Key

Chapter 2.1-2.5 Quiz Review

(Limit Definition of Derivative , Derivative Rules, Product & Quotient Rule)

No Calculators (answers can be left unsimplified)

Find the average rate of change of each function on the given interval. Use appropriate units if necessary.

1. $w(x) = \ln x; \quad 1 \le x \le 7$ slope b/t end points

$$\frac{w(7)-w(1)}{7-1} \rightarrow \frac{\ln 7 - \ln 1}{7-1} \rightarrow \frac{\ln 7 - 0}{6} = \boxed{\frac{\ln 7}{6}}$$

2. $s(t) = -t^2 - t + 4; \ [1,5]$
t represents seconds
s represents feet

$s(1) = 2$
$s(5) = -26$

$$\frac{s(5)-s(1)}{5-1} \rightarrow \frac{-26-2}{4} \rightarrow \frac{-28}{4} \rightarrow \boxed{-7\ ft/sec}$$

3. Find the derivative of $y = 2x^2 + 3x - 1$ by using the definition of the derivative. $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$f(x) = 2x^2+3x-1$
$f(\ ) = 2(\ )^2 + 3(\ ) - 1$
$f(x+h) = 2(x+h)^2 + 3(x+h) - 1$

$$f'(x) = \lim_{h\to 0} \frac{2(x+h)^2+3(x+h)-1-(2x^2+3x-1)}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{2(x^2+2xh+h^2)+3x+3h-1-2x^2-3x+1}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{\cancel{2x^2}+4xh+2h^2+\cancel{3x}+3h\cancel{-1}\cancel{-2x^2}\cancel{-3x}\cancel{+1}}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{\cancel{h}(4x+2h+3)}{\cancel{h}}$$

$$f'(x) = 4x + 2(0) + 3$$

$$\boxed{f'(x) = 4x+3}$$

4. For the function $h(t)$, h is the temperature of the oven in Fahrenheit, and t is the time measured in minutes.
   a. Explain the meaning of the equation $h(15) = 420$.

   The oven is 420°F after 15 minutes.

   b. Explain the meaning of the equation $h'(43) = -11$.

   The temperature in the oven is cooling by 11 degrees per minute on the 43rd minute.

Find the derivative of each function.

5. $f(x) = 4 - \frac{1}{2x^2}$

$f(x) = 4 - \frac{1}{2}x^{-2}$

$f'(x) = 0 - \frac{1}{2} \cdot -2x^{-3}$

$f'(x) = x^{-3}$

$\boxed{f'(x) = \frac{1}{x^3}}$

6. $g(x) = 3\sqrt{x} - \frac{6}{x^2} + 5\pi^3$

$g(x) = 3x^{1/2} - 6x^{-2} + 5\pi^3$

$g'(x) = \frac{1}{2} \cdot 3x^{-1/2} - -2 \cdot 6x^{-3} + 0$

$$\boxed{g'(x) = \frac{3}{2x^{1/2}} + \frac{12}{x^3} \quad \text{OR} \quad g'(x) = \frac{3}{2\sqrt{x}} + \frac{12}{x^3}}$$

7. $h(x) = 4e^x - 2\cos x$

$h'(x) = 4e^x - 2(-\sin x)$

$\boxed{h'(x) = 4e^x + 2\sin x}$

Find the derivative of each function.

8. $s(t) = \underbrace{t^2}_{f}\underbrace{\sin(t)}_{g}$ ↙ *product Rule

$$s'(t) = \overbrace{2t}^{f'} \cdot \overbrace{\sin(t)}^{g} + \overbrace{t^2}^{f} \cdot \overbrace{\cos(t)}^{g'}$$

$$\boxed{s'(t) = 2t\sin(t) + t^2\cos(t)}$$

9. $d(t) = \overbrace{3\sqrt{t}}^{f}\,\overbrace{\ln t}^{g}$

$$d(t) = \overbrace{3t^{1/2}}^{f} \cdot \overbrace{\ln(t)}^{g}$$

$$d'(t) = \overbrace{\tfrac{1}{2} \cdot 3t^{-1/2}}^{f'} \cdot \overbrace{\ln(t)}^{g} + \overbrace{3t^{1/2}}^{f} \cdot \overbrace{\tfrac{1}{t}}^{g'}$$

$$\boxed{d'(t) = \frac{3\ln(t)}{2t^{1/2}} + \frac{3}{t^{1/2}} \quad \text{or} \quad \frac{3\ln(t)}{2\sqrt{t}} + \frac{3}{\sqrt{t}}}$$

