10.2 Graphing Parametric Equations, Eliminating the Parameter

p. 703 #1-64

Parametric Equations: writing both x and y as functions of t

$x = f(t)$ and $y = g(t)$, t is the parameter

Eliminating Parameter

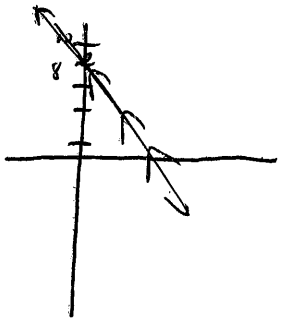
1) Parametric equation
2) solve for t
3) substitute into 2nd equation
4) write rectangular equation

a) sketch curve b) Write correspondiny rectangular equation

2) $x = 5 - 4t$

$y = 2 + 5t$ $\quad t = \frac{5-x}{4}$

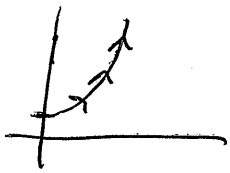
$y = 2 + 5\left(\frac{5-x}{4}\right) = -\frac{5}{4}x + \frac{33}{4}$



4) $x = 2t^2$ $\quad t = \sqrt{\frac{x}{2}}$

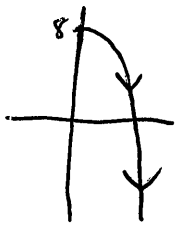
$y = t^4 + 1$

$y = \left(\sqrt{\frac{x}{2}}\right)^4 + 1$ $\quad y = \left(\frac{x}{2}\right)^2 + 1$ $\quad y = \frac{x^2}{4} + 1$



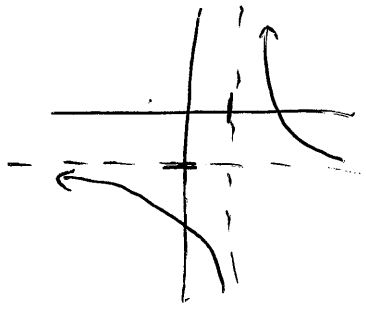
8) $x = \sqrt[4]{t}$ $\quad x^4 = t$

$y = 8 - t$ $\quad y = 8 - x^4$



10) $x = 1 + \frac{1}{t}$ $\quad t = \frac{1}{x-1}$

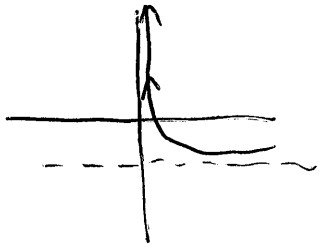
$y = t - 1$ $\quad y = \frac{1}{x-1} - 1$



14) $x = e^{-t} \quad x > 0$

$y = e^{2t} - 1$

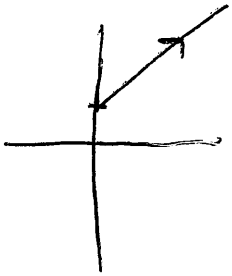
$y = x^{-2} - 1 = \frac{1}{x^2} - 1$



16) $x = \tan^2\theta \quad y = \sec^2\theta$

$y = x + 1 \quad x \geq 0$

$\sec^2\theta = \tan^2\theta + 1$



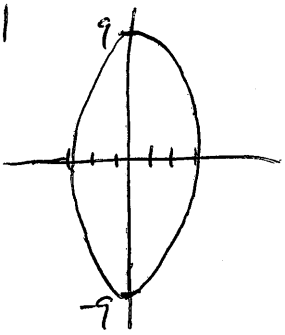
18) $x = 3\cos\theta \quad y = 7\sin\theta$

$\cos\theta = \frac{x}{3} \quad \sin\theta = \frac{y}{7}$

$\cos^2\theta + \sin^2\theta = 1$

$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{7}\right)^2 = 1$

$\frac{x^2}{9} + \frac{y^2}{49} = 1$



write rectangular equation

20) $x = \cos\theta \quad y = 2\sin 2\theta$

$y = 2 \cdot 2\sin\theta\cos\theta$

$y = 4\sin\theta\cos\theta$

$x = \cos\theta$

$\cos^2\theta + \sin^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$\sin^2\theta = 1 - x^2$

