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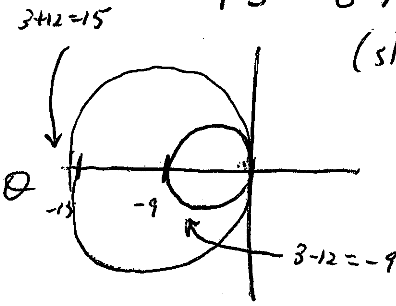
10.4b Polar Graphs, Equations, Derivatives p.722 43-87

(skip #54)

Graph polar equation

44) $r = 3(1 - 4\cos\theta)$ $0 \le \theta < 2\pi$



$r = a - b\cos\theta$

$a < b$

Limaçon (inner loop)

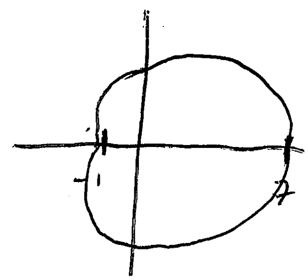
46) $r = 4 + 3\cos\theta$

$0 \le \theta < 2\pi$

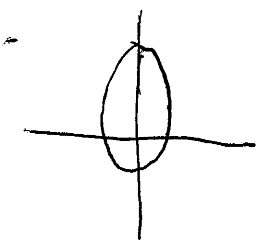
dimpled limaçon

$r = a + b\cos\theta$

$1 < \frac{a}{b} < 2$

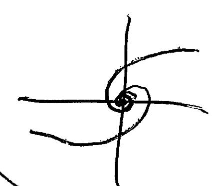


48) $r = \dfrac{2}{4 - 3\sin\theta}$ $0 \le \theta \le 2\pi$



50) $r = 3\sin\left(\frac{5\theta}{2}\right)$ $0 \le \theta < 4\pi$

52) $r^2 = \frac{1}{\theta}$ $r_1 = \frac{1}{\sqrt{\theta}}$ $r_2 = \frac{-1}{\sqrt{\theta}}$ $[0, \infty)$



$$\boxed{\text{distance formula: } d = \sqrt{r_1^2 + r_2^2 - 2r_1 r_2\cos(\theta_1 - \theta_2)}}$$

56) $(8, \frac{7\pi}{4}), (5, \pi)$ $d = \sqrt{8^2 + 5^2 - 2(8)(5)\cos\left(\frac{7\pi}{4} - \frac{4\pi}{4}\right)}$

$d = \sqrt{89 - 80\cos(3\pi/4)}$

$d = \sqrt{89 - 80\left(-\frac{\sqrt{2}}{2}\right)}$

$d = \sqrt{89 + 40\sqrt{2}} \approx \boxed{12.0652}$

58) $(4, 2.5)$ $(12, 1)$ $d = \sqrt{4^2 + 12^2 - 2(4)(12)\cos(2.5-1)}$

$\approx \boxed{12.3}$

60) Find $\frac{dy}{dx}$ and slopes of tangent lines

$r = 2(1-\sin\theta)$

$x = r\cos\theta$

$x = (2-2\sin\theta)\cos\theta$

$y = r\sin\theta$

$y = (2-2\sin\theta)(\sin\theta)$

$\frac{dy}{d\theta} = (-2\cos\theta)(\sin\theta) + (2-2\sin\theta)\cos\theta$

$\frac{dx}{d\theta} = (-2\cos\theta)\cos\theta + (2-2\sin\theta)(-\sin\theta)$

$\frac{dy/d\theta}{dx/d\theta} = \frac{dy}{dx}$

$\left.\frac{dy}{dx}\right|_{(2,0)}^{r,\theta} = \frac{(-2\cos 0)(0) + (2)(1)}{(-2)(1) + 0} = \frac{0+2}{-2} = \boxed{-1}$

$\left.\frac{dy}{dx}\right|_{(3, 7\pi/6)} = \frac{-2(\frac{\sqrt{3}}{2})(\frac{1}{2}) + (2-1)(\frac{\sqrt{3}}{2})}{(-2)(\frac{-\sqrt{3}}{2})(\frac{+\sqrt{3}}{2}) + (+3)(+\frac{1}{2})} = \frac{}{-\frac{3}{2} + \frac{3}{2}} = \frac{}{0}$ $\boxed{\text{undefined}}$

