AP Calculus – 2.3b Notes Derivatives of e^x, $\ln(x)$, $\sin x$, *and* $\cos x$ functions

Recall:

$\ln 1 = 0$ $\ln 0 =$ undefined $e^0 = 1$ $e^{\ln a} = a$ $\ln e^a = a$

Derivatives of Exponential Functions

$$\frac{d}{dx}e^x = e^x$$

Derivatives of Logarithmic Functions

$$\frac{d}{dx}\ln x = \frac{1}{x}$$

Find the value of the derivative at the given point.

4. If $f(x) = 3\ln x + e^x$, find $f'(5)$

$f'(x) = 3\left(\frac{1}{x}\right) + e^x$

$f'(x) = \frac{3}{x} + e^x$

$f'(5) = \frac{3}{5} + e^5$

5) What is the slope of the line tangent to the graph of $y = 2\ln(x)$ at the point $x = 8$?

(A) $\frac{1}{16}$ (B) $\frac{1}{8}$ (C) $\frac{1}{4}$ (D) 16 (E) 4

$y' = 2\left(\frac{1}{x}\right)$

$y' = 2\left(\frac{1}{8}\right) = \frac{1}{4}$

$y'(8) = \frac{1}{4}$

6) If $f(x) = 4\ln x - 3e^x + e$, find $f'(1)$

$f'(x) = 4\left(\frac{1}{x}\right) - 3e^x + 0$

$f'(x) = \frac{4}{x} - 3e^x$

$f'(1) = \frac{4}{1} - 3e^1$

$f'(1) = 4 - 3e$

Derivatives of cos x and sin x

$$\frac{d}{dx}\cos x = -\sin x \qquad \frac{d}{dx}\sin x = \cos x$$

Example: Find $f'(x)$ if $f(x) = 2\sin x - 5\cos x$

$f'(x) = 2\cos x - 5(-\sin x)$

$\boxed{f'(x) = 2\cos x + 5\sin x}$

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13. If $f(x) = 4e^x + 5\sin x$, find $f'(0)$

$f'(x) = 4e^x + 5\cos x$

$f'(0) = 4e^0 + 5\cos(0)$

$f'(0) = 4(1) + 5(1) = \boxed{9}$

14. If $f(x) = 2\cos x + e^x$, find $f'(\pi)$

$f'(x) = -2\sin x + e^x$

$f'(\pi) = -2\sin\pi + e^\pi$

$f'(\pi) = -2(0) + e^\pi$

$\boxed{f'(\pi) = e^\pi}$

Find the equation of the tangent line at the given x-value.

15. $f(x) = 3\cos x + x$ at $x = \frac{\pi}{2}$

~~$f'(x) = -3\sin x + 1$~~

$f'(\pi/2) = -3\sin\left(\frac{\pi}{2}\right) + 1 = -3(1) + 1$

$f'(\pi/2) = -2$

$f\left(\frac{\pi}{2}\right) = 3\cos\frac{\pi}{2} + \frac{\pi}{2} = 3(0) + \frac{\pi}{2}$

point: $(\pi, \pi/2)$

slope: $m = -2$

$y - (\pi/2) = -2(x - \pi)$

$\boxed{y - \frac{\pi}{2} = -2(x - \pi)}$

16. $f(x) = 4e^x - 3\sin x + x^2$ at $x = 0$

$f'(x) = 4e^x - 3\cos x + 2x$

$f'(0) = 4e^0 - 3\cos 0 + 2(0) \rightarrow 4 - 3 = 1$

$f'(0) = 1$

$f(0) = 4e^0 - 3\sin 0 + 0^2 = 4$

point: $(0, 4)$

slope: $m = 1$

$y - 4 = 1(x - 0)$

$\boxed{y - 4 = x}$

or

$y = x + 4$