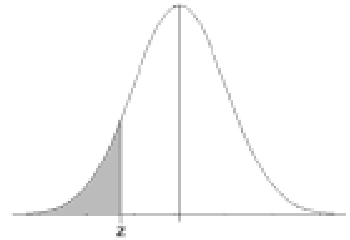
Name:____ **Period:** ___________

Accel. Pre-Calculus

Unit 9 Packet

Statistics Unit

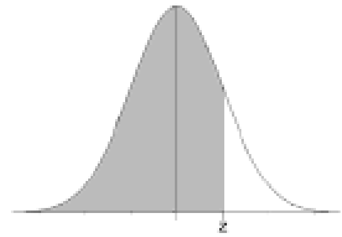
Standard Normal Cumulative Probability Table



Cumulative probabilities for NEGATIVE z-values are shown in the following table:

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Standard Normal Cumulative Probability Table



Cumulative probabilities for POSITIVE z-values are shown in the following table:

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Accelerated Pre-Calculus **Date:**

____

9.01: Review of Measures of Center and Spread

3 MEASURES OF CENTRAL TENDENCY		
Mean	**Median**	**Mode**
Denoted as $\bar{x}$, "x-bar" the average $\bar{x} = \frac{\sum x}{n}$	The number in the middle when the data is arranged in ascending order. If there are 2 numbers in the middle, then find their average.	The number which occurs most frequently. There does not have to be a mode. There can be more than one mode. Bimodal – 2 modes Trimodal – 3 modes

Example 1: Given scores from the latest test: 90, 89, 78, 81, 68, 100, 84, 83, 83, 74, 88, 80, 73, 89, 32

a) Find the measures of central tendency. Don't forget to put the data in ascending order!!!

Mean: ________________ Median: ________________ Mode: ________________

5 NUMBER SUMMARY				
Minimum (Lower Extreme)	**Lower (1st) Quartile** Q_1	**Median (2nd Quartile)** Q_2	**Upper (3rd) Quartile** Q_3	**Maximum (Upper Extreme)**
Smallest number	The median of the lower half. If there are 2 numbers find their average.	Divides the data into a lower and upper half.	The median of the upper half. If there are 2 numbers find their average.	Largest number

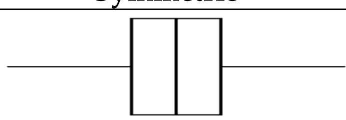
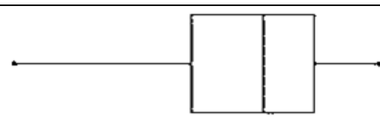
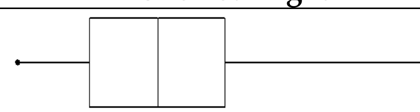
b) Find the 5-Number Summary of the test data above.

Min: _________ Q_1: __________ Median: __________ Q_3: __________ Max: __________

Box and Whisker Plot – A plot that displays the 5 number summary:

1. Draw a number line and scale it appropriately. Keep the minimum and maximum in mind.
2. Place points above the number line for each number in the 5 number summary.
3. Connect the minimum and Q_1 with a segment as well as Q_3 and the maximum.
4. Draw a box from Q_1 to Q_3.
5. Draw a vertical segment through the median.

c) Draw a box and whisker plot for the previous test data.

SHAPE OF A BOX AND WHISKER PLOT		
Symmetric	**Skewed Left**	**Skewed Right**
		

MEASURES OF DISPERSION (SPREAD)		
Range	**Interquartile Range**	**Mean Absolute Deviation**
The difference in the **maximum** and the **minimum**. (Max – Min)	The difference in the **upper quartile** and **lower quartile**. ($Q_3 - Q_1$)	$\text{MAD} = \frac{\sum \lvert x_i - \bar{x} \rvert}{n}$

d) Find the measures of spread for the given data set of test scores.

Range = ___________ IQR = ___________ MAD = ______________

Example 2:

a) List the number of pets from 8 of your classmates.

b) Calculate all measures of center, and the 5 number summary for the data.

$\bar{x}$ = ________________ Median: ________________ Mode: ________________

Min: __________ Q_1: __________ Median: __________ Q_3: __________ Max: __________

c) Construct a box plot and describe the shape of the data.

Shape: ______________________

d) Calculate the measures of spread.

Range = _______________ IQR = _______________ MAD = _______________

BEST MEASURE OF CENTER AND SPREAD	
SYMMETRIC WITH NO OUTLIERS	SKEWED or WITH OUTLIERS
Mean and Mean Absolute Deviation (MAD)	Median and Interquartile Range (IQR)

9.01 Homework: Statistics Review **Date:** ____________

1. Calculate all measures of center, spread, and the 5 number summary for the data provided. Construct a box plot and describe the shape of the data. Indicate if there are any outliers.

