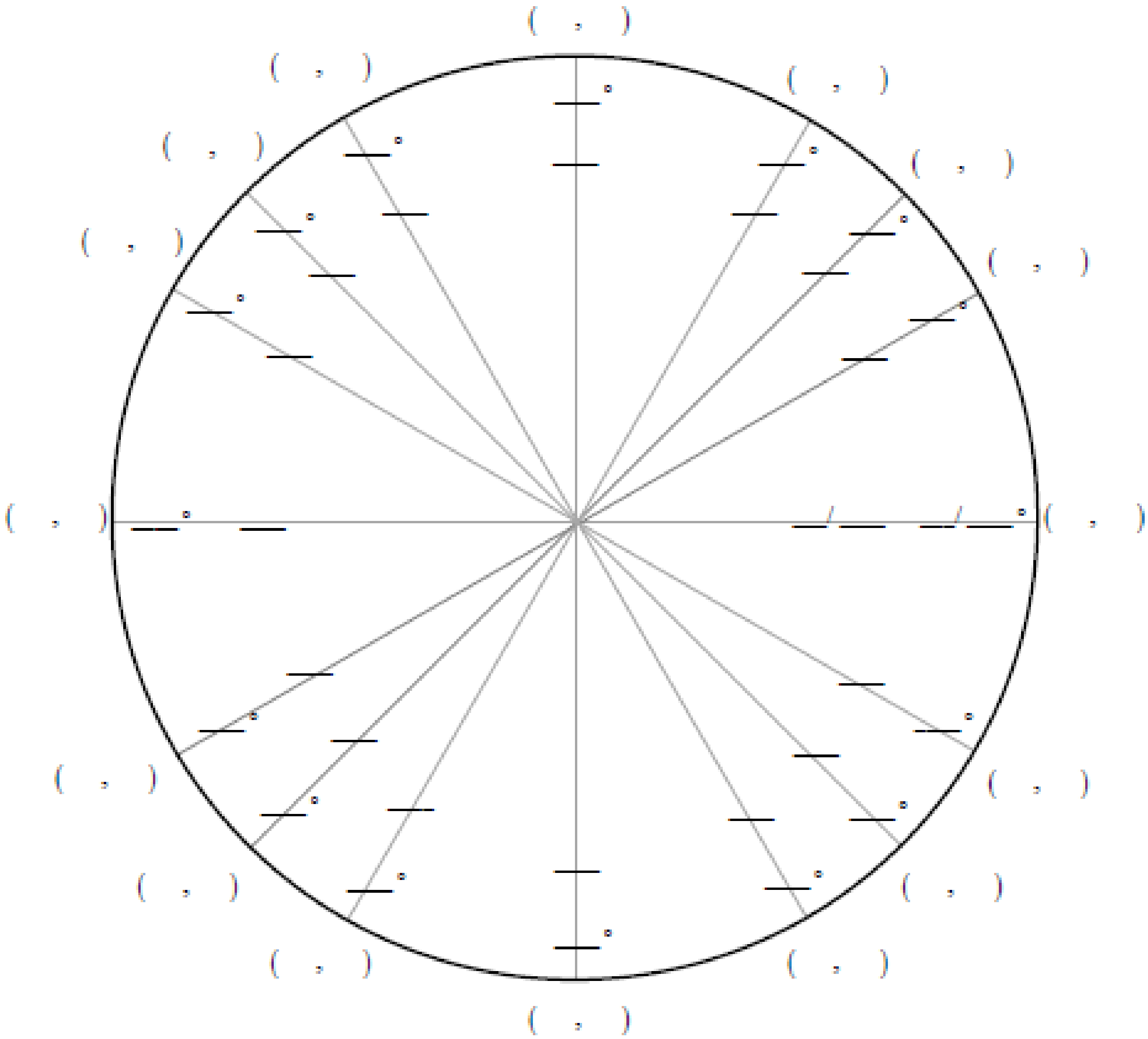
What about angles in other quadrants?

