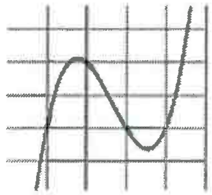
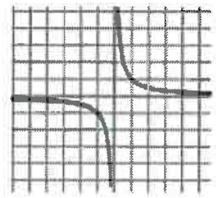
Key

## Continuity

continuous function

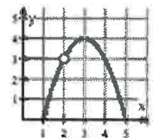




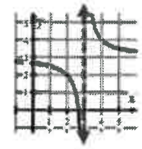
discontinuous function

## Types of Discontinuities:

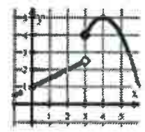
1. Removable (hole in graph)



2. Discontinuity due to vertical asymptote (nonremovable)



3. Jump discontinuity (nonremovable)



← piecewise functions with a break in between.

**For each function identify the type of each discontinuity and where it is located.**

1. $f(x) = \frac{x^2-8x+12}{x^2+3x-10} = 0$

$\frac{(x-2)(x-6)}{(x-2)(x+5)}$

$x=2$ is a hole

$x=-5$ : VA (nonremovable)

2. $g(x) = \frac{x+1}{x^4-1}$

$g(x) = \frac{x+1}{(x^2-1)(x^2+1)}$

$g(x) = \frac{x+1}{(x^2+1)(x+1)(x-1)}$

hole at $x=-1$

V.A. at $x=1$

* Find discontinuities from the denominator of Rational functions

Defining Continuity at a Point:

## Formal Definition of Continuity:

For $f(x)$ to be continuous at $x = c$, the following three conditions must be met:

1. $f(c)$ is defined
2. $\lim_{x\to c} f(x)$ exists $\left[\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x)\right]$
3. $\lim_{x\to c} f(x) = f(c)$

1. State whether the function $f(x) = \begin{cases} x^2-2x+1, & x<-1 \\ x+2, & -1 \le x < 2 \\ 2^x, & x \ge 2 \end{cases}$ is continuous at the given $x$ values. Justify your answers!

a. $x = -1$

$f(-1) = -1+2 = 1$

$\lim_{x\to -1^-} f(x) = (-1)^2 - 2(-1) + 1 = 4$

$\lim_{x\to -1^+} f(x) = -1+2 = 1$

$f(x)$ not continuous at $x=-1$ since $\lim_{x\to -1^+} f(x) \neq \lim_{x\to -1^-} f(x)$

Therefore limit does not exist.

b. $x = 2$

$f(2) = 2^2 = 4$

$\lim_{x\to 2^-} f(x) = (2)+2 = 4$

$\lim_{x\to 2^+} f(x) = 2^2 = 4$

Since $\lim_{x\to 2} f(x) = f(2)$

$f(x)$ is continuous at $x=2$.

**Recall:** What is a removable discontinuity?

$$\lim_{x \to c} f(x) \text{ exists, but } \lim_{x \to c} f(x) \neq f(c)$$

* Removing discontinuity means filling in the hole of graph.

1. $f(x) = \frac{x^2-1}{x-1} \rightarrow \frac{(x+1)(x-1)}{(x-1)}$

Find the $x$-value of the hole.

hole at $x = 1$

(Removable discontinuity at $x = 1$)

How do we find the $y$-value?

$\lim_{x \to 1} (x+1) = 2$

Hole exists at $(1, 2)$ so filling this graph with a point at $(1, 2)$ will Remove the discontinuity.

2. If the function $f$ is continuous for all real numbers and if $f(x) = \frac{x^2+6x+8}{x+4}$ when $x \neq -4$, then $f(-4) =$

Step through continuity conditions

i) $f(-4) = \_$

ii) $\lim_{x \to -4} \frac{x^2+6x+8}{x+4} \rightarrow \lim_{x \to -4} \frac{(x+2)(x+4)}{(x+4)} = -2$

iii) For $\lim_{x \to c} f(x) = f(c)$,

$\lim_{x \to -4} f(x) = f(-4) = -2$ ✓

3. Let $f$ be the function defined by $f(x) = \begin{cases} \frac{x^2-3x-18}{x-6}, & x \neq 6 \\ a, & x = 6 \end{cases}$. For what value of $a$ is $f$ continuous at $x = 6$?

$\rightarrow \begin{cases} \frac{x^2-3x-18}{x-6}, & -\infty < x < 6 \\ a, & x = 6 \\ \frac{x^2-3x-18}{x-6}, & 6 < x < \infty \end{cases}$

*continuity conditions:

i) $f(6) = a$

ii) $\lim_{x \to 6} \frac{x^2-3x-18}{x-6} \rightarrow \frac{0}{0} \rightarrow \lim_{x \to 6} \frac{(x-6)(x+3)}{(x-6)} = 9$

iii) $f(c) = \lim_{x \to 6} f(x) \rightarrow f(6) = 9$, $\boxed{a = 9}$

*step through continuity conditions

5. Let $f$ be the function defined by

$$f(x) = \begin{cases} \frac{x^2-2x-15}{x-5}, & x \neq 5 \\ a, & x = 5 \end{cases}.$$

For what value of $a$ is $f$ continuous at $x = 5$?

i) $f(5) = a$

ii) $\lim_{x \to 5} \frac{x^2-2x-15}{x-5} \rightarrow \frac{0}{0} \rightarrow \lim_{x \to 5} \frac{(x-5)(x+3)}{(x-5)} = 8$

iii) $f(5) = \lim_{x \to 5} f(x)$ $\boxed{a = 8}$

6. Let $f$ be the function defined by

$$f(x) = \begin{cases} \frac{x^2-16x+63}{x-7}, & x \neq 7 \\ b, & x = 7 \end{cases}.$$

For what value of $b$ is $f$ continuous at $x = 7$?

i) $f(7) = b$

ii) $\lim_{x \to 7} \frac{x^2-16x+63}{x-7} \rightarrow \frac{0}{0}$ $\lim_{x \to 7} \frac{(x-7)(x-9)}{(x-7)}$

$\lim_{x \to 7} (x-9) \rightarrow 7-9 = \boxed{-2}$

iii) $f(7) = \lim_{x \to 7} f(x)$

$\boxed{b = -2}$