Name:____ **Period:** __________

Accel. Pre-Calculus

Unit 7 Packet

Polar Graphs & Complex Numbers

7.01 Polar Coordinates **Date: __________________**

Opener: Plotting Polar Points in Desmos

1. Go to desmos.com/calculator
2. Click the wrench (upper right) and choose the polar grid
3. Put the angle setting in degrees (shocking, right?!?)
4. Equation 1: r = 5 from $-6 \le r \le 6$, scale of 1
   Suggestion: Turn off the graph by clicking the colored circle to the left of Equation 1
5. Equation 2: a = 15 from $-360 \le a \le 360$, scale of 15
6. Equation 3: (r cos a, r sin a)
7. Equation 4: (x_1, y_1) *shift-underscore makes subscripts*
8. Equation 5: $x_1 = 1$, with a slider
9. Equation 6: $y_1 = 1$, with a slider

Use the sliders to move the points around.

Points (pun definitely intended!) to consider:

- What happens when r is negative?

- What happens when a is negative?

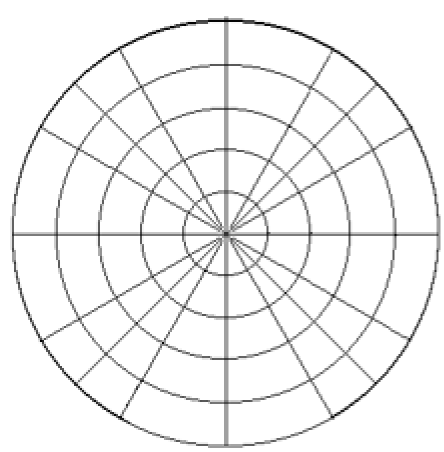
There is more than one way to plot a point:

Rectangular Graph: Polar Graph:

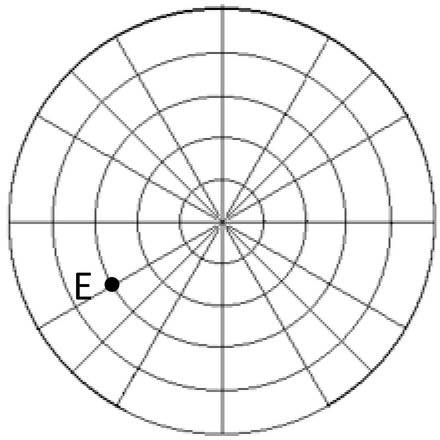
Example: Plot the Polar Points: (r, θ)

A (2, 135°)

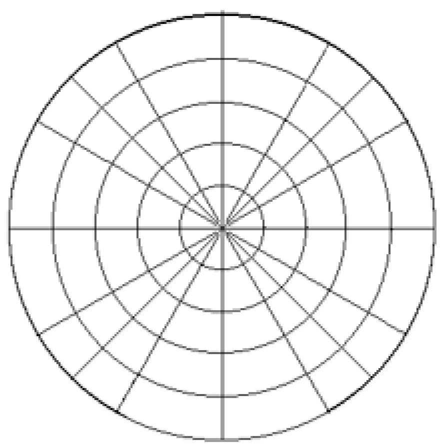
B $\left(1, \frac{7\pi}{6}\right)$



Example: Name the location of E in 4 different ways with $-2\pi \leq \theta \leq 2\pi$.



Example: Plot 3 points and determine different pairs of coordinates for them.



7.01 Practice:

Graph each point on a polar grid. (Examples 1 and 2)

1. $R(1, 120°)$

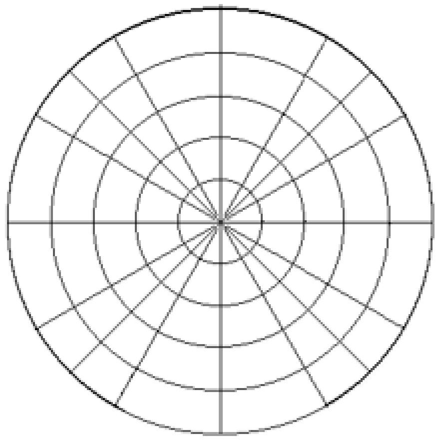
3. $F\left(-2, \frac{2\pi}{3}\right)$

5. $Q\left(4, -\frac{5\pi}{6}\right)$

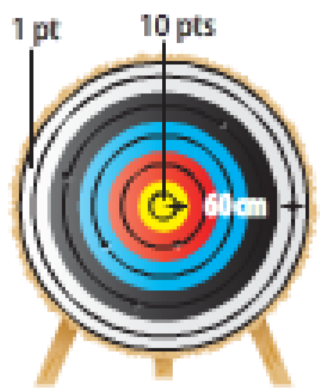
7. $D\left(-1, -\frac{5\pi}{3}\right)$

9. $C(-4, \pi)$

11. $P(4.5, -210°)$



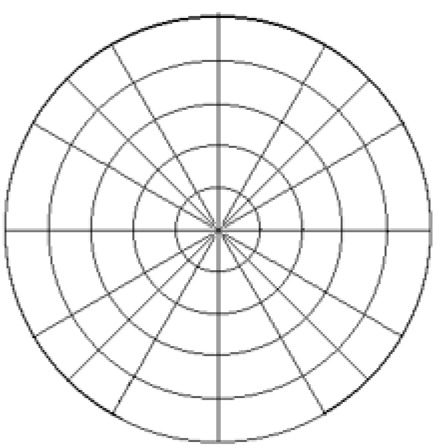
13. **ARCHERY** The target in competitive target archery consists of 10 evenly spaced concentric circles with score values from 1 to 10 points from the outer circle to the center. Suppose an archer using a target with a 60-centimeter radius shoots arrows at (57, 45°), (41, 315°), and (15, 240°). (Examples 1 and 2)



a. Plot the points where the archer's arrows hit the target on a polar grid.

b. How many points did the archer earn?

Find three different pairs of polar coordinates that name the given point if $-360° \leq \theta \leq 360°$ or $-2\pi \leq \theta \leq 2\pi$. (Example 3)



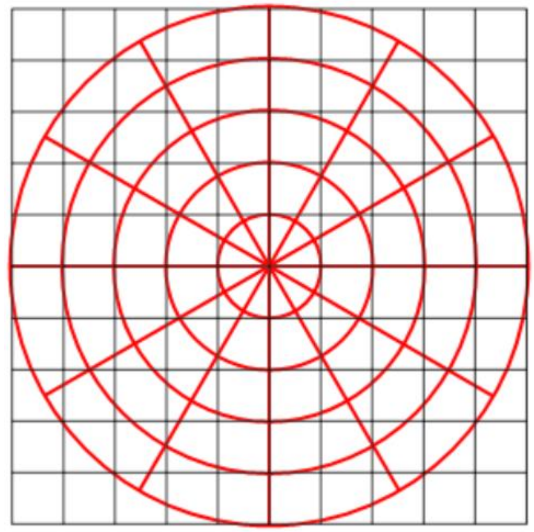
15. $(-2, 300°)$

17. $\left(-3, \frac{2\pi}{3}\right)$

19. $\left(-5, -\frac{4\pi}{3}\right)$

21. $(-1, -240°)$

7.02 Converting Polar Coordinates **Date: ________________**



The polar and rectangular grids do overlap so that a location can take on coordinates from either system.