7. $cos \frac{2\pi}{3} =$
8. $csc \frac{3\pi}{4} =$
9. $cot 210° =$
10. $sin \frac{5\pi}{3} =$
11. $tan 360° =$
12. $sec 330° =$

What about angles outside of 0 to 2π?

13. $cos \frac{11\pi}{3} =$
14. $sin - 390° =$
15. $tan 1305° =$
16. $csc \frac{19\pi}{6} =$
17. $sec - 135° =$
18. $cot - \frac{3\pi}{2} =$

1.13 Unit Circle Values HW Date: ____________________



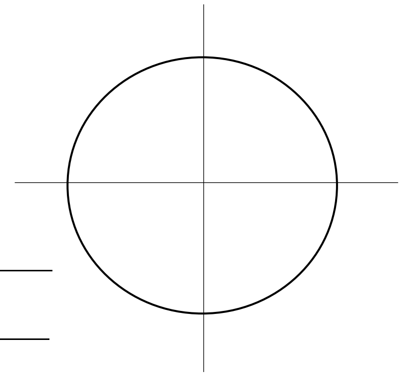
Degree	Radian	Coordinates	cos θ	sin θ	tan θ	sec θ	csc θ	cot θ
0°								
30°								
45°								
60°								
90°								

Accelerated Precalculus **Name** ________________________

1.14 Trig with the Unit Circle & Reference Angles Date _______________

To find values of trig functions anywhere on the unit circle:

a) Determine the quadrant of the given angle and reference angle.
b) Give the coordinates. Check signs!
c) Evaluate the trig function.



Remember: if the coordinates of any point on the unit circle are (x, y), then

cos Θ = _______ sin Θ = _______ tan Θ = _______

sec Θ = _______ csc Θ = _______ cot Θ = _______

Find the value of each using the steps outlined above.

1) cos 120° =

a) quadrant: _______

b) reference angle: _______

c) coordinates: ____________

2) tan $\frac{4\pi}{3}$ =

a) quadrant: _______

b) reference angle: _______

c) coordinates: ____________

3) sin 315°=

a) quadrant: _______

b) reference angle: _______

c) coordinates: ____________

4) tan $\frac{5\pi}{4}$ =

a) quadrant: _______

b) reference angle: _______

c) coordinates: ____________

5) cos 210°

a) quadrant: _______

b) reference angle: _______

c) coordinates: ____________

6) tan $\frac{5\pi}{6}$

a) quadrant: _______

b) reference angle: _______

c) coordinates: ____________

1.14 HW Complete the information for each angle provided:

Angle	Quadrant	Reference Angle	Check Signs	Coordinates	Trigonometric Ratio
$300°$					$\sin 300° =$
$\frac{7\pi}{6}$					$\csc \frac{7\pi}{6} =$
$240°$					$\tan 240° =$
$\frac{\pi}{6}$					$\sec \frac{\pi}{6} =$
$330°$					$\cot 330° =$
$\frac{\pi}{2}$					$\sec \frac{\pi}{2} =$
$\frac{11\pi}{6}$					$\cos \frac{11\pi}{6} =$
$\frac{\pi}{3}$					$\sec \frac{\pi}{3} =$
$135°$					$\cot 135° =$
$\frac{2\pi}{3}$					$\sec \frac{2\pi}{3} =$
$\frac{3\pi}{2}$					$\tan \frac{3\pi}{2} =$
$\frac{5\pi}{3}$					$\tan \frac{5\pi}{3} =$
$\frac{\pi}{4}$					$\cos \frac{\pi}{4} =$
π					$\csc \pi =$

We now know how to answer trigonometric problems that use the special angles found on the unit circle. Problems like:

1) sin 315°: Because this is a 45° family, the answer is either $\frac{\sqrt{2}}{2}$ or $-\frac{\sqrt{2}}{2}$. The angle 315° is found in the 4th quadrant, and sine is based on y which is negative in the 4th quadrant. So, the answer is the negative y-value: sin 315° = $-\frac{\sqrt{2}}{2}$