$\sin\theta = \sqrt{1-x^2}$

$y = 4\sqrt{1-x^2} \cdot x$

$\boxed{y = \pm 4x\sqrt{1-x^2}}$

Write rectangular equation

22) $x = -2 + 3\cos\theta$ $\quad y = -5 + 3\sin\theta$

$\frac{x+2}{3} = \cos\theta$ $\quad \frac{y+5}{3} = \sin\theta$

$\sin^2\theta + \cos^2\theta = 1$

$\left(\frac{y+5}{3}\right)^2 + \left(\frac{x+2}{3}\right)^2 = 1$

$\frac{(x+2)^2}{9} + \frac{(y+5)^2}{9} = 1$

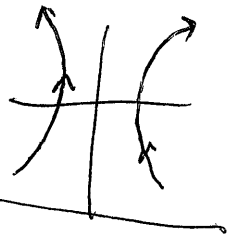
$\boxed{(x+2)^2 + (y+5)^2 = 9}$ circle

24) $x = \sec\theta$ $\quad y = \tan\theta$

$1 + \tan^2\theta = \sec^2\theta$

$1 + y^2 = x^2$

$x^2 - y^2 = 1$



26) $x = \cos^3\theta$ $\quad y = \sin^3\theta$

$\sqrt[3]{x} = \cos\theta$ $\quad \sqrt[3]{y} = \sin\theta$

$\sqrt[3]{x^2} = \cos^2\theta$ $\quad \sqrt[3]{y^2} = \sin^2\theta$

$\cos^2\theta + \sin^2\theta = 1$

$\sqrt[3]{x^2} + \sqrt[3]{y^2} = 1$

$\boxed{x^{2/3} + y^{2/3} = 1}$

28) $x = \ln 2t \rightarrow x = \log_e(2t)$

$y = t^2$ $\quad e^x = 2t$ $\quad t = \frac{1}{2}e^x$

$y = \left[\frac{1}{2}e^x\right]^2 = \boxed{y = \frac{1}{4}e^{2x}}$

30) $x = e^{2t}$ $\quad y = e^t$

$\sqrt{x} = e^t$

$\boxed{y = \sqrt{x}, \; x > 0}$

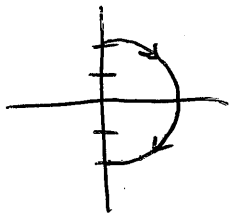
32) Compare differences $\cos^2\theta + \sin^2\theta = 1$

a) $x = 2\cos\theta$ $\quad \frac{x}{2} = \cos\theta$ $\quad \left(\frac{x}{2}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$

$y = 2\sin\theta$ $\quad \frac{y}{2} = \sin\theta$ $\quad \frac{x^2}{4} + \frac{y^2}{4} = 1$ $\quad \boxed{x^2 + y^2 = 4}$

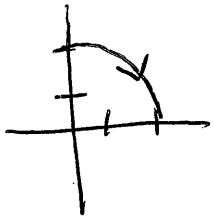
b) $x = \frac{\sqrt{4t^2 - 1}}{|t|}$ $\quad y = 1/t$

$x \geq 0,\ x \neq 2$ $\quad y \neq 0$



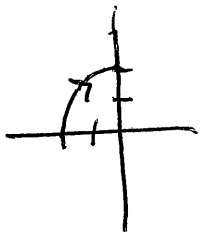
c) $x = \sqrt{t}$ $\quad y = \sqrt{4 - t}$

$x \geq 0$ $\quad y \geq 0$



d) $x = -\sqrt{4 - e^{2t}}$ $\quad y = e^t$

$-2 < x \leq 0$ $\quad y > 0$



34) a) $x = t + 1$ $\quad y = t^3$] orientations are reversed

b) $x = -t + 1$ $\quad y = (-t)^3$]

36) The set of points do not show orientation of the curve or restrictions on domain of original parametric equations.