If you know r and θ, how do you calculate x and y?

If you know x and y, how do you calculate r and θ?

Example: Find the rectangular coordinates for each point given in polar coordinates.

1. P (4, -60°)

2. Q $(-2, \frac{3\pi}{4})$

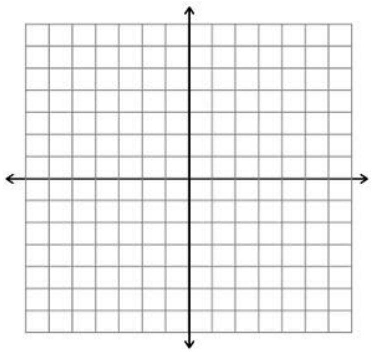
Example: For each point given in rectangular coordinates, find <u>four</u> unique polar coordinates with -2π ≤ θ ≤ 2π.

3. A (2, -5)

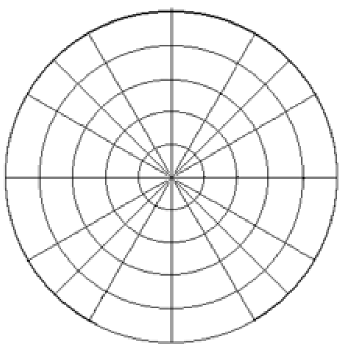
4. B (-9, -4)

Distance between 2 Points in the Polar Plane

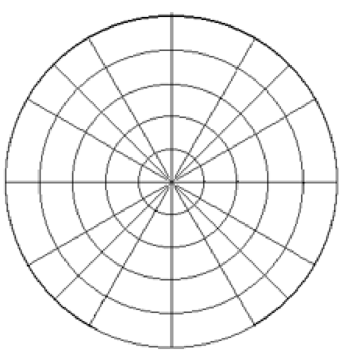
Review: How do we find the distance between two points in the Cartesian Plane?



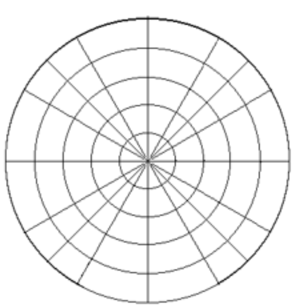
New method for distance using the Polar Plane:



Example: Find the distance between the points $\left(-3, \frac{\pi}{3}\right)$ *and* $(5, -\frac{11\pi}{6})$



Example: A radar detects 2 plane s at the same altitude. Their polar coordinates are (5 miles, 310°) and (2 miles, 192°). How far apart are the planes?



7.02 Practice: Complete the odd problems.

Find the rectangular coordinates for each point with the given polar coordinates. Round to the nearest thousandth if necessary. (Example 1)

1. $\left(2, \frac{\pi}{4}\right)$

2. $\left(\frac{1}{4}, \frac{\pi}{2}\right)$

3. $(5, 240°)$

4. $(2.5, 250°)$

5. $\left(-2, \frac{4\pi}{3}\right)$

6. $(-13, -70°)$

7. $\left(3, \frac{\pi}{2}\right)$

8. $\left(\frac{1}{2}, \frac{3\pi}{4}\right)$

9. $(-2, 270°)$

10. $(4, 210°)$

11. $\left(-1, -\frac{\pi}{6}\right)$

12. $\left(5, \frac{\pi}{3}\right)$

Find 4 pairs of polar coordinates for each point with the given rectangular coordinates for [-2π, 2π]. Round to the nearest thousandth if necessary. (Example 2)

13. $(7, 10)$

14. $(-13, 4)$

15. $(-6, -12)$

16. $(4, -12)$

17. $(2, -3)$

18. $(0, -173)$

19. $(a, 3a), a > 0$

20. $(-14, 14)$

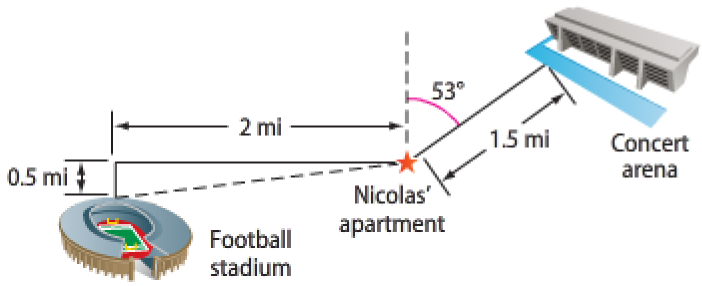
21. $(52, -31)$

22. $(3b, -4b), b > 0$

23. $(1, -1)$

24. $(2, \sqrt{2})$

25. **DISTANCE** Standing on top of his apartment building, Nicolas determines that a concert arena is 53° east of north. Suppose the arena is exactly 1.5 miles from Nicolas' apartment. (Example 3)



a. How many miles north and east will Nicolas have to travel to reach the arena?

b. If a football stadium is 2 miles west and 0.5 mile south of Nicolas' apartment, what are the polar coordinates of the stadium if Nicolas' apartment is at the pole?

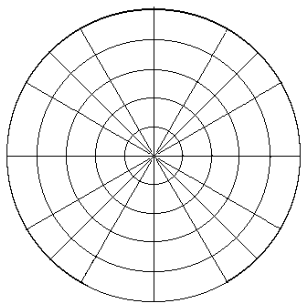
7.03 Quiz Review: Date ____________________

Polar Coordinates, Equations, and Distance

1) Graph each point on the polar grid. Find three other pairs of polar coordinates that name the point

if $-360° \leq \theta \leq 360°$ if $-2\pi \leq \theta \leq 2\pi$

a) $A(4, -120°)$



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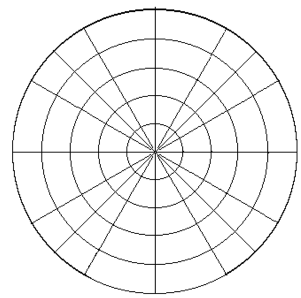
b) $B\left(2, \frac{7\pi}{4}\right)$



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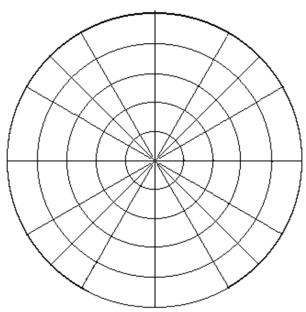
c) $C(-3, 210°)$