$\left.\frac{dy}{dx}\right|_{(4, 3\pi/2)} = \frac{(0)(-1) + (4)(0)}{(-0) + (4)(1)} = \frac{0}{4} = \boxed{0}$

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62) Find $\frac{dy}{dx}$ at θ. $r = 3 - 2\cos\theta$, $\theta = 0$

$x = r\cos\theta$

$x = (3-2\cos\theta)\cos\theta = 3\cos\theta - 2\cos^2\theta$

$\frac{dx}{d\theta} = -3\sin\theta - 4\cos\theta(-\sin\theta)$

$y = r\sin\theta$

$y = (3-2\cos\theta)(\sin\theta)$

$\frac{dy}{d\theta} = 2\sin\theta\sin\theta + (3-2\cos\theta)\cos\theta$

$\left.\frac{dy}{dx}\right|_{\theta=0} = \frac{0 + (3-2)(1)}{0 - 4(0)} = \frac{1}{0}$ undefined

At $\theta = 0$, $\frac{dy}{dx} = $ DNE (vertical tangent)

64) Find equation of tangent line $r=4,\ \theta=\pi/4$

$(r,\theta) \to (4, \pi/4)$

$x = r\cos\theta$

$x = 4\cos(\pi/4) = 4\left(\frac{\sqrt{2}}{2}\right) = 2\sqrt{2}$

$y = r\sin\theta$

$y = 4\sin(\pi/4) = 2\sqrt{2}$

$\frac{dy}{d\theta} = 4\cos\theta \qquad \frac{dx}{d\theta} = -4\sin\theta$

$\frac{dy}{dx} = \frac{4\cos\theta}{-4\sin\theta} = \frac{4\cos\pi/4}{-4\sin(\pi/4)} = -1$

point $(2\sqrt{2}, 2\sqrt{2})$
$m = -1$

$\boxed{y - 2\sqrt{2}) = \cancel{-}1(x - 2\sqrt{2})}$

66) Find horizontal, vertical tangents $r = a\sin\theta$

$y = r\sin\theta$

$y = a\sin^2\theta$

$\frac{dy}{d\theta} = 2a\sin\theta\cos\theta$

$0 = 2a\sin\theta\cos\theta$

$\theta = 0, \pi/2, \pi, 3\pi/2$

horiz. tangents
$(0,0)\ (a, \pi/2)$

$x = r\cos\theta \qquad x = a\sin\theta\cos\theta$

$\frac{dx}{d\theta} = a\cos\theta\cos\theta + a\sin\theta(-\sin\theta)$

$= a\cos^2\theta - a\sin^2\theta = a(\cos 2\theta)$

$0 = \cos 2\theta$

$2\theta = \pi/2, 3\pi/2, 5\pi/2, 7\pi/2$

$\theta = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$

vert. tangents: $\left(\frac{a\sqrt{2}}{2}, \pi/4\right), \left(\frac{a\sqrt{2}}{2}, 3\pi/4\right)$

68) Horizontal tangent $r = a\sin\theta\cos^2\theta$

$y = r\sin\theta$

$y = (a\sin\theta\cos^2\theta)(\sin\theta)$

$y = a\sin^2\theta\cos^2\theta$

$\frac{dy}{d\theta} = 2a\sin\theta\cos\theta\cdot\cos^2\theta + a\sin^2\theta\cdot 2\cos\theta(-\sin\theta)$

$= 2a\sin\theta\cos^3\theta - 2a\sin^3\theta\cos\theta$

$0 = 2a\sin\theta\cos\theta[\cos^2\theta - \sin^2\theta] = 0$

$0 = 2a\sin\theta\cos\theta\cdot\cos 2\theta = 0$

$\theta = 0, \pi \qquad 2\theta = \frac{\pi}{2}, 3\pi/2$

$\theta = \pi/4, 3\pi/4$

Horiz. tangents
$\left(\frac{\sqrt{2}a}{4}, \pi/4\right), \left(\frac{\sqrt{2}a}{4}, \frac{3\pi}{4}\right), (0,0)$

