2017

10.3b Arc Length, Surface Area

p. 712-714 # 46-86

$*$ Arc Length: $s = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

46) $x = 6t^2 \quad y = 2t^3 \quad 1 \le t \le 4$

$\frac{dx}{dt} = 12t \quad \frac{dy}{dt} = 6t^2 \quad \int_1^4 \sqrt{(12t)^2 + (6t^2)^2}\, dt = \int \sqrt{144t^2 + 36t^4}\, dt$

$\int \sqrt{36t^2(4+t^2)}\, dt$ | $u = 4+t^2$ | $\int 6t \cdot u^{1/2} \cdot \frac{du}{2t}$ | $\frac{3u^{3/2}}{3/2} = 2(4+t^2)^{3/2}\Big]_1^4$

$\int_1^4 6t\sqrt{4+t^2}\, dt$ | $\frac{du}{dt} = 2t$, $dt = \frac{du}{2t}$ | $3\int u^{1/2} du$ | $2\left(20^{3/2} - 5^{3/2}\right)$

$= \boxed{70\sqrt{5} \approx 156.525}$

48) $x = \arcsin t \quad y = \ln\sqrt{1-t^2} \quad 0 \le t \le 1/2$

$\ln(1-t^2)^{1/2} = \frac{1}{2}\ln(1-t^2)$

$\frac{dx}{dt} = \frac{1}{\sqrt{1-t^2}} \quad \frac{dy}{dt} = \frac{1}{2} \cdot \frac{-2t}{1-t^2} = \frac{-t}{1-t^2}$

$\int_0^{1/2} \sqrt{\frac{1}{1-t^2} - \frac{t^2}{(1-t^2)^2}}$

$\int_0^{1/2} \sqrt{\frac{1-t^2-t^2}{(1-t^2)^2}} = \int_0^{1/2} \sqrt{\frac{1}{(1-t^2)^2}}\, dt = \int_0^{1/2} \frac{1}{1-t^2}\, dt = \frac{A}{1-t} + \frac{B}{1+t}$

$(1-t)(1+t) \qquad t=1 \qquad t=-1 \qquad A = 1/2 \qquad B = 1/2$

$\int_0^{1/2} \frac{1}{2}\left(\frac{1}{1-t}\right) + \frac{1}{2}\left(\frac{1}{1+t}\right) dt = -\frac{1}{2}\ln|1-t| + \frac{1}{2}\ln|1+t|\Big]_0^{1/2}$

$-\frac{1}{2}\ln 1/2 + \frac{1}{2}\ln 3/2 - \left(\cancel{\frac{1}{2}\ln 1 + \frac{1}{2}\ln 1}\right)$

u-sub
$u = 1-t$
$\frac{du}{dt} = -1$

$= \frac{1}{2}\left(\ln 1/2 + \ln 3/2\right)$

$= \frac{1}{2}\left(\ln 3/2 - \ln 1/2\right)$

$= \frac{1}{2}\ln\left|\frac{3/2}{1/2}\right| = \boxed{\frac{1}{2}\ln|3| \approx 0.549}$

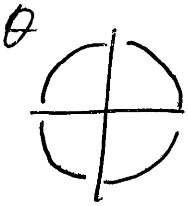
50) $x = t \quad y = \frac{t^5}{10} + \frac{1}{6t^3} = \frac{1}{10}t^5 + \frac{1}{6}t^{-3} \qquad 1 \le t \le 2$

$\frac{dx}{dt} = 1 \qquad \frac{dy}{dt} = \frac{1}{10} \cdot 5t^4 + \frac{-3}{6}t^{-4} \qquad S = \int_1^2 \sqrt{(1)^2 + \left[\frac{t^4}{2} - \frac{t^{-4}}{2}\right]^2}\, dt$

$S = \int_1^2 \sqrt{\left(\frac{t^4}{2} + \frac{1}{2t^4}\right)^2}\, dt = \int_1^2 \frac{t^4}{2} + \frac{1}{2t^4} = \frac{t^5}{10} - \frac{1}{6t^3}\Big]_1^2 = \boxed{\frac{779}{240}}$

52) <u>Arc Length</u> $\quad x = a\cos\theta \quad y = a\sin\theta \qquad 0 < \theta < 2\pi$