10. $y = \frac{4}{x} - \sec x$

$$f(x) = 4x^{-1} - \sec x$$

$$f'(x) = -4x^{-2} - \sec x \tan x$$

$$\boxed{f'(x) = \frac{-4}{x^2} - \sec x \tan x}$$

11. $h(x) = \dfrac{\overbrace{2-x}^{f}}{\underbrace{x+2}_{g}}$ *quotient Rule

$$h'(x) = \frac{\overbrace{(-1)}^{f'}\overbrace{(x+2)}^{g} - \overbrace{(2-x)}^{f}\overbrace{(1)}^{g'}}{\underbrace{(x+2)^2}_{g^2}} \rightarrow \frac{-x-2-2+x}{(x+2)^2}$$

$$\boxed{h'(x) = \frac{-4}{(x+2)^2}}$$

Find the equation of the tangent line of the function at the given x-value.

12. $f(x) = -2x^3 + 3x$ at $x = -1$.

$$f(-1) = -2(-1)^3 + 3(-1) = 2 - 3 = -1$$

$$f'(x) = -6x^2 + 3$$

$$f'(-1) = -6(-1)^2 + 3 = -3$$

point: $(-1, -1)$

slope: $m = -3$

$$\boxed{y + 1 = -3(x+1)}$$

13. $f(x) = 4\sin x - 2$ at $x = \pi$

$$f(\pi) = 4\sin\pi - 2 \rightarrow 4(0) - 2 = -2$$

$$f'(x) = 4\cos x$$

$$f'(\pi) = 4\cos\pi = 4(-1) = -4$$

point: $(\pi, -2)$

slope: $m = -4$

$$\boxed{y + 2 = -4(x - \pi)}$$

14. Find the equation for the normal line of $y = \frac{1}{2}x^2 + \frac{3}{4}x - 4$ at $x = -3$

$$y(-3) = \tfrac{1}{2}(-3)^2 + \tfrac{3}{4}(-3) - 4 = \frac{9}{2} - \frac{9}{4} - \frac{16}{4} = \frac{-7}{4}$$

point: $(-3, -7/4)$

$$y'(x) = 2 \cdot \tfrac{1}{2}x + \tfrac{3}{4}$$

$$y'(-3) = -3 + \frac{3}{4} = -\frac{9}{4}$$

slope (normal line): $m_2 = \frac{4}{9}$

$$\boxed{y + 7/4 = \tfrac{4}{9}(x+3)}$$

15. If $f(x) = 3\sin x - 2e^x$ find $f'(0)$. No calculator!

$$f'(x) = 3\cos x - 2e^x$$

$$f'(0) = 3\cos 0 - 2e^0$$

$$f'(0) = 3(1) - 2(1)$$

$$\boxed{f'(0) = 1}$$

16. Use the table below to estimate the value of $d'(120)$. Indicate units of measures.

Explain the meaning of d'(120) within context of this table.

t seconds	2	13	60	180	500
$d(t)$ feet	10	81	412	808	2,105

$d'(120) \approx \frac{d(180)-d(60)}{180-60} = \frac{808-412}{120} = \boxed{3.3\ ft/sec}$

$\boxed{\text{d'(120) means that the approximated rate of change of particle at t=120sec is 3.3 ft/sec.}}$

17. Is the function differentiable at $x = 2$?

$$f(x) = \begin{cases} 3x - 3x^2 - 5, & x < 2 \\ 7 - 9x, & x \geq 2 \end{cases}$$