70) Find tangents at pole

$r = 5\cos\theta$

$r^2 = 5r\cos\theta$

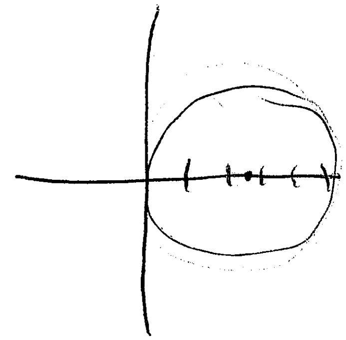
$r^2 = 5x$

$x^2 + y^2 = 5x$

$(5/2)^2$

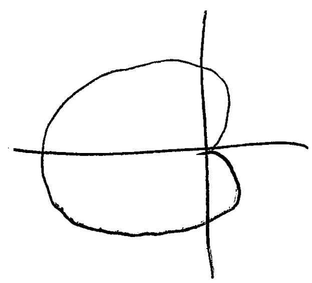
$x^2 - 5x + \_ + y^2 = 0 + \_$

$(x - 5/2)^2 + (y - 0)^2 = \frac{25}{4}$



Tangent at pole: $\theta = \pi/2$

72) $r = 3(1 - \cos\theta)$ cardioid



74) $r = -\sin(5\theta)$ rose curve w/ 5 petals

Symmetry about $\theta = \pi/2$

$\frac{dr}{d\theta} = -\cos 5\theta(5) = 0$

$5\theta = \pi/2, \frac{3\pi}{2}, 5\pi/2, 7\pi/2, 9\pi/2$

$\theta = \frac{\pi}{10}, \frac{3\pi}{10}, \frac{5\pi}{10}, \frac{7\pi}{10}, \frac{9\pi}{10}$

$0 = -\sin 5\theta$

$5\theta = \sin^{-1}(0)$

$5\theta = \pi, 2\pi, 3\pi, 4\pi, 5\pi$

$\theta = \pi/5, 2\pi/5, 3\pi/5, 4\pi/5$

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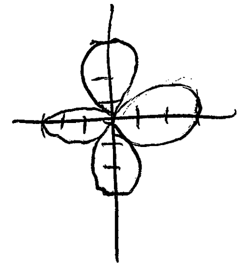
76) $r = 3\cos 2\theta$ Rose curve, 4 petals



Rel. extrema $\left(\frac{dr}{d\theta}=0\right)$ $(3,0)\ (-3, \pi/2), (-3, \frac{3\pi}{2}), (3, \pi)$

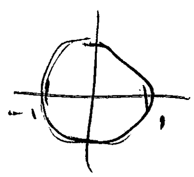
Tangents at poles: $0 = 3\cos 2\theta$

$2\theta = \cos^{-1}(0)$

$2\theta = \pi/2, 3\pi/2$

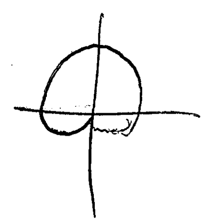
$\boxed{\theta = \pi/4, 3\pi/4}$

78) $r = 1$



circle

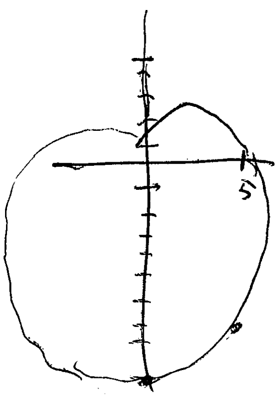
80) $r = 1 + \sin\theta$ cardioid



82) $r = 5 - 4\sin\theta$

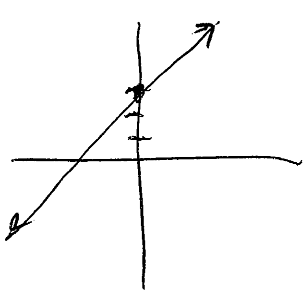
symmetric to $\theta = \pi/2$

θ	r
$-\pi/2$	9
$-\pi/6$	7
0	5
$\pi/6$	3
$\pi/2$	1



84) $r = \dfrac{6}{2\sin\theta - 3\cos\theta}$

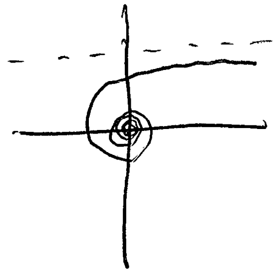
$2r\sin\theta - 3r\cos\theta = 6$



$2y - 3x = 6$

$y = \frac{3}{2}x + 3$

86) $r = \frac{1}{\theta}$



88) $r^2 = 4\sin\theta$ Lemniscate Rel. extrema $(\pm 2, \pi/2)$