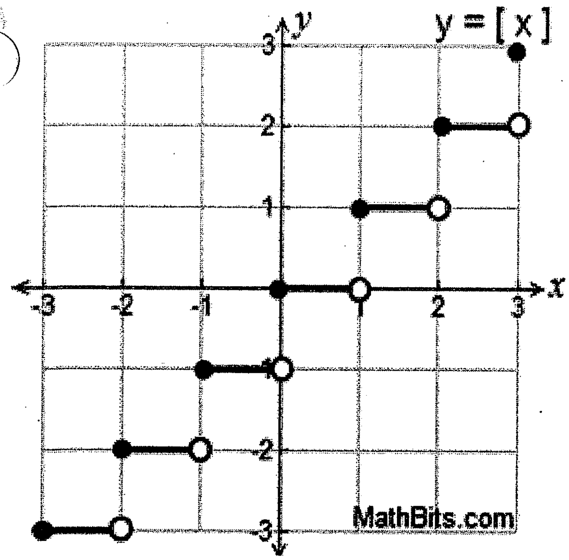
BC Ch. 1.4 Additional Notes: Greatest Integer Functions

The Greatest Integer Function $[\![x]\!]$ = largest integer $\leq x$ (also called floor functions)



Example 1:

$[\![4]\!] =$

$[\![4.8]\!] =$

$[\![-\pi]\!] =$

23. $\lim_{x \to 4^-} (5[\![x]\!] - 7)$

24. $\lim_{x \to 2^+} (2x - [\![x]\!])$

25. $\lim_{x \to 3} (2 - [\![-x]\!])$

26. $\lim_{x \to 1} \left(1 - \left[\!\!\left[-\frac{x}{2}\right]\!\!\right]\right)$

Example 2: Find x-values where f(x) is not continuous for f(x) = $[\![x - 8]\!]$

Example 3: Find x-values where f(x) is not continuous for f(x) = $[\![4x - 5]\!]$

1.4 Additional Problems:

Making a Function Continuous In Exercises 61–66, find the constant a, or the constants a and b, such that the function is continuous on the entire real number line.

63. $f(x) = \begin{cases} x^3, & x \leq 2 \\ ax^2, & x > 2 \end{cases}$

65. $f(x) = \begin{cases} 2, & x \leq -1 \\ ax + b, & -1 < x < 3 \\ -2, & x \geq 3 \end{cases}$

66. $g(x) = \begin{cases} \dfrac{x^2 - a^2}{x - a}, & x \neq a \\ 8, & x = a \end{cases}$

Testing for Continuity In Exercises 77–84, describe the interval(s) on which the function is continuous.

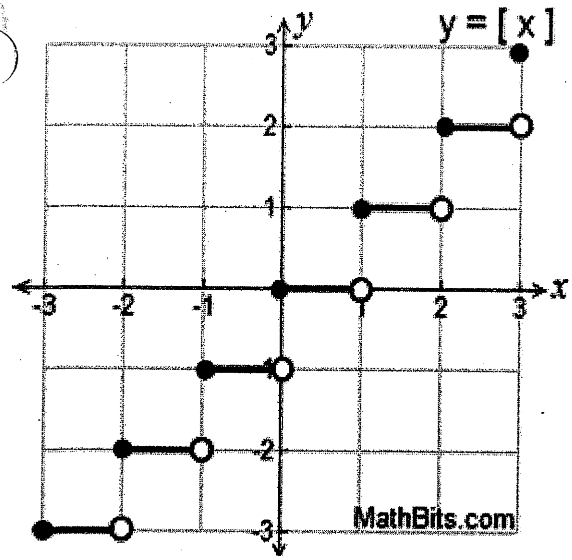
81. $f(x) = \sec \dfrac{\pi x}{4}$

BC Ch. 1.4 Additional Notes: Greatest Integer Functions

Key

The Greatest Integer Function $[\![x]\!]$ = largest integer $\leq x$ (also called floor functions)

*Round down to nearest integer

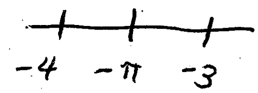


Example 1:

$[\![4]\!] = 4$

$[\![4.8]\!] = 4$

$[\![-\pi]\!] = -4$



23. $\lim_{x\to 4^-} (5[\![x]\!] - 7)$

$5[\![3.9]\!] - 7$

$5(3) - 7 = \boxed{8}$

24. $\lim_{x\to 2^+} (2x - [\![x]\!])$

$\lim_{x\to 2^+} 2x - \lim_{x\to 2^+} [\![x]\!]$

$= 4 - [\![2.1]\!]$

$= 4 - 2 = \boxed{2}$

25. $\lim_{x\to 3} (2 - [\![-x]\!])$

$\lim_{x\to 3^+} (2 - [\![-x]\!]) = 2 - [\![-3.1]\!] = 2 - (-4) = 6$

$\lim_{x\to 3^-} (2 - [\![-x]\!] = 2 - [\![-2.9]\!] = 2 - (-3) = 5$

since $\lim_{x\to 3^-} f(x) \neq \lim_{x\to 3^+} f(x)$, $\lim_{x\to 3} f(x) = \boxed{DNE}$