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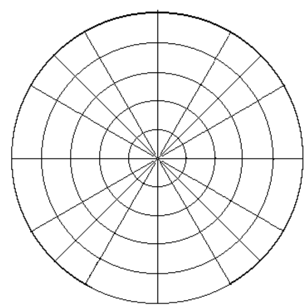
d) $D\left(-5, -\frac{\pi}{6}\right)$



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2) Given the polar distance formula between two points *A* (r_1, θ_1) and *B* (r_2, θ_2):

$AB = \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_2 - \theta_1)}$, find the distance between *A* and *B*.

a) $A(4, 200°)$ $B(-3, 60°)$

b) $A\left(-7, \frac{5\pi}{6}\right)$ $B\left(2, -\frac{4\pi}{3}\right)$

a) AB = ________________

b) AB = ________________

3) Find the rectangular coordinates for each point with the given polar coordinates. Answer in exact form.

a) $\left(5, \frac{5\pi}{3}\right)$ b) $\left(-6, \frac{3\pi}{4}\right)$

a) ________________ b) ________________

c) $\left(8, \frac{7\pi}{6}\right)$ d) $\left(-12, -\frac{3\pi}{2}\right)$

c) ________________ d) ________________

4) Find four unique polar coordinates for each point given as rectangular coordinates.
Use $-360° \leq \theta \leq 360°$. Round to the nearest thousandths.

a) (-1, 5) b) (3, -7)

a) ____________ ____________ b) ____________ ____________

____________ ____________ ____________ ____________

Use $-2\pi \leq \theta \leq 2\pi$. Round to the nearest thousandths.

c) $(-5\sqrt{3}, -5)$ d) (4, 3)

c) ____________ ____________ d) ____________ ____________

____________ ____________ ____________ ____________

7.04 Complex Numbers Review **Date:** ________________________

Recall that the imaginary number i is defined such that $i^2 = -1$.

1. i = _______ 2. i^2 = ______ 3. i^3 = _______ 4. i^4 = _______

5. i^{11} = _______ 6. i^{18} = _______ 7. i^{67} = _______ 8. i^{724} = _______

A complex number has two parts: the real part and the imaginary part.

9. The standard form for complex number is _____________________.

Perform the given operation. Write your answer in standard form of a complex number.

10. $(-4 + 7i) + (2 - 3i)$ = ___________________ 11. $(7 - 12i) - (4 + 9i)$ = ___________________

12. $(5 + 8i) \cdot (2 - 10i)$ = ___________________ 13. $(3 + 4i) \cdot (-8 + 2i)$ = ___________________

14. $(3 - 5i) \cdot (3 - 5i)$ = ___________________ 15. $(3 - 5i) \cdot (3 + 5i)$ = ___________________

#14: two factors that are the exact same multiplied together (just like a binomial squared). #15: two factors that only have the sign in the middle changed. They are <u>conjugates.</u>

Which product resulted in an <u>entirely real</u> value having <u>no imaginary part</u>? ________________

Generalize it as a formula by simplifying: $(a + bi) \cdot (a - bi)$ =

State the conjugate $(a - bi)$ of the given complex number $(a + bi)$.

16. $9 + 4i$ ____________________ 17. $5 - 2i$ ____________________ 18. $-3 + 7i$ ____________________

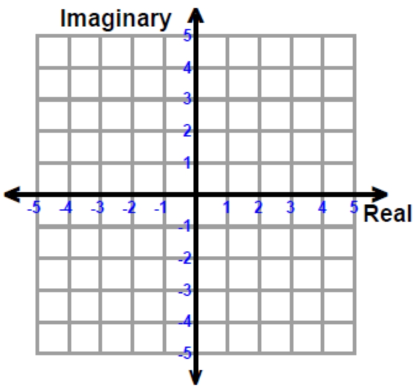
Use the conjugate of the denominator to <u>rationalize</u> the following fractions.

19. $\frac{1+i}{5-2i}$ = ____________________

20. $\frac{5-6i}{-3+7i}$ = ____________________

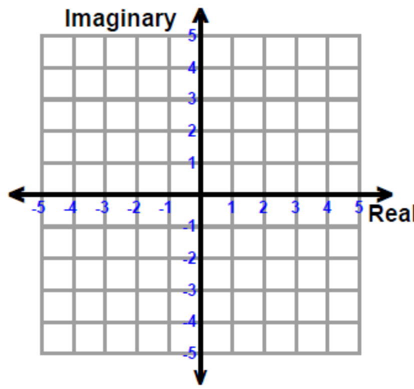
Graph the number on the complex plane and find its absolute value (distance from zero).

21. $4 - 3i$



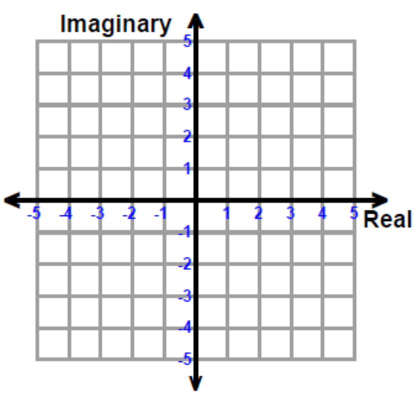
$|4 - 3i|$ = ________________

22. $-5 - 5i$



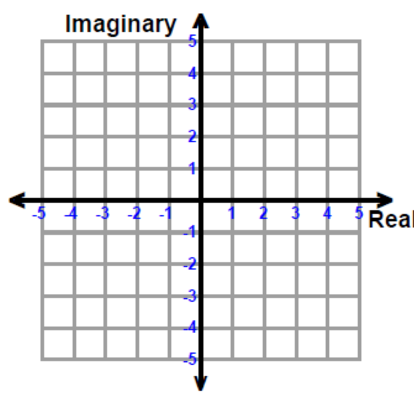
$|-5 - 5i|$ = ________________

23. $-2 + 4i$



$|-2 + 4i|$ = ________________

24. $5 - i$

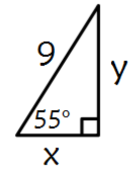


$|5 - i|$ = ________________

7.05 Complex Numbers in Rectangular Form **Date:** ____________________

Opener: where we have been this year?

1. From <u>right triangle trigonometry</u>: In the triangle to the right, find x and y.



2. From <u>vectors</u>: For a bird flying 20m West and 35m North, find the resulting magnitude and direction (measured in standard position) of its flight.