Homerun distances (in feet) to center field in 13 ballparks:
387, 400, 400, 410, 410, 410, 414, 415, 420, 420, 421, 457, 461

$\bar{x}$ = ________________ Median: ________________ Mode: ________________

Min: __________ Q_1: __________ Median: __________ Q_3: __________ Max: __________

Range = __________ IQR = __________ MAD = __________

Describe the Shape/Skew/Outliers:

2. Suppose that the numbers of unnecessary procedures recommended by five doctors in a 1-month period are 2, 2, 8, 10, and 18. If we ask a 6th doctor and find out that they recommend 35 procedures.

   (a) How will the Median and Mean be affected?

   (b) How will the IQR and Mean Absolute Deviation be affected?

3. Suppose the salaries (in dollars) of six employees are: 8000, 10000, 15000, 16000, 20000 and 39000.

   a. What are the Median and Mean salaries?

   b. Why are they such different numbers?

   c. Which measure of center is the better pick for this data? Why?

   d. Find the Mean Absolute Deviation.

4. Based solely on the given mean and median, decide on the shape of each distribution:

   (a) Mean = 100 Median = 98 Shape:

   (b) Mean = 20 Median = 41 Shape:

   (c) Mean = 934 Median = 850 Shape:

5. Give a set of numbers that would have a Mean Absolute Deviation of 0 units.

9.02 Standard Deviation Notes **Date** ________________

Statisticians use the Standard Deviation to discuss dispersion (spread) of data rather than the Mean Absolute Deviation (MAD).

The average of the squared differences of the mean is the ________________________.

The _____________________ _______________________ is the average distance from the mean. It tells us how tightly the data values are clustered around the mean.

MORE MEASURES OF DISPERSION (SPREAD)		
Variance	**Standard Deviation for the whole population**	**Standard Deviation for a sample**
$\sigma^2 = \frac{\sum(x_i - \bar{x})^2}{n}$	$\sigma = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n}}$	$s = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n-1}}$

Example 1: Compare the spread of the data for the two sets.

Falcons Football
Points Scored in 2008
Regular Season Games

Home Games	Away Games
34	9
38	9
22	27
34	14
20	24
45	22
13	25
31	24

a. Calculate the range of each set: **Home:** _________________

Away: _________________

b. The range of the home games is _____ the range of the away games. **Explain what this means:**

c. Calculate the Mean of each set:

$\bar{x}$ **(Home games) =** __________

$\bar{x}$ **(Away games) =** __________

d. Calculate the IQR for each set:

Home: _____________________

Away: _____________________

e. The IQR of the home games is _____ the IQR of the away games. Explain what this means:

To calculate the <u>Standard Deviation</u> of a data set, find how far (the difference) each data value ($x_1, x_2, x_3, \ldots, x_n$) is from the mean ($\bar{x}$). These are the <u>deviations</u> from the mean. Square the differences and then find the <u>average</u> by adding them together and dividing by the number of data values for a population (n) or by (n – 1) for a sample. This number is the variance: σ^2. The Standard Deviation, σ, is the square root of the variance.

Ex 1 (cont'd) Falcons Points Scored in 2008 Regular Season Games

Home: $\bar{x} =$	**Away:** $\bar{x} =$
$(34 - \quad)^2 =$	$(9 - \quad)^2 =$
$(38 - \quad)^2 =$	$(9 - \quad)^2 =$
$(22 - \quad)^2 =$	$(27 - \quad)^2 =$
$(34 - \quad)^2 =$	$(14 - \quad)^2 =$
$(20 - \quad)^2 =$	$(24 - \quad)^2 =$
$(45 - \quad)^2 =$	$(22 - \quad)^2 =$
$(13 - \quad)^2 =$	$(25 - \quad)^2 =$
$(31 - \quad)^2 =$	$(24 - \quad)^2 =$
Sum of Dev^2:	Sum of Dev^2:
To calculate the variance, σ^2, divide the sum of deviation^2 by n (since this is the whole population and not sample)	
$\sigma^2 =$	$\sigma^2 =$
To calculate the standard deviation, σ, take the square root of the variance.	
$\sigma =$	$\sigma =$

f. σ of home games:

g. σ of away games:

σ (home) _____ σ (away)

h. Explain the difference in σ:

Example 2: Use test scores (from the previous lesson): **90, 89, 78, 81, 68, 100, 84, 83, 83, 74, 88, 80, 73, 89, 32** to calculate the following:

Show calculations for standard deviation here:

$\bar{x}$ = ____________ σ = ____________

Median = _________ IQR = _____________

Which measure of center and spread should be used to describe the data? Justify your response.

