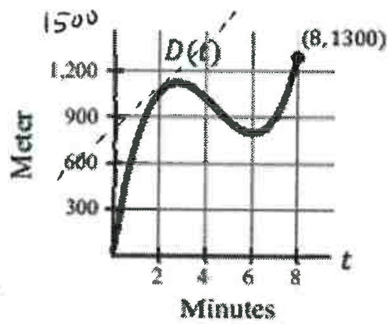
AP Calculus – 1.1 Notes – Exploring Rates of Change & Defining Limits

Key

Can Change occur at an instant?

1. Mr. Brust's distance from his house is modeled by the function $D(t)$. While riding his bike to the store, he realizes he dropped his wallet and turns around to find it. After finding his wallet, he finishes his ride to the store.



   a. What is his average speed (rate of change) for his trip to the store if he arrives after 8 minutes?

   $\frac{D(8)-D(0)}{8-0} = \frac{1300-0}{8-0} = \frac{1300}{8} = 162.5$ meters per minute

   b. What was his average rate of change between 2 and 6 minutes?

   $\frac{D(6)-D(2)}{6-2}$

   c. What was his average rate of change between 2 and 3 minutes?

   $\frac{D(3)-D(2)}{3-2}$

Is it possible to know how fast he was going at an instant?

Instantaneous rate of change

   d. Give an example of how to calculate a rate of change that would give a close estimate to the instantaneous rate of change at $t = 2$.

   $\frac{D(2)-D(1.999)}{2-1.999}$ → slope of the tangent line approximation

   e. Give a rough estimate of the instantaneous rate of change at $t = 2$.

   $\frac{1500-1200}{4-2} \rightarrow \frac{300}{2} \rightarrow 150$ meters/min

2. $b(t)$ represents the buffalo population in the United States where t is measured in years since 1800.

   a. What does $b(90)$ represent?

   The number of buffalo in US in 1890

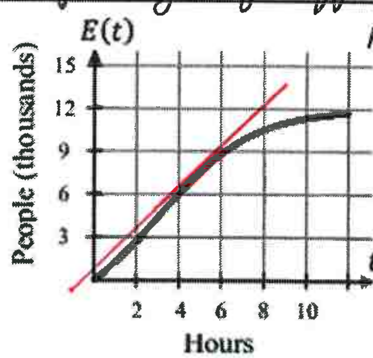
   b. What does $\frac{b(50)-b(0)}{50-0}$ represent?

   The Average Rate of Change of buffalo population per year from 1800 to 1850

   c. What does $\frac{b(32)-b(31.999)}{32-31.999}$ represent?

   A good estimate of the rate of change of buffalo population per year in 1832.

3. The number of people who have entered an amusement park is modeled by the function E, where $E(t)$ gives the number of people in thousands who have entered the park and t gives the number of hours since 10:00 a.m. for $0 \le t \le 11$. The graph of the function E is shown to the right.



   a. Draw a tangent line at $t = 3$.

   b. Give a rough estimate of the instantaneous rate of change at $t = 3$.

   1500 people per hour

   c. Give an example of how to calculate a rate of change that would give a close estimate to the instantaneous rate of change at $t = 6$.

   $\frac{E(6)-E(5.999)}{6-5.999}$

4. A basketball player's free throw attempts can be modeled by f, where $f(g)$ is the total number of made free throws during the season and g is the number of games for $0 \le g \le 82$.

   a. What does $f(50)$ represent?

   The number of free throws made through 50 games.

   b. What does $\frac{f(50)-f(0)}{50-0}$ represent?

   The avg rate of change of FT's made per game in first 50 games

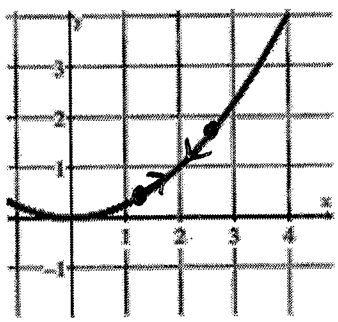
   c. What does $\frac{f(50)-f(49.999)}{50-49.999}$ represent?

   → Approximate rate of change of FT's

Defining Limits:

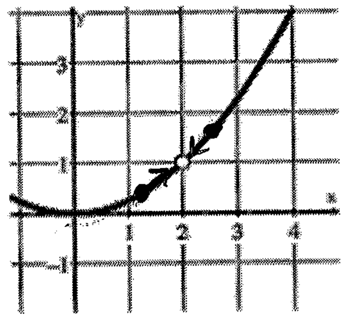
Limits

As x approaches 2, $f(x)$ approaches 1.