26. $\lim_{x\to 1} \left(1 - \left[\!\!\left[-\frac{x}{2}\right]\!\!\right]\right)$

$\lim_{x\to 1^-} \left(1 - \left[\!\!\left[-\frac{x}{2}\right]\!\!\right]\right) = 1 - \left[\!\!\left[\frac{-0.9}{2}\right]\!\!\right] = 1 - (-1) = 2$

$\lim_{x\to 1^+} \left(1 - \left[\!\!\left[-\frac{x}{2}\right]\!\!\right] = 1 - \left[\!\!\left[\frac{-1.1}{2}\right]\!\!\right] = 1 - (-1) = 2$ ✓

$= \boxed{2}$

Example 2: Find x-values where f(x) is not continuous for $f(x) = [\![x - 8]\!]$

x	y
0	-8
0.1	-8
0.5	-8
0.8	-8
0.9	-8
1	-7

$\boxed{\text{f(x) is not continuous at k where k is an integer.}}$

Example 3: Find x-values where f(x) is not continuous for $f(x) = [\![4x - 5]\!]$

* Test decimal values to determine what increments of x will push the expression into the next integer value

x	y
0	-5
*0.25	-4
0.3	-3.8 → -4
*0.5	-3

x	y
0.6	-2.6 → -3
*0.75	-2

$$\boxed{f(x) \text{ is not continuous at } \tfrac{1}{4}k \text{ where } k \text{ is an integer.}}$$

1.4 Additional Problems:

Making a Function Continuous In Exercises 61–66, find the constant a, or the constants a and b, such that the function is continuous on the entire real number line.

Continuity conditions:

i) $f(c)$ is defined

ii) $\lim_{x\to c} f(x)$ exists $\left[\lim_{x\to c^+} f(x) = \lim_{x\to c^-} f(x)\right]$

iii) $f(c) = \lim_{x\to c} f(x)$

63. $f(x) = \begin{cases} x^3, & x \le 2 \\ ax^2, & x > 2 \end{cases}$ $c = 2$

i) $f(2) = 2^3 = 8$

ii) $\lim_{x\to 2^-} x^3 = 8$ $\lim_{x\to 2^+} ax^2 = 4a$

$\lim_{x\to 2^-} 8 = \lim_{x\to 2^+} 4a \rightarrow 4a = 8,\ a = 2$

iii) $f(2) = \lim_{x\to 2} f(x) = 8$ $\boxed{a=2}$

65. $f(x) = \begin{cases} 2, & x \le -1 \\ ax + b, & -1 < x < 3 \\ -2, & x \ge 3 \end{cases}$

At $x=-1$

$2 = ax + b$

$2 = a(-1) + b$

$2 = -a + b$

$a = b - 2$

At $x = 3$

$ax + b = -2$

$3a + b = -2$

$3(b-2) + b = -2$

$3b - 6 + b = -2$

$4b = 4$ $\boxed{b=1}$

$a = b - 2 = 1 - 2 = \boxed{-1}$

66. $g(x) = \begin{cases} \dfrac{x^2 - a^2}{x - a}, & x \ne a \\ 8, & x = a \end{cases}$ $c = a$

i) $f(a) = 8$

ii) $\lim_{x\to a} \dfrac{x^2 - a^2}{x - a} = \dfrac{0}{0}$ $\lim_{x\to a} \dfrac{\cancel{(x-a)}(x+a)}{\cancel{(x-a)}} = a + a = 2a$

iii) $f(a) = \lim_{x\to a} f(x)$, $8 = 2a$

$\hookrightarrow \boxed{a=4}$

Testing for Continuity In Exercises 77–84, describe the interval(s) on which the function is continuous.

81. $f(x) = \sec \dfrac{\pi x}{4}$

* The secant graph has vertical asymptote where $\cos x = 0$

$x = \ldots -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$

$\frac{\pi x}{4} = \frac{\pi}{2} \rightarrow x = 2$

$\frac{\pi x}{4} = \frac{3\pi}{2} \rightarrow x = 6$

$\frac{\pi x}{4} = \frac{5\pi}{2} \rightarrow x = 10$

f(x) is continuous on:

$\ldots(-6,-2),(-2,2),(2,6),(6,10]\ldots$