3. From <u>polar coordinates</u>: convert (-2, -2) from rectangular form into polar form.

Complex Numbers:

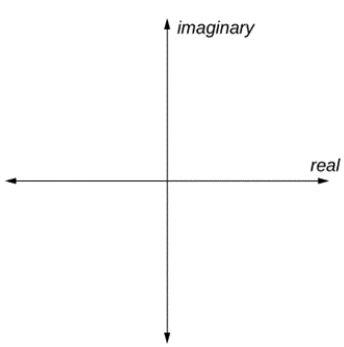
Rectangular Form, also known as *Standard Form*:

Graphing a complex number:

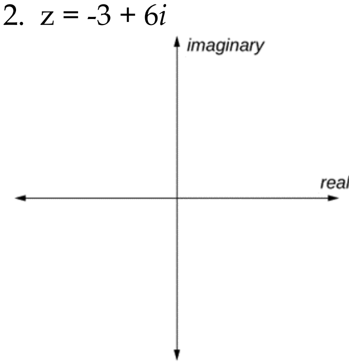
Absolute Value of a complex number, also known as the *modulus*:

Examples: Graph each number in the complex plane and find its modulus.

1. $z = 8 - i$



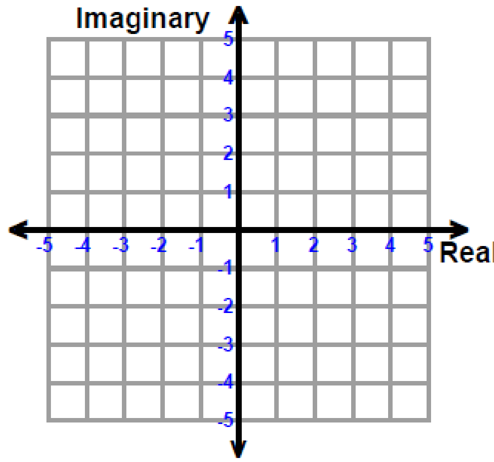
2. $z = -3 + 6i$



Distance & Midpoint between Complex Numbers

Investigation: Find the distance between complex numbers $z_1 = 3 + i$ and $z_2 = -4 + 3i$.

First, a visual usually helps, so plot the complex numbers.



How would you find the distance between those two points?

Formula: The distance between two complex numbers is

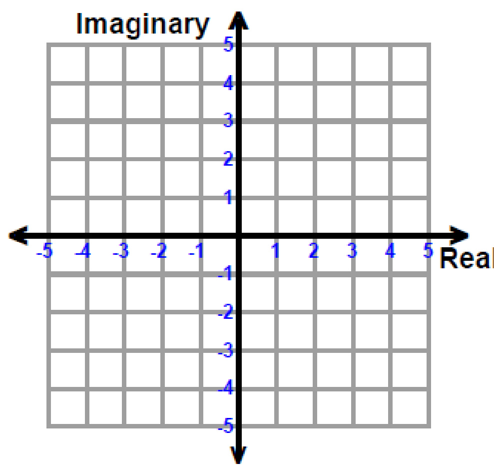
Examples: Find the distance between the two complex numbers.

1) $z_1 = 5 - 3i$ and $z_2 = -1 - 8i$

2) $z_1 = -8 + 4i$ and $z_2 = 1 + 7i$

Investigation: Find the midpoint between complex numbers $z_1 = 3 + i$ and $z_2 = -4 + 3i$.

Again, plot the complex numbers so that you can "see" this.



How would you find the midpoint between the two points?

Formula: The midpoint between two complex numbers is

Example: Find the midpoint between the two complex numbers

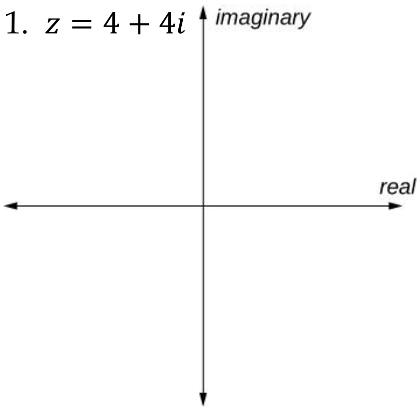
3) $z_1 = 5 - 3i$ and $z_2 = -1 - 8i$

4. $z_1 = -8 + 4i$ and $z_2 = 1 + 7i$

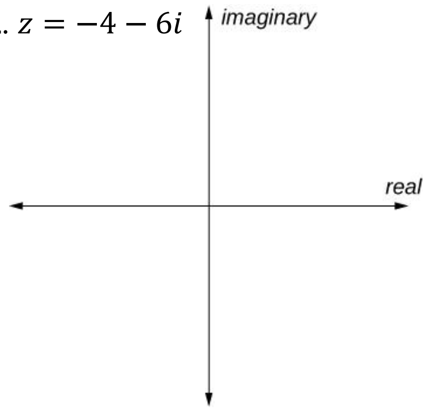
7.05 Homework: Rectangular Form of Complex Numbers

Plot each complex number and find its modulus.

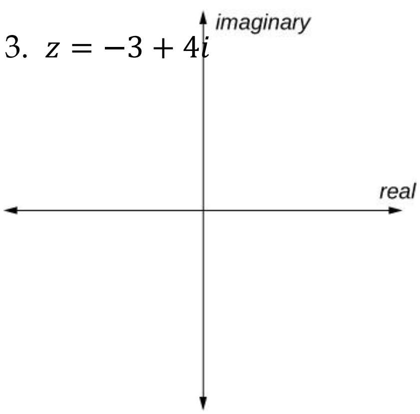
1. $z = 4 + 4i$



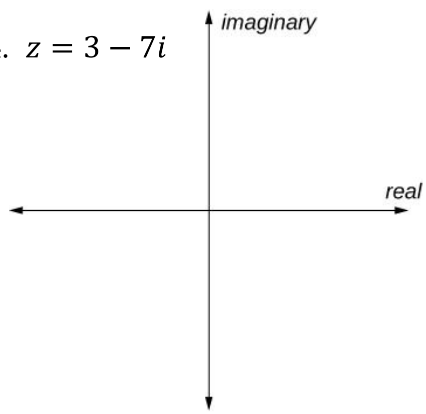
2. $z = -4 - 6i$



3. $z = -3 + 4i$



4. $z = 3 - 7i$



Find the distance between the points in the complex plane.

5. $1 + 2i$, $-1 + 4i$

6. $-5 + i$, $-2 + 5i$

7. $6i$, $3 - 4i$

8. $-7 - 3i$, $3 + 5i$

Find the midpoint of the segment connecting the points in the complex plane.

9. $2 + i$, $6 + 5i$

10. $-3 + 4i$, $1 - 2i$

11. $7i$, $9 - 10i$

12. $-1 + \frac{1}{2}i$, $\frac{1}{2} + \frac{1}{4}i$

7.06 Adding & Subtracting Complex Numbers Geometrically **Date:** ____________________

Recall that complex numbers take the form a + b*i*.