9.02 Standard Deviation Homework **Date _________________**

Mrs. Durand has a problem and needs your input. She has to give one math award this year to a deserving student, but she can't make a decision. Here are the test grades for her two best students:

Emily: 90, 90, 80, 100, 99, 81, 98, 82 **Jacob: 90, 90, 91, 89, 91, 89, 90, 90**

1. Which of the two students should get the math award? Discuss why he/she should be the recipient.

2. Calculate the *mean deviation*, *variance*, and *standard deviation* of Emily's distribution. The formulas are below. Fill out the table to help calculate by hand.

   Mean deviation: $\frac{\sum |x_i - \bar{x}|}{n}$ variance: $\sigma^2 = \frac{\sum (x_i - \bar{x})^2}{n}$ standard deviation: $\sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}$

   The mean of Emily's test scores: _____________

x_i for Emily	$x_i - \bar{x}$	$\lvert x_i - \bar{x} \rvert$	$(x_i - \bar{x})^2$
90			
90			
80			
100			
99			
81			
98			
82			

   a. Mean deviation for Emily:

   b. Variance for Emily:

   c. Standard deviation for Emily:

d. What do these measures of spread tell you?

3. Calculate the *mean deviation*, *variance*, and *standard deviation* of Jacob's distribution. The formulas are below. Fill out the table to help calculate by hand.

Mean deviation: $\frac{\sum |x_i - \bar{x}|}{n}$ variance: $\sigma^2 = \frac{\sum (x_i - \bar{x})^2}{n}$ standard deviation: $\sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}$

The mean of Jacob's test scores: ____________

x_i for Jacob	$x_i - \bar{x}$	$\vert x_i - \bar{x} \vert$	$(x_i - \bar{x})^2$
90			
90			
91			
89			
91			
89			
90			
90			

a. Mean deviation for Jacob:

b. Variance for Jacob:

c. Standard deviation for Jacob:

d. What do these measures of spread tell you?

4. Based on this information about the spread of test scores for Emily and Jacob, which of the two students should get the math award and discuss why he/she should be the one to receive it.

9.03 Using Technology to Calculate Standard Deviation **Date:** ______________

Mean, standard deviation, and many other statistical measurements can be calculated using a scientific or graphing calculator. Use the steps that apply to your type of calculator:

TI-30XS MultiView or TI-36X Pro

1. Hit [data]
2. To clear a list, hit [data] again and select the list to clear.
3. Enter your data in a list (under L1).
4. Hit [2^{nd}] [data] and then select "1-Var Stats".
5. Select the List your data is stored in (L1), and the frequency for each value recorded in that list (FRQ:One), and then enter "CALC".

TI Graphing

1. Hit [STAT] and select "1:Edit…".
2. To clear a list, arrow up to the list name above the list "L1", hit [CLEAR] and then [ENTER].
3. Enter your data under L1.
4. Hit [STAT] again, move over to "CALC" and then select "1-Var Stats".
5. Indicate the list you put your data in, like L1. If you need to specify a different list, press [2nd] and [1] or [2] or [3] etc, depending on the list name needed.
6. Select "Calculate" or hit [ENTER] (wording depends on OS).

You will receive a list of the following info:

- $\bar{x}$ = ______ mean
- $\sum x$= ______ sum of all data
- $\sum x^2$ = ______ sum of all squared data
- Sx = ______ standard deviation of a SAMPLE
- σx = ______ standard deviation of a POPULATION
- n = ______ how many pieces of data in the set
- minX = ______ minimum
- Q1 = ______ lower quartile
- Med = ______ median
- Q3 = ______ upper quartile
- maxX = ______ maximum

Example 1. Find the mean, median, range, interquartile range, and standard deviation for the following data sets.

a. 35, 27, 39, 41, 41, 38, 28, 33, 35, 37, 40

Mean	
Median	
Range	
IQR	
Standard Deviation	

b. 84, 85, 105, 76, 73, 93, 81, 74, 84, 80, 72

Mean	
Median	
Range	
IQR	
Standard Deviation	

Example 2. Compare the mean and standard deviation of the following data sets.

a. mpg of hybrids: 50, 37, 42, 40, 39, 38, 41
Mean: _______ Std. Dev.: _________

b. mpg of sedans: 28, 19, 24, 22, 18, 24, 26
Mean: _______ Std. Dev.: _________

What are the implications of these statistics?

9.03 Practice: Find the range and standard deviation of each data set.

1. 22, 18, 19, 25, 27, 21, 24

2. 38, 46, 37, 42, 39, 40, 48, 42

3. 8.4, 7.7, 8.6, 7.5, 8.9, 7.8, 8.6, 9.1, 8.0

4. 1.25, 3.69, 5.67, 4.89, 0.12, 4.35, 2.78

5. 515, 720, 635, 895, 585, 690, 770, 840

6. 116, 105, 117, 124, 107, 112, 117, 125, 110, 113

Find the mean, median, mode, range, and standard deviation of each data set.