$\lim_{x\to 2} f(x) =$ 1

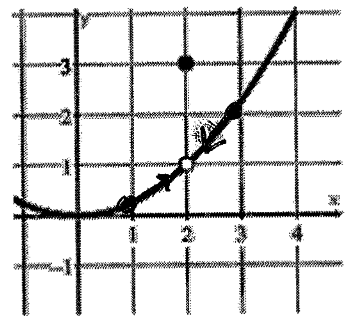
$f(2) =$ 1



$\lim_{x\to 2} f(x) =$ 1

$f(2) =$ undefined

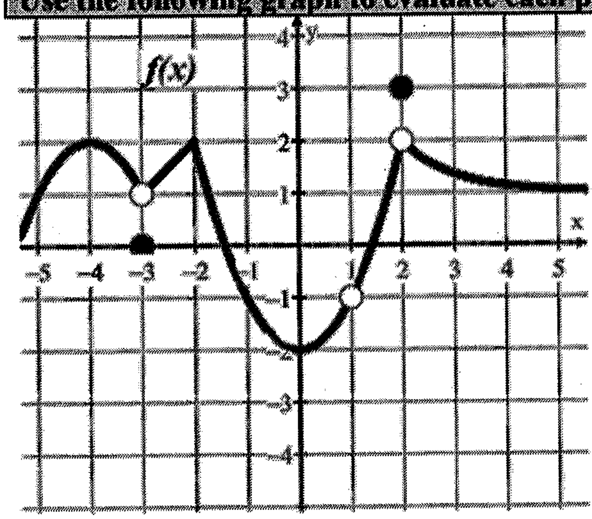
(dne) does not exist



$\lim_{x\to 2} f(x) =$ 1

$f(2) =$ 3

Use the following graph to evaluate each problem.



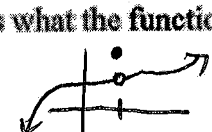
1. $\lim_{x\to 1} f(x) =$ -1
2. $f(-3) =$ 0
3. $\lim_{x\to 2} f(x) =$ 2
4. $f(2) =$ 3
5. $f(1) =$ undefined (dne)
6. $f(-2) =$ 2
7. $\lim_{x\to 0} f(x) =$ -2
8. $\lim_{x\to -3} f(x) =$ 1

9. Give an interpretation of the statement $\lim_{x\to 7} f(x) = 10$

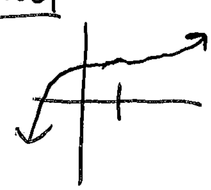
As x approaches 7, f(x) approaches 10.

A limit does NOT tell us the value of $f(x)$. It just tells us what the function approaches!

True or false? $f(1) = \lim_{x\to 1} f(x)$ in all cases. $\boxed{\text{False}}$



True or false? $f(1) \neq \lim_{x\to 1} f(x)$ in all cases $\boxed{\text{False}}$

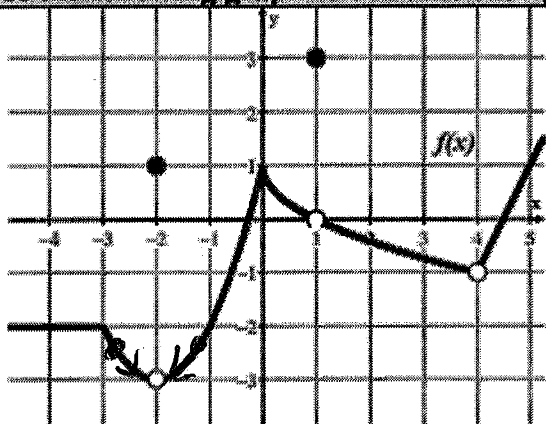


Practice Problems:

Give an interpretation of each statement.

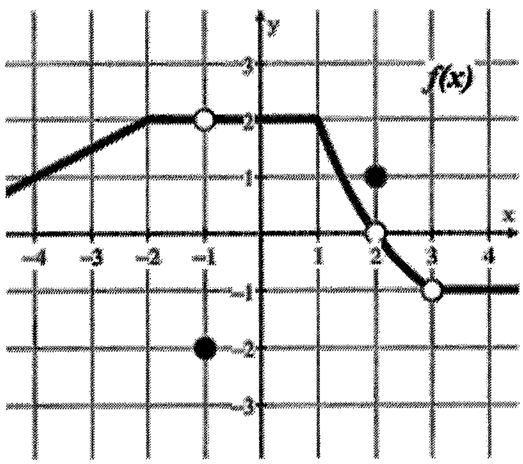
1. $\lim_{x\to 1} f(x) = 9$	2. $\lim_{x\to -2} f(x) = 3$	3. $\lim_{x\to 4} f(x) = -8$
As x approaches 1, f(x) approaches 9 (y-value)	As x approaches -2, f(x) approaches 3	As x approaches 4, f(x) approaches -8

Use the following graph to evaluate each problem.