38) Circle $x = h + r\cos\theta$ $\quad y = k + r\sin\theta$

$\cos\theta = \frac{x - h}{r}$ $\quad \sin\theta = \frac{y - k}{r}$

$\cos^2\theta + \sin^2\theta = \frac{(x-h)^2}{r^2} + \frac{(y-k)^2}{r^2} = 1$ $\quad \boxed{(x-h)^2 + (y-k)^2 = r^2}$

40) Hyperbola: $x = h + a\sec\theta$ $\quad y = k + b\tan\theta$

$\frac{x-h}{a} = \sec\theta$ $\quad \frac{y-k}{b} = \tan\theta$

$1 + \tan^2\theta = \sec^2\theta$

$\sec^2\theta - \tan^2\theta = 1$

$$\left(\frac{x-h}{a}\right)^2 - \left(\frac{y-k}{b}\right)^2 = 1$$

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

42) Write parametric equation

Line: passes through $(1, 4)$ and $(5, -2)$
$x_1\ y_1$ $\quad x_2\ y_2$

$x = x_1 + t(x_2 - x_1)$ $\quad y = y_1 + t(y_2 - y_1)$

$t = \frac{x - x_1}{x_2 - x_1}$

$x = 1 + t(5-1)$ $\quad y = 4 + t(-2-4)$

$$\boxed{\begin{aligned} x &= 1 + 4t \\ y &= 4 - 6t \end{aligned}}$$

$$y = y_1 + \left(\frac{x - x_1}{x_2 - x_1}\right)(y_2 - y_1)$$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

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44) Write parametric equations Circle: center $(-6, 2)$ radius: 4
$h\ k$ $\quad r$

$x - h = r\cos\theta$ $\quad x = h + r\cos\theta$

$y - k = r\sin\theta$ $\quad y = k + r\sin\theta$

$$\boxed{\begin{aligned} x &= -6 + 4\cos\theta \\ y &= 2 + 4\sin\theta \end{aligned}}$$

46) Ellipse: vertices $(4,7), (4,-3)$ foci: $(4,5), (4,-1)$

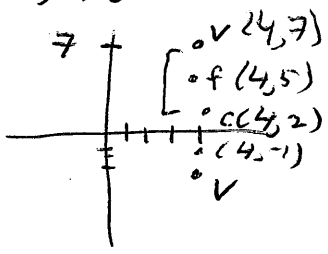
h k
center $(4,2)$

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

$a=5$
$c=3$
$b=4$

$a^2 - c^2 = b^2$

$25 - 9 = 4^2 = 16 \quad b=4$



$$\frac{(x-4)^2}{4^2} + \frac{(y-2)^2}{5^2} = 1$$

$x = h + r\cos\theta \rightarrow x = 4 + 5\cos\theta$

$y = k + r\sin\theta \quad y = 2 + 4\sin\theta$

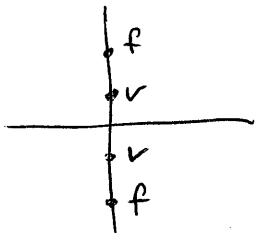
48) Hyperbola vertices: $(0,1), (0,-1)$ foci: $(0,2), (0,-2)$

$a^2 + b^2 = c^2$

h k
center $(0,0)$ $a=1$ $b=\_$ $c=2$

$a^2 + b^2 = c^2$

$1^2 + b^2 = 2^2 \quad b = \sqrt{3}$



$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1 \qquad \frac{(y-0)^2}{1} - \frac{(x-0)^2}{3} = 1$$

$\begin{cases} x = h + a\sec\theta \\ y = k + b\tan\theta \end{cases}$ $\quad x = 0 + \sqrt{3}\sec\theta \quad y = 0 + 1\tan\theta$

x and y interchanged

$\begin{cases} y = h + a\sec\theta \\ x = k + b\tan\theta \end{cases}$ $\quad y = 0 + 1\sec\theta \quad x = 0 + \sqrt{3}\tan\theta$

50) $y = \frac{4}{x-1}$

- parametric equations

$x = t$

$y = \frac{4}{t-1}$

or

$x = t + 1$

$y = \frac{4}{t}$

52) $y = x^2$

$x = t$ | $x = t^3$

$y = t^2$ | $y = t^6$

54) $y = 4x + 1$, $t = -1$ at point $(-2, -7)$

$x = -1 + t$

$y = 4(-1 + t) + 1 = 4t - 3$

56) $y = 4 - x^2$ $t = 1$ at $(1, 3)$

$x = t$

$y = t^2$

58) $x = \theta + \sin\theta$ $y = 1 - \cos\theta$ not smooth at $x = (2n-1)\pi$

60)

62) smooth everywhere

64)