7. Price (in dollars) of 7 different cordless phone models at an electronics store: 35, 50, 60, 60, 75, 65, 80

8. Number of homeruns for the 10 batters who hit the most homeruns during the 2014 MLB regular season: 32, 35, 40, 37, 35, 36, 32, 34, 37, 36

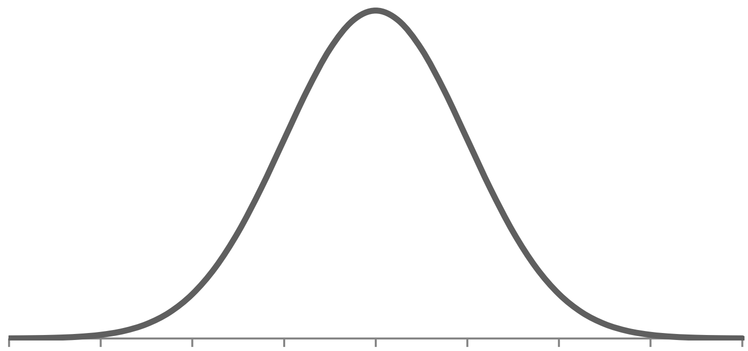
9. Waiting times (in minutes) for several people at a Georgia Department of Driver Services office: 11, 7, 14, 2, 8, 13, 3, 6, 10, 3, 8, 4, 8, 4, 7

10. Calories in a 1-ounce serving of several breakfast cereals: 135, 115, 120, 110, 110, 100, 105, 110, 125

9.04 Normal Distribution - The Empirical Rule Date ____________________

Normal Distribution is modeled by a normal (bell) curve and is symmetric about the mean.

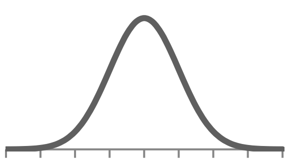
- It is formed using the mean and standard deviation.
- The total area under the curve is _____ because it represents all of the probability = ______ %
- Empirical Rule (68–95–99.7 Rule): the percent (probability) of the area under the curve for each standard deviation is shown on the graph below.



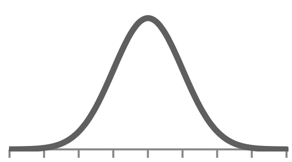
The normal distribution curve is used to find probability. It must be a normal distribution in order to use the above percentages (probabilities).

Example 1: For a normal curve, find the following probabilities of a randomly selected x-value from the distribution. If may be helpful to use a sketch.

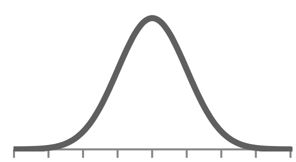
a. $P(x \leq \bar{x} - 2\sigma)$



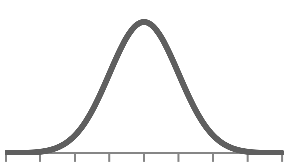
b. $P(x \geq \bar{x} + \sigma)$



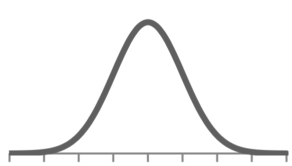
c. $P(\bar{x} - 2\sigma \leq x \leq \bar{x} + \sigma)$



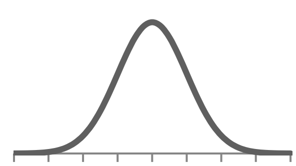
d. $P(x \geq \bar{x})$



e. $P(x \geq \bar{x} + 3\sigma)$



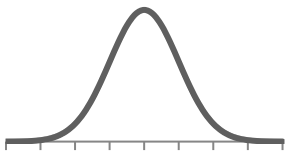
f. $P(\bar{x} - \sigma \leq x \leq \bar{x} + 3\sigma)$



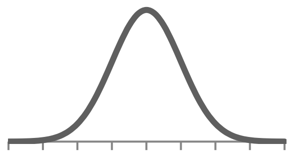
9.04 Homework – Normal Distribution & the Empirical Rule

1. Find the indicated probability for a randomly selected x-value from the distribution.

a. $P(x \leq \bar{x} + \sigma)$



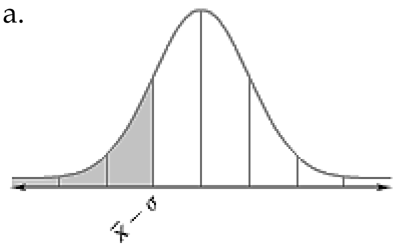
b. $P(x \geq \bar{x} - 2\sigma)$



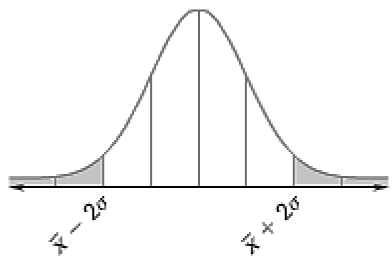
c. $P(\bar{x} - \sigma \leq x \leq \bar{x} + \sigma)$