2) tan $\frac{7\pi}{6}$: Because this is the "over 6" family (30° family) but in the 3rd quadrant, the coordinates are $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$. The formula for tangent is $\frac{y}{x}$, here that means $\frac{-1/2}{-\sqrt{3}/2}$. Both the negatives and the denominators of 2 cancel out, leaving $\frac{1}{\sqrt{3}}$. This has to be rationalized. So, the answer is: tan $\frac{7\pi}{6} = \frac{\sqrt{3}}{3}$

<u>What if the angle is bigger than 1 revolution around the circle?</u>

3) cos 840°: As before, if values are too large to know, use coterminal angles to bring them down to a value we already know. Subtract 360° enough times so that the angle is now between 0° and 360°: 840° – 360° – 360° = 120°. The coterminal angle will have the same cosine value, so consider it as cos 120°. This angle is in the 60° family in Quadrant 2 with coordinates $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$. Cosine uses the x-value of the coordinates.

So, the answer is: cos 840° = cos 120° = $-\frac{1}{2}$

<u>What if the angle is negative?</u>

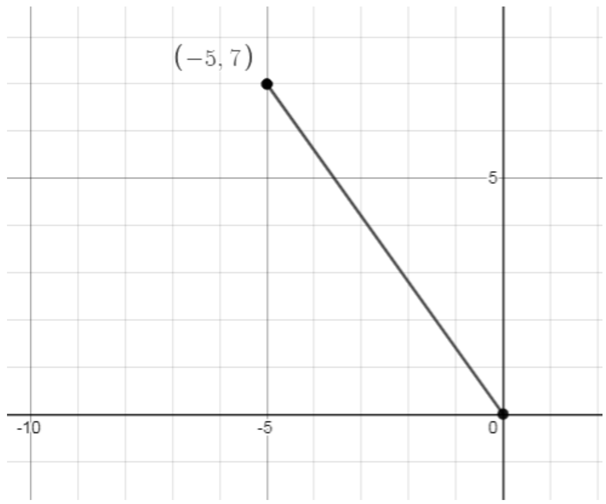
4) csc $\left(-\frac{4\pi}{3}\right)$: The angle is from the "over 3" family (60° family). Cosecant is the flip of sine, and sine is the y-value. The y-values for all "over 3" angles are $\frac{\sqrt{3}}{2}$ or $-\frac{\sqrt{3}}{2}$. To determine the quadrant, find the coterminal angle by adding 2π:

$-\frac{4\pi}{3} + \frac{6\pi}{3} = \frac{2\pi}{3}$. The y-value in Quadrant 2 will be positive, y = $\frac{\sqrt{3}}{2}$. The cosecant value requires the reciprocal. So csc $\left(-\frac{4\pi}{3}\right)$ = csc $\frac{2\pi}{3} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

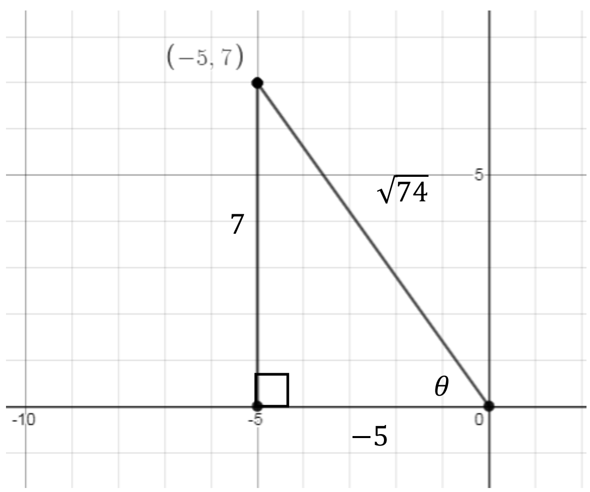
What if the location is not even *on* the unit circle? Sometimes we have to fall back on **right triangle trigonometry** rather than using the unit circle.