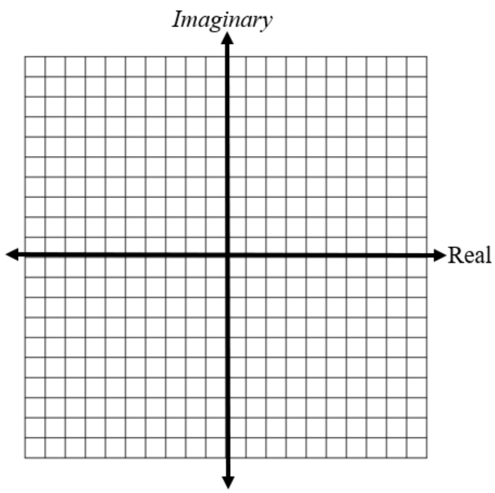
When adding or subtracting complex numbers algebraically, real parts are added together or subtracted then imaginary parts are added together or subtracted - similar to combining like terms.

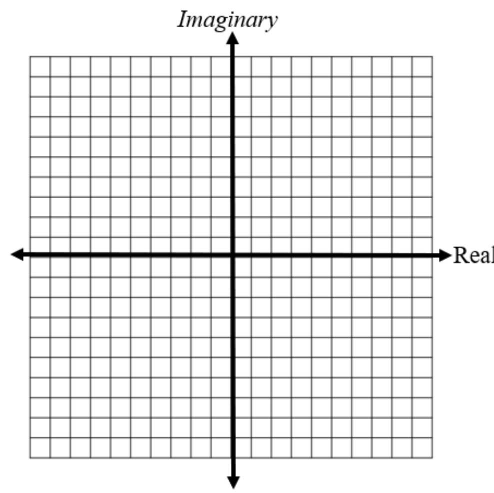
Complex numbers can also be added or subtracted geometrically/graphically by plotting the points in the complex plane and creating vectors with them. Then using geometric vector addition or subtraction.

Examples: 1. $(5 + 2i) + (3 + 6i) =$ 2. $(-3 + 4i) + (-10i) =$

Algebraically:

Geometrically:

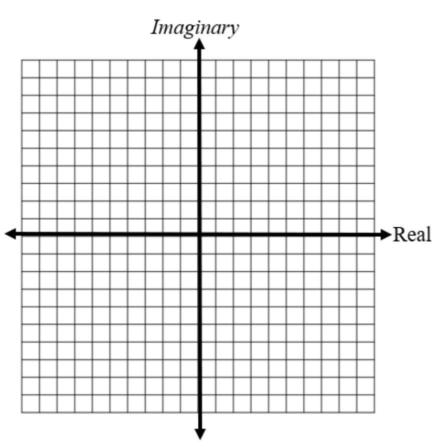


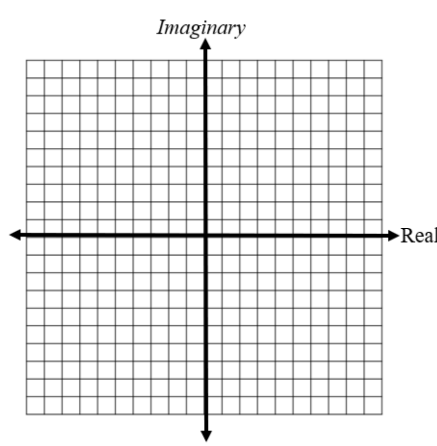


3. $(5 - i) - (6 - 5i) =$ 4. $(-8 - 2i) - (-10 - 8i) =$

Algebraically:

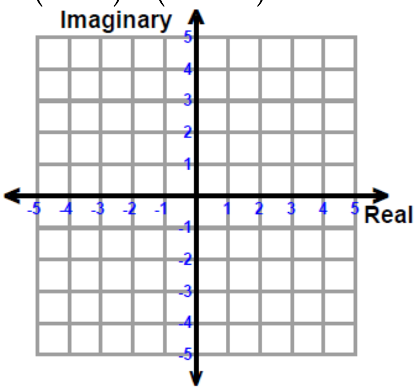
Geometrically:



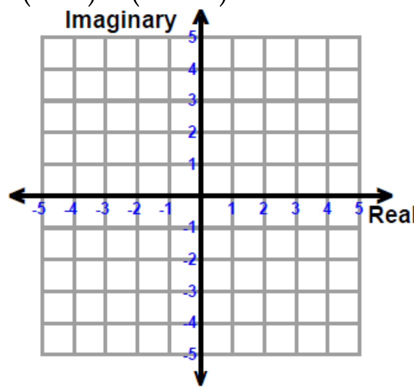


7.06 Practice: Evaluate each sum or difference geometrically, then verify your answer using algebra.

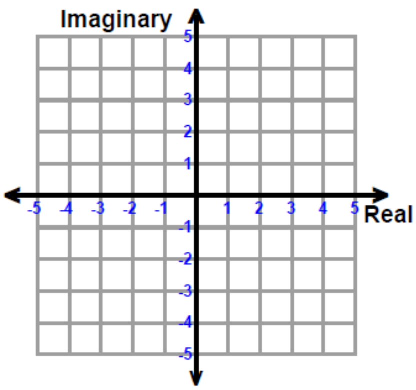
1. $(5 + 3i) + (-10 - 7i)$



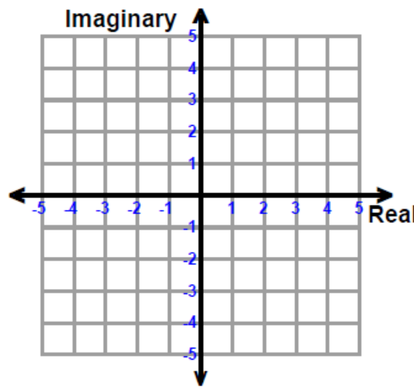
2. $(-5 - i) + (-3 + 6i)$



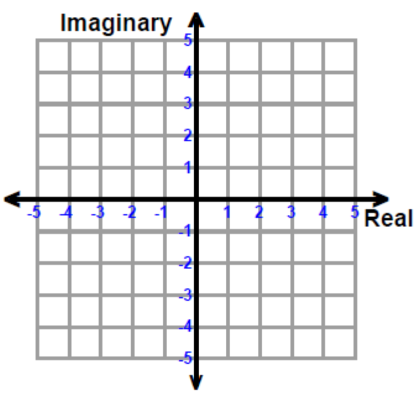
3. $(4 - 2i) + (-6 - 2i)$



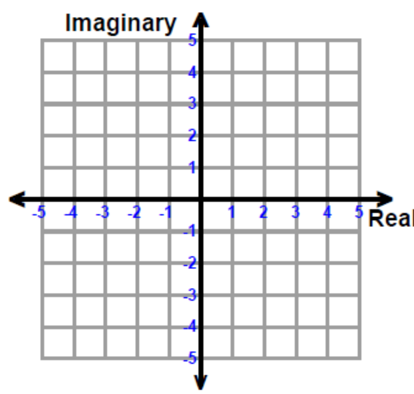
4. $-4i + (3 - i)$



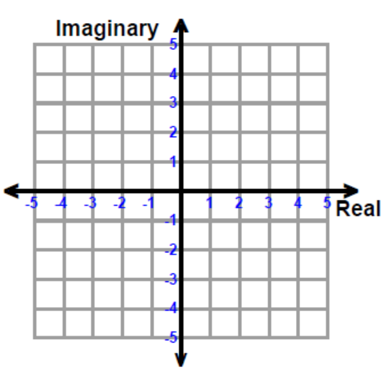
5. $(1 - 5i) - (3 - 8i)$



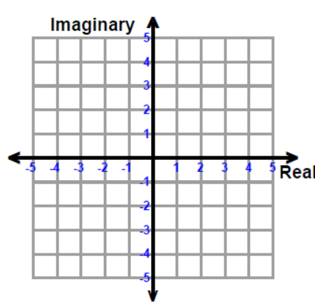
6. $4i - (4 + i)$



7. $(2 + 3i) - (-3 + 3i)$



8. $(-5 - 5i) - (-4 - 2i)$

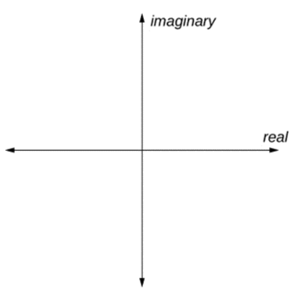


7.07 Complex Numbers in Polar Form **Date:** ____________________

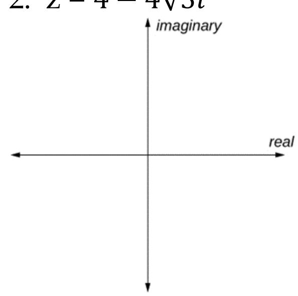
Polar Form of Complex Numbers, also know as *Trigonometric Form*:

Examples: Graph each complex number on the rectangular plane. Then, find its polar form, where 0 ≤ θ ≤ 2π. Be exact.

1. z = $10i$

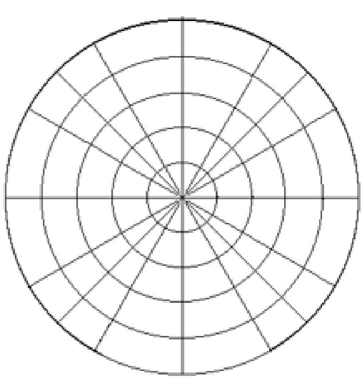


2. z = $4 - 4\sqrt{3}i$

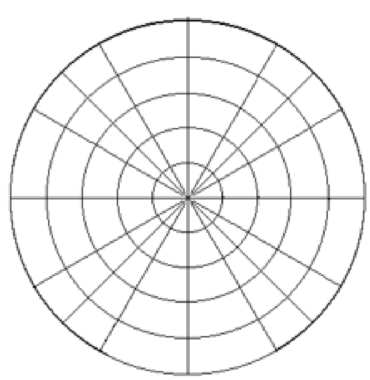


Examples: Graph each complex number on the polar plane. Then, find its rectangular form. Be exact.

3. $5(\cos 120° + i \sin 120°)$



4. $-2\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right)$



7.07 Homework: Directions: Be exact. Work these problems without using a calculator!

Write the polar form of each complex number where $0 \le \theta \le 2\pi$.

1. $z = 2 - 2i$

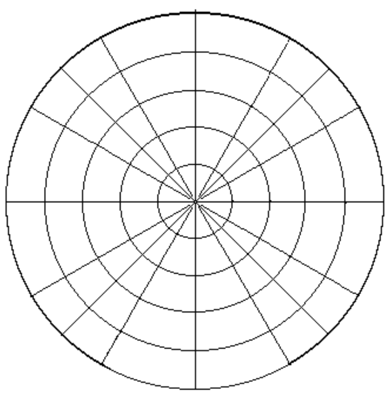
2. $z = 3 + 3i$

3. $z = -\sqrt{3} + i$

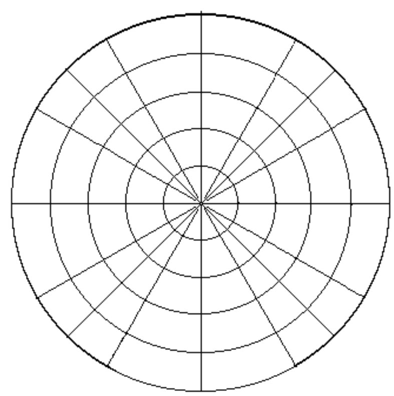
4. $z = -5 - 5\sqrt{3}i$

Graph each number on a polar grid. Then express it in rectangular form.

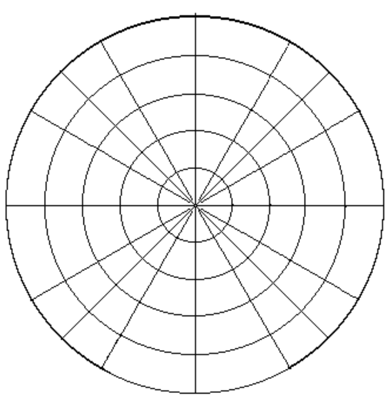
5. $z = 3\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$



6. $z = 2\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)$



7. $z = \left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right)$



7.08 Operations with Complex Numbers in Polar Form **Date:** ____________________

Find the *product* of two complex numbers in polar form: derive the formula.

For $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$

$z_1 \cdot z_2 =$

Examples: Find the product of the complex numbers in polar form. Answer in <u>both</u> polar form and rectangular form.

1. $z_1 = 4(\cos 225° + i\sin 225°)$ and $z_2 = 3(\cos 90° + i\sin 90°)$

2. $z_1 = \sqrt{2}\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$ and $z_2 = \frac{1}{5}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

Division is the opposite operation from Multiplication. How do you think the pattern changes when we divide two complex numbers in polar form?

For $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$

$z_1 \div z_2 = \frac{z_1}{z_2} =$

Example: Find the quotient of the complex numbers in polar form: $\frac{z_1}{z_2}$. Write the answer in <u>both</u> polar form and rectangular form.

3. $z_1 = 2(\cos 210° + i\sin 210°)$ and $z_2 = 8(\cos 60° + i\sin 60°)$

4. $z_1 = \frac{2}{5}\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$ and $z_2 = \frac{1}{2}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)$

7.08 Practice: Simplify. Express answers in **both** polar form and in rectangular form. Match angle measurement units to the problem, where $0° \leq \theta \leq 360°$ or $0 \leq \theta \leq 2\pi$.

