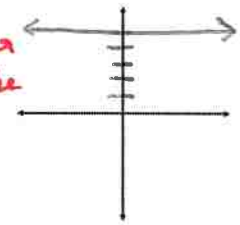
Key

AP Calculus – 2.3 Notes – Derivatives of Polynomials (Power Rule)

1. Constant Rule: If $f(x) = c$, then $f'(x) = 0$

Example: $f(x) = 5$, so $f'(x) = 0$ slope of the line is 0.



2. Power Rule: If $f(x) = x^n$, then $f'(x) = n * x^{n-1}$

Steps *a*) Bring Exponent down as coefficient in front of the variable

b) Subtract 1 from the original exponent value

Power Rule Conditions:

i) Convert radicals to rational exponents (ex: $\sqrt{x^5} = x^{\frac{5}{2}}$)

ii) Bring variable to the numerator before applying power rule

iii) Expand terms: resolve parentheses & fractional terms before applying Power Rule

**Important Note*: Be sure the function is in the appropriate form (all conditions met!) before applying Power Rule

Example 1: Find Derivatives of the following:

a) $y = x^7$ → $\boxed{y' = 7x^6}$

b) $g(x) = \sqrt[3]{x}$ $\quad g'(x) = \frac{1}{3}x^{-2/3}$ $\quad \boxed{g'(x) = \frac{1}{3x^{2/3}}}$

$g(x) = x^{1/3}$

c) $y = \frac{4}{x^5}$ → $y = 4x^{-5}$ $\quad y' = -20x^{-6}$ $\quad \boxed{y' = \frac{-20}{x^6}}$

d) $y = 8x^{2/3} - \sqrt[5]{x} + \frac{2}{3\sqrt{x}} + 0.875$ $\quad \boxed{y' = \frac{16}{3x^{1/3}} - \frac{1}{5x^{4/5}} - \frac{1}{3x^{3/2}}}$

$y = 8x^{2/3} - x^{1/5} + \frac{2}{3}x^{-1/2} + 0.875$

$y' = 8 \cdot \frac{2}{3}x^{-1/3} - \frac{1}{5}x^{-4/5} + \frac{2}{3} \cdot \frac{-1}{2}x^{-3/2} + 0$

Example 2: If $f(x) = \frac{1}{x^2}$ $\quad$ $find\ f'(2)$

$f(x) = x^{-2}$ | $f'(x) = -2x^{-3}$

$f'(x) = \frac{-2}{x^3}$

$f'(2) = \frac{-2}{2^3} = \boxed{\frac{-1}{4}}$

Example 3: $If\ f(x) = \sqrt[3]{x^2}\ , write\ the\ tangent\ line\ equation\ to\ f(x) at\ x = 1$

$f(x) = x^{2/3}$

$f'(x) = \frac{2}{3}x^{-1/3} = \frac{2}{3x^{1/3}}$

$f'(1) = \frac{2}{3(1)^{1/3}} = \frac{2}{3}$

$f(1) = 1^{2/3} = 1$

point: $(1,1)$

slope: $m = 2/3$

$y - y_1 = m(x - x_1)$

$\boxed{y - 1 = \frac{2}{3}(x - 1)}$

Example 4: Find $f'(x)$ if $f(x) = \frac{x^4 - 3x^2 + 4(\sqrt[3]{x})}{2\sqrt{x}}$

$f(x) = \frac{x^4}{2x^{1/2}} - \frac{3x^2}{2x^{1/2}} + \frac{4x^{1/3}}{2x^{1/2}}$

$f(x) = \frac{1}{2}x^{7/2} - \frac{3}{2}x^{3/2} + 2x^{-1/6}$

$f'(x) = \frac{1}{2} \cdot \frac{7}{2}x^{5/2} - \frac{3}{2} \cdot \frac{3}{2}x^{1/2} + 2 \cdot \frac{-1}{6}x^{-7/6}$

$\boxed{f'(x) = \frac{7}{4}x^{5/2} - \frac{9}{4}x^{1/2} - \frac{1}{3x^{7/6}}}$

Example 5: Find $f'(x)$ if $f(x) = 3x(x + 1)^2$

*expand first

$f(x) = 3x(x+1)(x+1)$

$f(x) = 3x(x^2 + 2x + 1)$

$f(x) = 3x^3 + 6x^2 + 3x$

$\boxed{f'(x) = 9x^2 + 12x + 3}$