5) Find sec θ where the terminal side of θ passes through (-5, 7).

Step 1: Plot the point and connect it to the origin.



Step 2: Connect the point perpendicularly to the closest x-axis, making a reference triangle. Label θ as the reference angle. Use Pythagorean Theorem to determine the length of the hypotenuse. Label all 3 side lengths.



Step 3: Use SohCahToa and the reciprocal relations to complete the problem.

$$\sec\theta = \frac{hypotenuse}{adjacent} = \frac{\sqrt{74}}{-5}$$

1.15 Practice

Page 251: #5-31 odd and #43-51 odd

The given point lies on the terminal side of an angle θ in standard position. Find the values of the six trigonometric functions of θ. (Example 1)

5. $(1, -8)$

7. $(-8, 15)$

Find the exact value of each trigonometric function, if defined. If not defined, write *undefined*. (Example 2)

9. $\sin \frac{\pi}{2}$

11. $\cot(-180°)$

13. $\cos(-270°)$

15. $\tan \pi$

Sketch each angle. Then find its reference angle. (Example 3)

17. $135°$

19. $\frac{7\pi}{12}$

21. $-405°$

23. $\frac{5\pi}{6}$

Find the exact value of each expression. (Example 4)

25. $\cos \frac{4\pi}{3}$

27. $\sin \frac{3\pi}{4}$

29. $\csc 390°$

31. $\tan \frac{11\pi}{6}$

Find the exact value of each expression. If undefined, write *undefined*. (Examples 7 and 8)

43. $\sec 120°$

45. $\cos \frac{11\pi}{3}$

47. $\csc 390°$

49. $\csc 5400°$

51. $\cot\left(-\frac{5\pi}{6}\right)$

1.16 Unit Circle Trigonometry Extension Worksheet

The given point lies on the terminal side of an angle θ in standard position. Find the values of the six trigonometric functions of θ.

1. (1, -8)

2. (-8, 15)

State the quadrant or axis where the terminal side of θ is found.

3. sin θ < 0 and cos θ < 0

4. tan θ > 0 and sec θ > 0

5. cos θ > 0 and cot θ < 0

6. sec θ < 0 and sin θ = 0

7. cos θ = 0 and csc θ > 0

8. cot θ < 0 and cos θ < 0

First, state the quadrant or axis where the terminal side of θ is found. Then, find the exact value of the specified trigonometric function using the given information.

9. Find cos θ if sin θ = $\frac{1}{2}$ and tan θ < 0.

Quadrant: ______

cos θ = _________

10. Find tan θ if cos θ = $-\frac{\sqrt{2}}{2}$ and sin θ < 0.

Quadrant: ______

tan θ = _________

11. Find sin θ if sec θ is undefined and csc < 0.

Quadrant: ______

sin θ = _________

12. Find cot θ if sec θ = 2 and csc θ < 0.

Quadrant: ______

cot θ = _________

13. Find csc θ if tan = $\sqrt{3}$ and sec θ > 0

Quadrant: ______

csc θ = _________

14. Find sec θ if cot θ = -1 and sin θ > 0

Quadrant: ______

sec θ = _________

15. Find sec θ and csc θ if tan θ = $-\frac{4}{3}$ and cos θ < 0.

Quadrant: ______

sec θ = _________

csc θ = _________

16. Find csc x and cos x if sec θ = $\frac{17}{8}$ and sin θ < 0.

Quadrant: ______

csc θ = _________

cos θ = _________

17. Find cos θ and cot θ if sin θ = $-\frac{1}{4}$ and tan θ < 0.

Quadrant: ______

cos θ = _________

cot θ = _________

18. Find sin θ and cos θ if cot θ = $\frac{3}{7}$ and sec θ < 0.

Quadrant: ______

sin θ = _________

cos θ = _________

1.17 Unit Circle Trigonometry Extension Worksheet

Evaluate each without using a calculator.