1. $6\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) \cdot 4\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

2. $5(\cos 135° + i\sin 135°) \cdot 2(\cos 45° + i\sin 45°)$

3. $3\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right) \div \frac{1}{2}(\cos\pi + i\sin\pi)$

4. $2(\cos 90° + i\sin 90°) \cdot 2(\cos 270° + i\sin 270°)$

5. $3\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) \div 4\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

6. $4\left(\cos\frac{9\pi}{4} + i\sin\frac{9\pi}{4}\right) \div 2\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$

7. $\frac{1}{2}(\cos 60° + i\sin 60°) \cdot 6(\cos 150° + i\sin 150°)$

8. $6\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right) \div 2\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

9. $5(\cos 180° + i\sin 180°) \cdot 2(\cos 135° + i\sin 135°)$

10. $\frac{1}{2}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) \div 3\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

7.09 More Operations with Complex Numbers in Polar Form **Date:** ______________

Powers are the shorthand for repeated Multiplication. How do you think the pattern changes when we raise a complex number in polar form to an exponent?

For $z = r(\cos\theta + i\sin\theta)$

$z^n =$

Examples: Find the power of the complex number in polar form. Answer in <u>both</u> polar form and rectangular form.

1. $z^5 = \left[3\sqrt{2}\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)\right]^5$

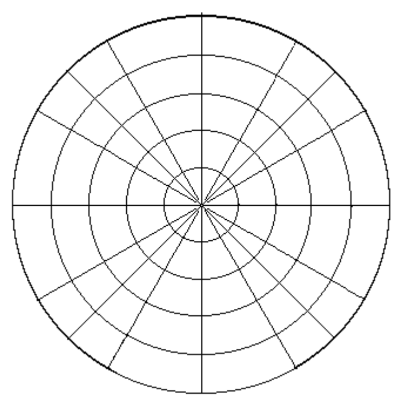
Investigation:

Use multiplication (in rectangular form) **<u>or</u>** the power rule (in polar form): $(-1 + \sqrt{3}i)^3$

Again, use either method listed above: $(-1 - \sqrt{3}i)^3$

What do you notice?

What is $\sqrt[3]{8}$ equivalent to? Plot your answers in the complex plane.



Can we use DeMoivre's Theorem (the Power Rule above) to derive a formula for evaluating roots of complex numbers in polar form?

Example: Find all distinct fourth roots of $-5+12i$.

7.09 Practice: Simplify. Express answers in **both** polar form and in rectangular form. Match angle measurement units to the problem, where $0° \leq \theta \leq 360°$ or $0 \leq \theta \leq 2\pi$.

1. $\left[8\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)\right]^3$

2. $\left[4\sqrt{3}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)\right]^6$

3. $\left[5\sqrt{2}(\cos 120° + i\sin 120°)\right]^5$

Convert each complex number into polar form. Then find each power. Answer in polar form where $0 \leq \theta \leq 2\pi$.

4. $\left(4\sqrt{3} - 4i\right)^3$

6. $(-1 - i)^6$

7. $\left(-1+\sqrt{3}i\right)^5$

Convert each complex number into polar form. Then find all distinct roots of the complex number. Answer in polar form where $0 \leq \theta \leq 2\pi$.

8. Sixth roots of i

9. Fourth roots of $4\sqrt{3}-4i$

10. Fifth roots of unity (1)

7.10 More Practice with Operations of Complex Numbers

Find the product $z_1 \cdot z_2$ and the quotient $\frac{z_1}{z_2}$. Express answers in both polar and rectangular form. Match angle measurement units to the problem, where $0° \leq \theta \leq 360°$ or $0 \leq \theta \leq 2\pi$.

1. Let $z_1 = 7\left(\cos\frac{9\pi}{8} + i\sin\frac{9\pi}{8}\right)$ and $z_2 = 2\left(\cos\frac{\pi}{8} + i\sin\frac{\pi}{8}\right)$

2. Let $z_1 = 4(\cos 200° + i\sin 200°)$ and $z_2 = 25(\cos 150° + i\sin 150°)$

Convert each complex number into polar form. Then find each power. Answer in polar form where $0 \leq \theta \leq 2\pi$.

3. $\left(1 - \sqrt{3}i\right)^4$

4. $\left(-\sqrt{2} + \sqrt{2}i\right)^5$

Convert each complex number into polar form. Then find all distinct roots of the complex number. Answer in polar form where $0 \leq \theta \leq 2\pi$.

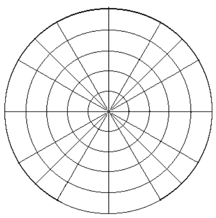
5. Fifth roots of 32

6. Fourth roots of $-81i$

7.11: Test Review Date ____________________

First plot each point, given as polar coordinates. Then, determine 3 other coordinates for the same point. Use -360° ≤ θ ≤ 360° if in degrees, or use -2π ≤ θ ≤ 2π if in radians. ***No calculator**

1. A = (-4, 135°)



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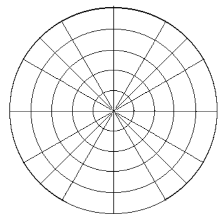
2. B = $\left(2, \frac{\pi}{3}\right)$



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Find the distance between the polar points. Use the polar method: $\sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_2 - \theta_1)}$

3. (-6, 210°) and (4, -45°)

4. $\left(1, \frac{2\pi}{3}\right)$ and $\left(-5, -\frac{7\pi}{6}\right)$

Convert the given rectangular coordinates into polar coordinates, where 0 ≤ Θ ≤ 2π.

5. (-3, 3) ***No calculator**

6. $(-4\sqrt{5}, -2)$

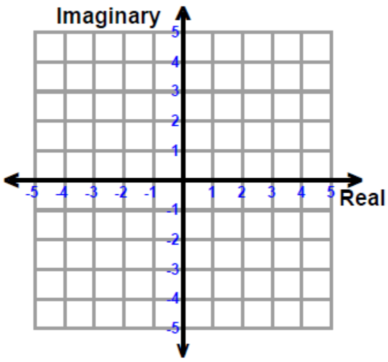
Convert the given polar coordinates into rectangular coordinates.