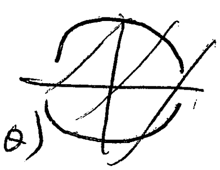
$\frac{dx}{d\theta} = -a\sin\theta \qquad \frac{dy}{d\theta} = a\cos\theta \qquad S = \int_0^{2\pi} \sqrt{a^2\sin^2\theta + a^2\cos^2\theta}\, d\theta$



$S = \int \sqrt{a^2(\sin^2\theta + \cos^2\theta)}\, d\theta = \int a\sqrt{1}\, d\theta = 4 \cdot \int_0^{\pi/2} a\, d\theta = 4a\theta\Big]_0^{\pi/2}$

$= 4a(\pi/2) - 0 = \boxed{2\pi a}$

54) $x = \cos\theta + \theta\sin\theta \qquad y = \sin\theta - \theta\cos\theta$



$\frac{dx}{d\theta} = -\sin\theta + 1\sin\theta + \theta\cos\theta \qquad \frac{dy}{d\theta} = \cos\theta + -\cos\theta + -\theta(-\sin\theta)$

$\frac{dx}{d\theta} = \theta\cos\theta \qquad \frac{dy}{d\theta} = \theta\sin\theta$

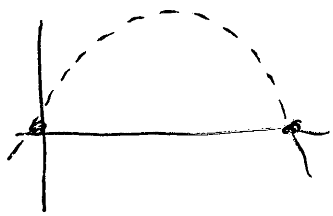
$S = \int_0^{2\pi} \sqrt{(\theta\cos\theta)^2 + (\theta\sin\theta)^2}\, d\theta = \sqrt{\theta^2(\cos^2\theta + \sin^2\theta)}\, d\theta = \theta\sqrt{1}\, d\theta$

$= \int_0^{2\pi} \theta\, d\theta = \frac{\theta^2}{2}\Big]_0^{2\pi} = \frac{(2\pi)^2}{2} - 0 = \frac{4\pi^2}{2} = \boxed{2\pi^2}$

2017

10.36

56) Path of Projectile $x = (90\cos\theta)t$ $y = (90\sin\theta)t - 16t^2$



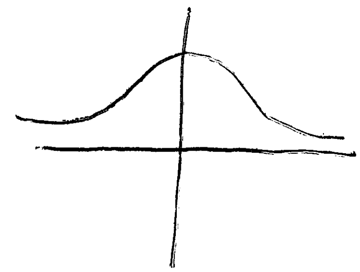
* set $y=0$ to find endpts $90\sin\theta\, t - 16t^2 = 0$

$t(90\sin\theta - 16t) = 0$

$t=0,\ t = \frac{90}{16}\sin\theta$

58) $x = 4\cot\theta$ $\quad y = 4\sin^2\theta$ $\quad -\pi/2 \le \theta \le \pi/2$

$\frac{dx}{d\theta} = -4\csc^2\theta$ $\quad \frac{dy}{d\theta} = 8\sin\theta\cos\theta$



horizontal tangent: $\frac{dy}{d\theta} = 0$

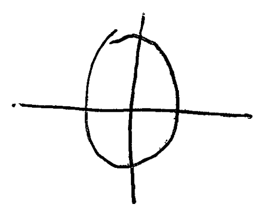
$0 = 8\sin\theta\cos\theta$

$\theta = -\pi/2, 0, \pi/2$

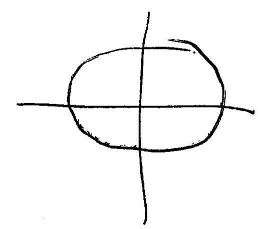
horizontal tangents: $(0,4)$ at $\theta = \pm\pi/2$

c) $\displaystyle\int_{\pi/4}^{\pi/2} \sqrt{(-4\csc^2\theta)^2 + [8\sin\theta\cos\theta]^2}\, d\theta \approx \boxed{4.5183}$

60) a) $x = 3\cos t$ $\quad y = 4\sin t$



~~b)~~ $x = 4\sin t$ $\quad y = \cos t$



b) 4 pts. of intersection

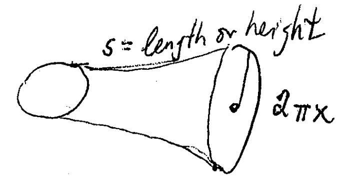
c) same place at same time for $t = 0.6435, 3.785$ for $\tan t = 3/4$
$(2.4, 2.4)$ $(-2.4, -2.4)$

d) curves intersect twice, not at same time.