2. Give the percent of the area under the normal curve represented by the shaded region.

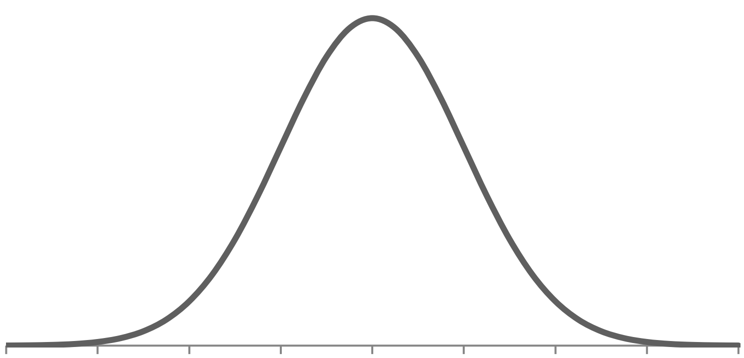
a.



b.



3. A normal distribution has a mean of 25 and a standard deviation of 5. Find the probability that a randomly selected x-value from the distribution is in the given interval. Label the x-axis and the probabilities under the curve below.



a. Between 20 and 30

b. Between 10 and 25

c. Between 15 and 35

d. At least 20

e. At least 35

f. At most 30

9.05 Applications of the Empirical Rule **Date: __________________**

A) Comparing the SAT and ACT: college admissions offices need to compare scores of students who take the Scholastic Aptitude Test (SAT) with those who take the American College Test (ACT). Suppose that for recent college applicants who took the SAT, scores have a mean of 1497 (out of 2400) and a standard deviation of 350. Further, suppose that for recent college applicants who took the ACT, scores have a mean of 21 (out of 36) and a standard deviation of 5.6.

1. Sketch normal curves for both the SAT and ACT listing values for 1, 2, and 3 standard deviations on each side of the mean.

Apply the empirical rule to approximate the following:

2. About 95% of SAT takers score between what two values?
3. About 95% of ACT takers score between what two values?
4. What is the proportion of students who score between 1147 and 1847 on the SAT?
5. What is the proportion of students who score between 15.4 and 32.2 on the ACT?
6. If John scored at the 84th percentile on the ACT, what score did he achieve?
7. College Board reports that 1,672,395 students took the SAT in 2014. About how many students achieved a score of at least 2197?
8. ACT, Inc. reports that 1,845,787 students took the ACT in 2014. About how many students achieved a score of at most 21?

B) Last spring, 250 students took the Algebra 2 final exam. The scores were distributed normally with a mean of 70 and a standard deviation of 5.

9. Sketch the normal curve for the final exam scores, listing values for 1, 2, and 3 standard deviations on each side of the mean.

Apply the empirical rule to approximate the following:

10. What percentage of scores is between scores 65 and 75?

11. What percentage of scores is between scores 60 and 70?

12. What percentage of scores is between scores 60 and 85?

13. What percentage of scores is less than a score of 55?

14. What percentage of scores is at least a score of 80?

15. How many Algebra 2 students achieved a score between 70 and 80?

16. How many Algebra students achieved a score of at most 75?

C) Statistics kept for NFL football teams regarding the number of injuries suffered by NFL players during their careers showed the distribution is approximately normal with the mean number of injuries per player to be 9 with a standard deviation of 2. If there are 1696 NFL players in the current season, determine how many players will have the following number of injuries:

17. Less than 9 injuries in their career.

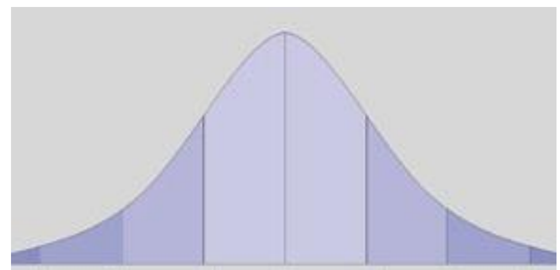
18. At least 7 injuries in their career.

19. More than 5 but less than 11 injuries in their career.

9.06 Standard Normal Distributions Date ___________________

What do you do when you are looking for the probability of an x-value in a normal distribution, but that value does not fall on one of the standard deviations?