7. (14, 210°) ***No calculator**

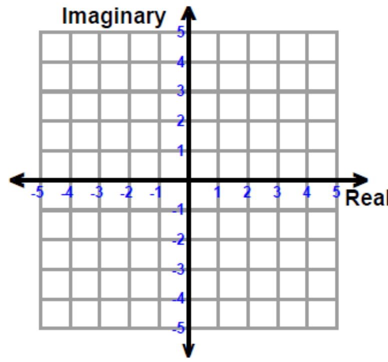
8. $\left(2\sqrt{3}, \frac{11\pi}{7}\right)$

Simply each expression using geometric methods. ***No calculator**

9. $(3 + 2i) + (1 - 5i)$



10. $(-4 + i) - (1 - 2i)$



Find the distance between the complex numbers. ***No calculator**

11. $(13 + 2i)$ and $(9 - 5i)$

12. $(-8 + 5i)$ and $(-2 - i)$

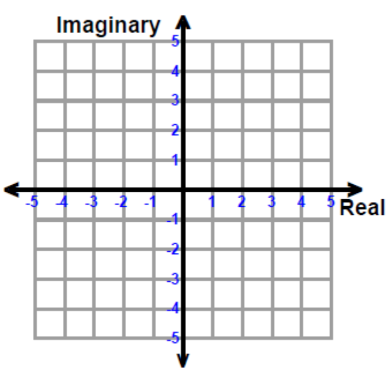
Find the midpoint between the complex numbers. ***No calculator**

13. $(13 + 2i)$ and $(9 - 5i)$

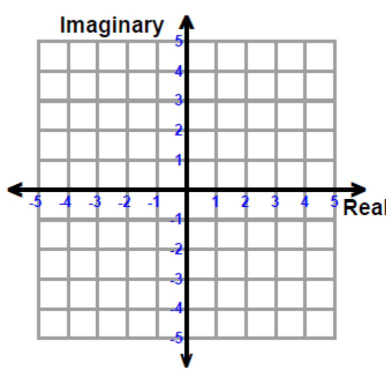
14. $(-8 + 5i)$ and $(-2 - i)$

Graph each complex number, find its modulus (absolute value) and argument (direction), and then write in polar form, where $0 \leq \theta \leq 2\pi$. ***No calculator**

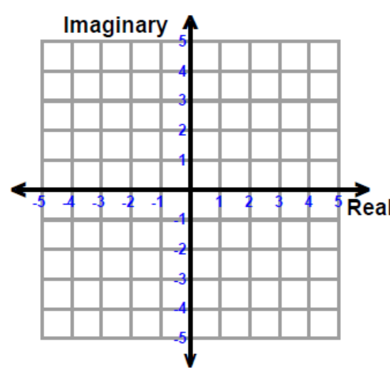
15. $z = -1 + \sqrt{3}i$



16. $z = 2 - 2i$



17. $z = -5i$



Modulus:

Modulus:

Modulus:

Argument:

Argument:

Argument:

Polar: Polar: Polar:

18. Convert $z = -5 + 12i$ to polar form, where $0 \leq \Theta \leq 2\pi$.

19. Convert $z = 4\sqrt{3}(\cos 30° + i \sin 30°)$ to rectangular form. ***No calculator**

Simplify each expression using polar methods. Answer in polar form, where $0 \leq \Theta \leq 2\pi$.
***No calculator**

Given: $z_1 = 3\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)$, $z_2 = 4\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$, $z_3 = 2\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

20. $z_1 \cdot z_2$

21. $z_2 \cdot z_3$

22. $\frac{z_1}{z_2}$

23. $\frac{z_3}{z_2}$

24. $(z_2)^4$

25. $(z_3)^3$

26. Find the cube roots of z_2.

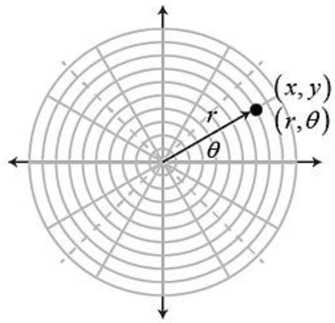
27. Find the fourth roots of z_1

Accelerated Pre-Calculus
February & March 2022
Unit 7 – Polar Graphs & Complex Numbers

Monday	Tuesday	Wednesday	Thursday	Friday
Feb 21 **No School President's Day**	22 7.01 Polar Coordinates • Plot points • Multiple representations HW: 7.01 Practice	23 7.02 Polar Coordinates • Convert btw Rectangular & Polar • Multiple representations • Distance Formula HW: 7.02 Practice	24 7.03 Polar Coordinate Review HW: Polar Review	25 **7.04 Quiz- Polar Coordinates & Converting Points with Rectangular System**
28 **Early Release Day** 7.05 Complex Numbers in Rectangular Form • Absolute Value • Modulus • Distance Between • Midpoint HW: 7.05 Practice	Mar 1 7.06 Adding & Subtracting Complex Coordinates Geometrically HW: 7.06 Practice	2 **Check-In Quiz** 7.07 Complex Numbers in Polar Form • Modulus and Argument HW: 7.07 Practice	3 7.08 Operations with Complex Numbers in Polar Form • Product • Quotient HW: 7.08 Practice	4 7.09 More Complex Number Operations • Power • Roots HW: 7.09 Practice
7 7.10 More Practice with Operations HW: 7.11 Review	8 7.11 Review HW: Study!	9 **Test: Polar and Complex**	10 TASK: Battleship - Star Wars Edition!	11 **No School Teacher Work Day**

Polar Coordinates, (r, θ):

$x = r\cos\theta$
$y = r\sin\theta$



$r = \sqrt{x^2 + y^2}$
$\theta = \tan^{-1}\left(\frac{y}{x}\right)$ if $x > 0$
or $\theta = \tan^{-1}\left(\frac{y}{x}\right) + \pi$, if $x < 0$

Distance between two points on the polar plane: $\sqrt{{r_1}^2 + {r_2}^2 - 2r_1r_2\cos(\theta_2 - \theta_1)}$

Complex Numbers, Rectangular (Standard) form: z = a + bi

Absolute value (modulus): $|z| = \sqrt{a^2 + b^2}$

Distance between 2 complex numbers is the modulus of their difference: $|z_1 - z_2|$

Midpoint between 2 complex numbers is the average of the values: $\frac{z_1+z_2}{2}$

Polar (Trigonometric) Form of a complex number: $\boldsymbol{z = r(\cos\theta + i\sin\theta)}$ **or** $\boldsymbol{r\ cis\ \theta}$
Where $a = r\cos\theta$, $b = r\sin\theta$, $r = \sqrt{a^2 + b^2}$, and $\tan\theta = \frac{b}{a}$ (remember to add π if a < 0)

Multiplication of Complex Numbers
$z_1 \cdot z_2 = r_1r_2[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)]$

Division of Complex Numbers
$\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)], r_2 \neq 0$

De Moivre's Theorem (Powers of a Complex Number)
$z^n = [r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$

nth Roots of a Complex Number
$\sqrt[n]{r}\left(\cos\frac{\theta + 2\pi k}{n} + i\sin\frac{\theta + 2\pi k}{n}\right), k = 0,1,2, \ldots, n-1$