* f(x) is differentiable if $\lim_{x \to c^-} f'(x) = \lim_{x \to c^+} f'(x)$

$f'(x) = \begin{cases} 3-6x, & x<2 \\ -9, & x \geq 2 \end{cases}$

$\lim_{x \to 2^-} 3-6x = -9$

$\lim_{x \to 2^+} -9 = -9$ ✓

* and continuous at $x=2$ $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$

$\lim_{x \to 2^-} 3x-3x^2-5 = -11$

$\lim_{x \to 2^+} 7-9x = -11$ ✓

$\boxed{\text{yes}}$ ✓

18. What values of a and b would make the function differentiable at $x = 4$?

$$f(x) = \begin{cases} a\sqrt{x} + bx^2 - 1, & x < 4 \\ \frac{16}{x} + bx, & x \geq 4 \end{cases}$$

* set equations equal

* set derivatives equal

$\rightarrow a\sqrt{4} + b(16) - 1 = \frac{16}{4} + 4b$

$2a = 5 - 12b$

$f'(x) = \begin{cases} \frac{1}{2}ax^{-1/2} + 2bx, & x<4 \\ -16x^{-2} + b, & x \geq 4 \end{cases}$

$\frac{a}{2\sqrt{4}} + 8b = -\frac{16}{4^2} + b$

$\rightarrow 4\left(\frac{a}{4} + 8b = -1 + b\right)$

$a + 32b = -4 + 4b \rightarrow a = -4 - 28b$

$2(-4-28b) = 5-12b$

$-8 - 56b = 5 - 12b$

$\boxed{b = -\frac{13}{44}}$

$\boxed{a = \frac{47}{11}}$

Each limit represents the instantaneous rate of change of a function. Identify the original function, and the x-value of the instantaneous rate of change.

19. $\lim_{x \to 4} \frac{\boxed{(x^2-3x)}-(4)}{x-4}$

Function: $f(x) = \boxed{f(x) = x^2 - 3x}$

Instantaneous rate at $x = \boxed{4}$

$f'(c) = \lim_{x \to c} \frac{\boxed{f(x)} - f(c)}{x - c}$

$f'(x) = 2x - 3$

$f'(4) = 2(4) - 3 = \boxed{5}$

20. $\lim_{h \to 0} \frac{\boxed{9(5+h)-10(5+h)^2}+(205)}{h}$

Function: $f(x) = \boxed{f(x) = 9x - 10x^2}$

Instantaneous rate at $x = \boxed{x=5}$

$f'(x) = \lim_{h \to 0} \frac{\boxed{f(x+h)} - f(x)}{h}$

$f(x) = 9x - 10x^2$

$f'(x) = 9 - 20x$

$f'(5) = 9 - 20(5) = \boxed{-91}$

Use the table to find the value of the derivatives of each function.

21.

x	$h(x)$	$h'(x)$	$r(x)$	$r'(x)$
-2	-3	2	-2	4

a. $f(x) = \overbrace{-h(x)}^{f}\overbrace{r(x)}^{g}$
Find $f'(-2)$.

$f'(x) = \overbrace{-h'(x)}^{f'}\cdot\overbrace{r(x)}^{g} + \overbrace{-h(x)}^{f}\cdot\overbrace{r'(x)}^{g'}$

$f'(-2) = -h'(-2)\cdot r(-2) - h(-2)\cdot r'(-2)$

$f'(-2) = -2(-2) - (-3)(4)$

$= 4 + 12$

$\boxed{f'(-2) = 16}$

b. $g(x) = \dfrac{\overbrace{h(x)+r(x)}^{f}}{\underbrace{r(x)}_{g}}$ *quotient Rule
Find $g'(-2)$.