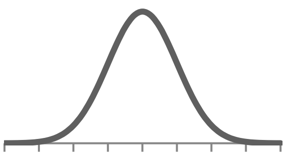
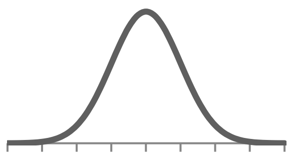
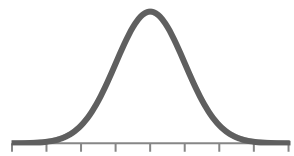
Standard Normal Distribution:



- Formed using a mean of 0 and a standard deviation of 1.
- Used when the x-value does not fall on a standard deviation.
- The Empirical Rule still applies to a standard normal distribution.
- To change an x-value from a normal distribution with mean $\bar{x}$ and standard deviation σ use the **z-score formula**: $z = \frac{x-\bar{x}}{\sigma}$
- The z-score is the number of standard deviations the x-value lies away from the mean.

Example 1: For a standard normal distribution, find the following probabilities of a randomly selected x-value from the distribution. If may be helpful to draw a sketch.

a. $P(z \le -1)$ b. $P(z \ge 1)$ c. $P(-2 \le z \le 3)$

Converting to a z-score:

Example 2: Consider a normal distribution with a mean of 72 and a standard deviation of 3. Convert the following into z-scores.

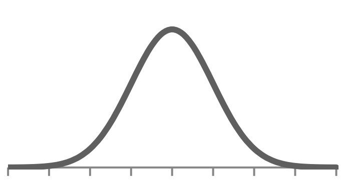
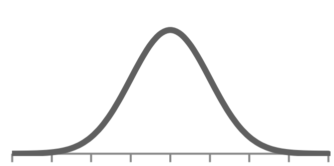
a. x = 65 b. x = 81 c. x = 76

WHAT'S THE POINT???

Z-scores enable us to find any probability with a table of values. The probability given on the z-score table is the probability that is **less than the z-score**. BE CAREFUL! We are not always looking for less than!!!!

Example 3: Consider a normal distribution with a mean of 72 and a standard deviation of 3. Sketch a graph and find the following:

a. $P(x \le 65)$ b. $P(x \ge 81)$ c. $P(76 \le x \le 81)$

9.06 Standard Normal Distribution Homework

1) A normal distribution has a mean of 70 and a standard deviation of 10.
Calculate the z-score and use the z-score table to find the indicated probability.

a. $P(x \le 65)$

b. $P(x \ge 47)$

c. $P(54 \le x \le 83)$

d. $P(x \le 91)$

e. $P(x \ge 39)$

f. $P(79 \le x \le 101)$

2) The scores on an Intelligence Quotient (IQ) test are normally distributed with a mean of 100 and a standard deviation of 15.

a. Draw and label the normal distribution.

b. What is the probability that a person will score at least a 121?

c. What is the probability that a person will score no more than 79?

d. What is the probability that a person will score between 92 and 111?

e. What minimum score would someone need to score higher than 80% of those taking an IQ test?

9.07 Standard Normal Distribution Applications **Date** ______________

All data in the following exercises is normally distributed.

1. Jack took a standardized achievement test with a mean of 125 and a standard deviation of 15. Jack's score was 148.
   a. What is the percentile rank for Jack's score on this test? Percentile rank means that Jack scored as well or better than that percentage of students taking the test.

   b. If Jill scored at the 67^{th} percentile, what was her score on the test?

2. The average number of absences for 1^{st} graders is 15 with a standard deviation of 6.
   a. What is the probability of a 1^{st} grader having fewer than 6 absences?

   b. What is the probability of a 1^{st} grader having more than 20 absences?

   c. If a student is absent more often than 30.85% of other 1^{st} graders, how many days did she miss?

3. On the Scholastic Aptitude Test (SAT), scores have a mean of 500 and a standard deviation of 100.
   a. Mo scored 600 on the math section. What percentile did he achieve?

   b. Larry scored 750 on the math section. What percentile did he achieve?

   c. Curley scored better than 97.72% of students on the math section of the SAT. What was his score?

4. A patient recently diagnosed with Alzheimer's disease took a cognitive abilities test. The mean score on this test is 52 with a standard deviation of 5.
   a. If the patient scored a 45 on the test, what is his percentile rank?

   b. If a patient scored higher than 87.7% of other Alzheimer's patients, what was her score on the test?

5. Mrs. Durand's Algebra 2 unit test scores have a mean of 82 with a standard deviation of 5.5.
   a. What is the probability that a student will score at least a 92?

   b. What is the probability a student will get a B?

   c. What is the probability a student will fail?

6. The diameter of maple trees in a Canadian forest have a mean of 10 inches and standard deviation of 3.2 inches. What is the percentage of trees with a diameter between 8 and 15 inches?