1. $\sin x = \frac{\sqrt{3}}{2}$ $\quad \frac{\pi}{2} \le x \le \pi$

2. $\cos x = \frac{-\sqrt{2}}{2}$ $\quad \pi \le x \le \frac{3\pi}{2}$

3. $\tan x = \sqrt{3}$ $\quad \pi \le x \le \frac{3\pi}{2}$

4. $\cot x$ = undefined $\quad 0 \le x \le \pi$

5. $\csc x = -2$ $\quad -\pi \le x \le -\frac{\pi}{2}$

6. $\sec x = -\frac{2\sqrt{3}}{3}$ $\quad -\frac{3\pi}{2} \le x \le -\pi$

Evaluate each. Give two solutions.

7. $\sin x = \frac{-1}{2}$ $\quad 0^{o} \le x \le 360^{o}$

8. $\cot x = -1$ $\quad 0^{o} \le x \le 360^{o}$

9. $\tan x = 0$ $\quad 0^{o} \le x \le 360^{o}$

10. $\sec x = -\sqrt{2}$ $\quad 0 \le x \le 2\pi$

11. $\csc x$ = undefined $\quad 0 \le x \le 2\pi$

12. $\cos x = \frac{\sqrt{3}}{2}$ $\quad 0 \le x \le 2\pi$

Complete each trigonometric expression.

13. $\cos 60^{o} = \sin$ ______

14. $\tan \frac{\pi}{4} = \sin$ ______

15. $\sin \frac{2\pi}{3} = \cos$ ______

16. $\cos \frac{7\pi}{6} = \sin$ ______

17. $\sin(-45^{o}) = \cos$ _____

18. $\cos \frac{5\pi}{3} = \sin$ ______

1.19 Review Worksheet: Intro to Trigonometry

Part 1: Be prepared to complete these problems without the use of a calculator!
Determine the exact values of the six trigonometric functions of the angle θ.

1. $\theta = \frac{3\pi}{4}$

2. θ = 240°

3. (5, -12) is on the terminal side of an angle in standard position. Determine the exact values of sin, cos, and tan of the angle.

4. State the quadrant in which θ lies.
   a) sin θ < 0, cos θ < 0
   b) sin θ > 0, tan θ < 0
   c) cot θ > 0, cos θ > 0

5. Use the given information to determine sine, cosine, and tangent ofθ, *unless already given*.
   a) $\tan \theta = -\frac{15}{8}$; sin θ < 0.
   b) sec θ = -2 ; 0 ≤ θ ≤ π.
   c) sin θ = 0 ; sec θ = -1.

6. Evaluate the trigonometric function of the quadrant angle.
   a) sec π
   b) $\cot\left(\frac{\pi}{2}\right)$

Evaluate each without using a calculator.

7. tan 225°

8. cos (-750°)

9. sin (-240°)

10. $\tan\left(\frac{5\pi}{3}\right)$

11. $\sin\left(-\frac{\pi}{6}\right)$

12. $\cos\left(\frac{11\pi}{4}\right)$

13. Find the acute angle that satisfies the given equation. Answer in both degrees and radians.

a) $\cos \theta = \frac{1}{2}$

b) $\cot \theta = \frac{\sqrt{3}}{3}$

<u>Part 2:</u> You will be able to use a calculator for problems like these.

1. Convert 260° to radian measure.

2. Convert $\frac{5\pi}{8}$ to degree measure.

State the quadrant in which the terminal side of the angle lies, and find its reference angle. Answer the question in the units that it is given.

3. 105°

4. -310°

5. $\frac{9\pi}{5}$

6. DME (Distance Measuring Equipment) is standard avionic equipment on a commercial airplane. This equipment measures the distance from a plane to a radar station. If the distance from a plane to a radar station is 220 miles and the angle of depression is 38˚, find the number of ground miles from a point directly below the plane to the radar station.

7. A pendulum is 22.9 centimeters long, and the bob at the end of the pendulum travels 10.5 centimeters. Find the degree measure of the angle through which the pendulum swings.