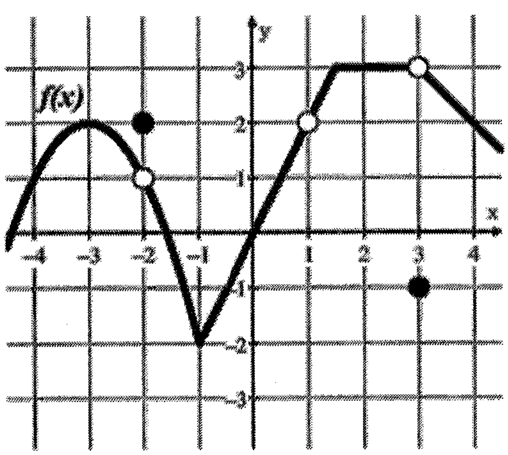
4. $f(-2) =$ 1	5. $\lim_{x\to 1} f(x) =$ 0
6. $\lim_{x\to -2} f(x) =$ -3	7. $\lim_{x\to 0} f(x) =$ 1
8. $f(4) =$ undefined	9. $\lim_{x\to 4} f(x) =$ -1
10. $\lim_{x\to -4} f(x) =$ -2	11. $f(1) =$ 3

Use the following graph to evaluate each problem.



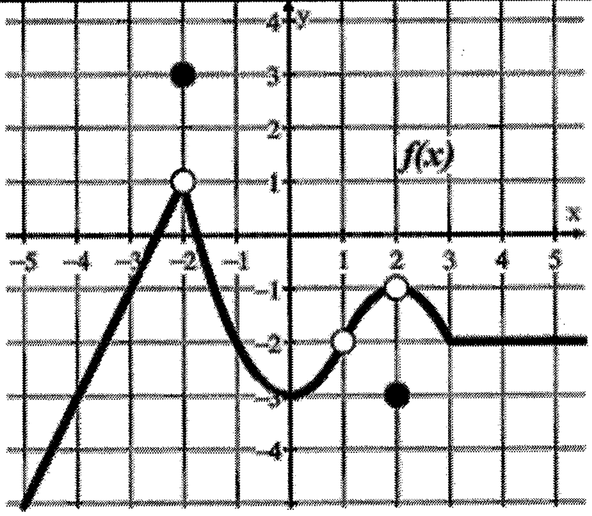
12. $\lim_{x\to -1} f(x) =$ 2	13. $\lim_{x\to 3} f(x) =$ -1
14. $f(2) =$ 1	15. $\lim_{x\to -2} f(x) =$ 2
16. $\lim_{x\to 1} f(x) =$ 2	17. $f(3) =$ undefined
18. $f(-1) =$ -2	19. $\lim_{x\to 2} f(x) =$ 0

Use the following graph to evaluate each problem.



20. $\lim_{x\to 2} f(x) =$ 3	21. $f(1) =$ undefined
22. $\lim_{x\to 3} f(x) =$ 3	23. $\lim_{x\to -2} f(x) =$ 1
24. $\lim_{x\to 1} f(x) =$ 2	25. $f(-2) =$ 2
26. $\lim_{x\to -3} f(x) =$ 2	27. $f(3) =$ -1

Use the following graph to evaluate each problem.



28. $\lim_{x\to -2} f(x) =$ 1	29. $\lim_{x\to 1} f(x) =$ -2
30. $\lim_{x\to 2} f(x) =$ -1	31. $f(-2) =$ 3
32. $f(1) =$ undefined	33. $\lim_{x\to 0} f(x) =$ -3
34. $\lim_{x\to -4} f(x) =$ -3	35. $f(2) =$ -3

36. Let f be a function that is defined for all real numbers x. Of the following, which is the best interpretation of the statement $\lim_{x\to 4} f(x) = 8$.

(A) The value of the function f at $x = 4$ is 8.

(B) The value of the function f at $x = 8$ is 4.

(C) As x approaches 4, the values of $f(x)$ approach 8.

(D) As x approaches 8, the values of $f(x)$ approach 4.

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37. Let f be a function that is defined for all real numbers x. Of the following, which is the best interpretation of the statement $\lim_{x\to -1} f(x) = 2$.

(A) As x approaches 2, the values of $f(x)$ approach -1

(B) The value of the function f at $x = -1$ is 2.

(C) The value of the function f at $x = 2$ is -1.

(D) As x approaches -1, the values of $f(x)$ approach 2.