7. The weights of 1800 fish in a lake have a mean of 3 kilograms and standard deviation of .5 kilograms.
   a. What percentage of fish weigh at least 3.75 kilograms?

   b. Approximately how many fish in the lake weigh at least 3.75 kilograms?

9.09 Confidence Intervals **Date: ___________**

Opener: We plan to meet Saturday morning for a fun day at Six Flags. If I tell you that I will be there at 10:30, what time do you expect me to arrive? ________ Would any other times also be reasonable? If so, what are they? ______________________________

Would you be more confident that I will arrive "on time" if you make my window of arrival times wider or narrower? Why?

Population vs. Sample:

Population: includes all elements of a set of data

example:

Sample: includes a portion of a set of data

example:

Parameter: a number relating to the population

example:

Statistic: a number relating to the sample

example:

Identify the population, sample, and statistic for each of the following scenarios:

A survey of 1300 American households found that 32% of those households have basements.

Population: Sample: Statistic:

The average bill from every 6th person getting food at Chipotle in a 3-hour period was $19.61.

Population: Sample: Statistic:

Confidence Intervals are intervals of plausible values for estimating a parameter, with a given percent confidence. We use a sample mean to estimate the population mean. We use a sample proportion to estimate a population proportion.

Consider this: The Milton Parks and Recreation Department wants to build a new park in Crabapple. To allocate funds to build the park, they need to determine if residents in the area want one. They mail a survey to residents within 1 mile of the proposed location and find that 78% of residents who responded are in favor of building the new park. They'll find confidence intervals to project what all residents in the area may think of the new park.

Confidence Interval for Proportion:	**Confidence Interval for Mean:**
$\hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$	$\bar{x} \pm z\frac{\sigma}{\sqrt{n}}$
$\hat{p} =$	$\bar{x} =$
$z =$	$\sigma =$
$n =$	$z =$
Margin of error =	$n =$
$p =$	Margin of error =
	$\mu =$

Examples:

1. A survey of 1150 people found that 84.6% of respondents believed a toilet paper roll should roll over (not under). Construct the following confidence intervals for the proportion of people whose toilet paper rolls over and state the margin of error for each.
   90%

   95%

2. In a sample of 2500 people, 770 people separate their Skittle's by color before eating them. Construct an 85% confidence interval for the proportion of people who "taste the rainbow" with colors separated.

3. A recent survey of 133 Milton students found their average daily screen time is 5.402 hours. If the population standard deviation is 1.565 hours, construct the following confidence intervals for the average daily screen time for all Milton students and state the margin of error for each.
   80%

   99%

4. A recent survey found that Milton students get an average of 6.303 hours of sleep each night. Given the sample size of 540 students and population standard deviation of 0.926 hours, construct an 88% confidence interval for the average amount of sleep by Milton students.

9.09 Homework:

1. Students who take the SAT Mathematics exam a second time generally score higher than their first try. The change in score has a Normal distribution with standard deviation $\sigma = 50$ points. A random sample of 1000 students gain an average of $\bar{x} = 22$ points on their second try.
   a) Construct a 95% confidence interval for the mean score gain μ in the population.

   b) Construct a 90% confidence interval for μ.

   c) Construct a 99% confidence interval for μ.

   d) What is the margin of error for each of the confidence intervals calculated above?
   in part a)

   in part b)

   in part c)

2. The National Survey of Student Engagement found that 87% of 400 students reported their peers at least "sometimes" copy information from the Internet in their reports without citing the source. Construct an 88% confidence interval for the proportion of students who would report the issue and find the margin of error.

3. A recent survey of 1366 adults found that 1127 of those respondents like hot sauce on their eggs. Construct an 80% confidence interval for the proportion of adults that prefer spicy eggs.

4. A national opinion poll found that 44% of all American adults agree that parents should be given vouchers good for education at any public or private school of their choice. The result was based on a small sample. How large of a random sample is required to obtain a margin of error of 0.03 in a 95% confidence interval for the proportion of American adults who agree?

5. A radio talk show invites listeners to enter a dispute about a proposed pay increase for city council members. "What yearly pay do you think council members should get? Call us with your number." In all, 958 people call. The mean pay they suggest is $8740 per year. Assume that the standard deviation of all citizens is $1125. Calculate a 90% confidence interval for the mean pay μ that all citizens would propose for council members.

9.10 More Confidence Intervals Practice **Date:** ____________

1. A town takes a poll of its residents to find out how many people would be willing to pay a new tax to repair the town's sidewalks. Out of 1100 people polled, only 130 said that they would be willing to pay.
   (a) Find a 90% confidence interval for the proportion of the whole town that would be willing to pay the extra tax.

   (b) If 15,000 people live in this town, then we are 90% confident that between ______________ and ______________ will be willing to pay this tax. (Fill in with numbers of people.)