Surface Area

* About y-axis $S_y = 2\pi \int_a^b x \sqrt{x'(t)^2 + y'(t)^2}\, dt$

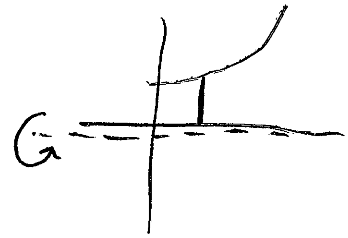
About x-axis $S_x = 2\pi \int_a^b y \sqrt{x'(t)^2 + y'(t)^2}\, dt$



cylinder: $S = 2\pi r h$

62) $x = \frac{1}{4}t^2$ $y = t + 3$ $0 \le t \le 3$ (About x-axis)

$x'(t) = \frac{2}{4}t$ $y'(t) = 1$



$S_x = 2\pi \int_0^3 (t+3)\sqrt{\left(\frac{1}{2}t\right)^2 + (1)^2}\, dt \approx \boxed{114.1999}$

64) $x = \theta + \sin\theta$ $y = \theta + \cos\theta$ $0 \le \theta \le \pi/2$

$x'(\theta) = 1 + \cos\theta$ $y'(\theta) = 1 - \sin\theta$ (x-axis)

$S_x = 2\pi \int_0^{\pi/2} (\theta + \cos\theta)\sqrt{(1+\cos\theta)^2 + (1-\sin\theta)^2}\, d\theta \approx \boxed{23.2433}$

66) $x = t$ $y = 4 - 2t$ $0 \le t \le 2$ $x'(t) = 1$ $y'(t) = -2$

a) x-axis: $S_x = 2\pi \int_0^2 4 - 2t\sqrt{1^2 + 2^2}\, dt = 2\pi \int_0^2 (\sqrt{5})(4-2t)\, dt$

$2\pi\sqrt{5} \cdot 4t - \frac{2t^2}{2}\Big]_0^2 = \boxed{8\pi\sqrt{5}}$

b) y-axis: $S_y = 2\pi \int_0^2 t\sqrt{1^2 + 2^2}\, dt = 2\pi\sqrt{5} \cdot \frac{t^2}{2}\Big]_0^2 = \boxed{4\pi\sqrt{5}}$

Surface Area

68) $x = \frac{1}{3}t^3 \quad y = t+1 \quad 1 \le t \le 2 \quad$ (y-axis)

$x'(t) = \frac{1}{3}\cdot 3t^2 \quad y'(t) = 1$

$$S_y = 2\pi \int_1^2 \frac{1}{3}t^3 \sqrt{(t^2)^2 + 1^2}\,dt = \frac{\pi}{9}\left(x^4+1\right)^{3/2}\Big]_1^2 = \frac{\pi}{9}\left[17^{3/2} - 2^{3/2}\right]$$

$$\approx \boxed{23.48}$$

70) $x = a\cos\theta \quad y = b\sin\theta \quad 0 \le \theta \le 2\pi$

a) x-axis $\quad x'(\theta) = -a\sin\theta \quad y'(\theta) = b\cos\theta$

78) $x = r\cos\phi \quad y = r\sin\phi \quad$ (x-axis) $\quad x'(\phi) = -r\sin\phi$

$y'(\phi) = +r\cos\phi$

$$S = 2\pi\int_0^\theta r\sin\phi\sqrt{r^2\sin^2\phi + r^2\cos^2\phi}\,d\phi$$

$$= 2\pi r^2\int_0^\theta \sin\phi\,d\phi = -2\pi r^2\cos\phi\Big]_0^\theta = \boxed{2\pi r^2 - 2\pi r^2\cos\theta}$$

80) Find Area of region:

$x = 2\cot\theta \quad y = 2\sin^2\theta$

$x'(\theta) = -2\csc^2\theta \quad 0<\theta<\pi$

from #77

Integration by Substitution

$$\int_a^b y\,dx = \int_{t_1}^{t_2} y \cdot x'(t)\,dt$$

$$2\int_{\pi/2}^0 (2\sin^2\theta)(-2\csc^2\theta)\,d\theta = -8\int_{\pi/2}^0 d\theta = -8\theta\Big]_{\pi/2}^0 = \boxed{4\pi}$$

82) Area of asteroid: $\boxed{(b)\ \frac{3}{8}\pi a^2}$

84) area of deltoid: $\boxed{(c)\ 2\pi a^2}$

86) area of teardrop: $\boxed{(e)\ 2\pi ab}$