$g'(x) = \dfrac{\overbrace{(h'(x)+r'(x))}^{f'}\overbrace{r(x)}^{g} - \overbrace{(h(x)+r(x))}^{f}\cdot\overbrace{r'(x)}^{g'}}{\underbrace{r(x)^2}_{g^2}}$

$g'(-2) = \dfrac{[h'(-2)+r'(-2)]\,r(-2) - [h(-2)+r(-2)]\,r'(-2)}{[r(-2)]^2}$

$g'(-2) = \dfrac{[2+4](-2) - (-3-2)(4)}{(-2)^2} \rightarrow \dfrac{-12+20}{4} \rightarrow \dfrac{8}{4} = \boxed{2}$

c. $w(x) = \overbrace{(4 - 2h(x))}^{f}\overbrace{(1 - r(x))}^{g}$
Find $w'(-2)$.

$w'(x) = -2h'(x)[1-r(x)] + (4-2h(x))\cdot -r'(x)$

$w'(-2) = -2h'(-2)[1-r(-2)] + (4-2h(-2))\cdot -r'(-2)$

$= -2(2)(1+2) + (4+6)(-4)$

$= -4(3) + 10(-4)$

$= -52$

$\boxed{w'(-2) = -52}$

22. At what x-value(s) does the function $f(x) = \frac{x^4}{4} - 3x^3 + 9x^2 + 7$ have a horizontal tangent? *find horizontal tangent by setting (numerator) of $f'(x) = 0$.

$f'(x) = 4\cdot\frac{1}{4}x^3 - 9x^2 + 18x$

$f'(x) = x^3 - 9x^2 + 18x$

$0 = x(x^2 - 9x + 18)$

$0 = x(x-6)(x-3)$

$\boxed{x = 0,\ x = 3,\ x = 6}$

23. If $f(x) = \cos x + \sin x$, find $f'\left(\frac{\pi}{3}\right)$

$f'(x) = -\sin x + \cos x$

$f'\left(\frac{\pi}{3}\right) = -\sin\left(\frac{\pi}{3}\right) + \cos\left(\frac{\pi}{3}\right)$

$f'(\pi/3) = -\left(\frac{\sqrt{3}}{2}\right) + \frac{1}{2} = \frac{-\sqrt{3}+1}{2}$

$\boxed{f'(\pi/3) = \frac{1-\sqrt{3}}{2}}$

24. $S(x)$ is the number of students in Mr. Kelly's class and x is the number of years since 2015.

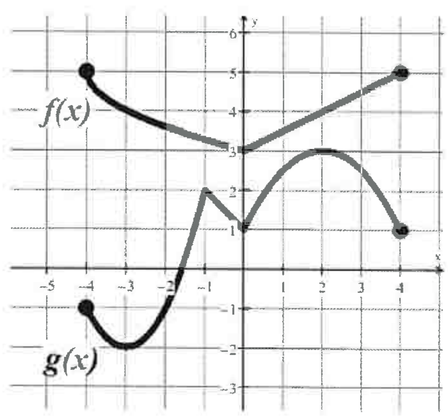
a. Explain the meaning of $S(3) = 127$.

In 2018, there are 127 students in Mr. Kelly's class.

b. Explain the meaning of $S'(3) = 4$.

In 2018, the number of students in Mr. Kelly's class is increasing by 4 students per year.

25. **Use the graphs of f and g to find the following.**



a. $h(x) = f(g(x))$. Find the average rate of change on the interval [2,4].

$g(4) = 1$
$g(2) = 3$

$\dfrac{f(g(4)) - f(g(2))}{4-2} \rightarrow \dfrac{f(1) - f(3)}{4-2} \rightarrow \dfrac{3.5-4.5}{2} = \boxed{\dfrac{-1}{2}}$

b. $j(x) = g(f(x))$. Find the average rate of change on the interval $[-3,2]$.

$f(2) = 4$
$f(-3) = 4$

$\dfrac{g(f(2)) - g(f(-3))}{2-(-3)} \rightarrow \dfrac{g(4) - g(4)}{5} \rightarrow \dfrac{1-1}{5} \rightarrow \dfrac{0}{5} \rightarrow \boxed{0}$