2. A company that produces white bread is concerned about the distribution of the amount of sodium in its bread. The company takes a simple random sample of 100 slices of bread and computes the sample mean to be 103 milligrams of sodium per slice.
   (a) Construct a 99% confidence interval for the unknown mean sodium level assuming that the population standard deviation is 10 milligrams.

   (b) Interpret the 99% confidence interval found in (a).

3. You work for a consumer advocate agency and want to find the mean repair cost of a washing machine. In the past, the standard deviation of the cost of repairs for washing machines has been $17.50. As part of your study, you randomly select 40 repair costs and find the mean to be $100.00.
   (a) Calculate a 85% confidence interval for the population mean.

   (b) Interpret the interval found in (a).

4. You want to estimate the mean fuel efficiency for all Ford Focus cars with 99% confidence and a margin of error of no more than 1 mile per gallon. Preliminary data suggests that $\sigma = 2.4$ miles per gallon is a reasonable standard deviation for all cars of this make and model. How large a sample do you need?

5. The actual time it takes to cook a 10–pound turkey is a Normal random variable with a mean of 2.8 hours and a standard deviation of 0.24 hours. Suppose that a random sample of 35 10–pound turkeys is taken.
   (a) What is the probability that a randomly selected 10-pound turkey will take less than 3.1 hours to cook?

   (b) What is the probability that the average cooking time of a 10–pound turkey will take between 2.7 and 2.95 hours to cook?

   (c) Given that an average of 2.9 hours was found for a sample of 35 turkeys, calculate an 80% confidence interval for the average cooking time of a 10–pound turkey.

6. Weight Watchers takes a poll of 250 members and finds that 95 of them include exercise with their diet program, while the rest do not. Find a 99% confidence interval for the proportion of all members that do exercise.

7. A magazine polls 395 readers and finds that 95 of them bought the magazine in the store, while the rest had a subscription. Find an 87% confidence interval for the proportion of all readers who have a subscription.

8. Fill in the blanks with one of the following for how the margin of error is impacted: *increases, decreases,* or *stays the same* where $ME = z\left(\frac{\sigma}{\sqrt{n}}\right)$.

   As the sample size (n) increases, the margin of error (ME) ______________________________.
   As the confidence level (C%) increases, the margin of error (ME) ________________________.
   As the standard deviation (σ) increases, the margin of error (ME) _______________________.

9.12 Review **Date:** ________________

1) A sample of 16 students finds that the average age is 22 years. All student ages have a standard deviation of 6 years. Construct a 95% confidence interval for the average age of students.

2) Construct a 99% confidence interval for the population mean lifetime of fluorescent lightbulbs. Assume the population has a Normal distribution with a standard deviation of 31 hours. A sample of 16 fluorescent light bulbs have a mean life of 645 hours.

3) A sample of 100 bean cans showed an average weight of 13 ounces. If all been cans have a standard deviation of 0.8 ounces, construct an 85% confidence interval for the mean weight of the population.

4) A researcher wants to know the percentage of Columbus residents who would favor a two cent increase in the gasoline tax to fund road repairs. A random sample of 900 residents finds 278 favor the increase.

   a. Specify the parameter and statistic for this problem.

   b. Find an 80% confidence interval for the parameter.

5) A random sample of female college students has a mean height of **64.5** inches, which is greater than the **63-**inch mean height of all adult American women. Determine if each bold-faced number is a parameter or a statistic.

6) In a certain Normal distribution of scores, the mean is 20 and the standard deviation is 3.

   a. Find the z-score corresponding to a score of 24.

   b. Find the percentile for a score of 24.

7) The Jackson triplets, Jenny, John, and James are in different math classes at City High. On their final exams, Jenny scored 82 on a test with a mean of 76 and a standard deviation of 7.5; John scored 77 on a test with a mean of 72 and a standard deviation of 10.5; and James scored 78 on a test with a mean of 66 and a standard deviation of 10.5. Who had the best z-score **and** what does this say about that triplet's their test score in relation to their peers?

8) Some test scores were Normally distributed with a mean of 55 and a standard deviation of 5. Approximately what percentage of the scores lie between 45 and 65?

9) The heights of a certain group of adult parrots were found to be Normally distributed. The mean height is 36 cm with a standard deviation of 8 cm. In a group of 1000 of these birds, how many would be at most 28 cm tall?

10) The life expectancy (in hours) of a fluorescent tube is normally distributed with a mean of 5000 and a standard deviation of 500. Find the probability that a tube lasts for at least 5650 hours.

11) A potato chip company sells a small snack bag of chips. The volume of the snack bag is Normally distributed with a mean of 1.75 ounces and a standard deviation of 0.15 ounces. What is the probability that a bag contains between 1.63 and 1.84 ounces?