Key

AP Calculus – 2.5 Notes - Derivatives of Trig Functions

Trig Derivatives

$\frac{d}{dx}\sin x = \cos x$

$\frac{d}{dx}\tan x = \sec^2 x$

$\frac{d}{dx}\sec x = \sec x \tan x$

$\frac{d}{dx}\cos x = -\sin x$

$\frac{d}{dx}\cot x = -\csc^2 x$

$\frac{d}{dx}\csc x = -\csc x \cot x$

Common struggles for students dealing with trig derivatives:

- Memorizing.
- Unit Circle values.
- Simplifying/manipulating trig expressions.
- Trig reciprocals in a calculator.

1. Find the derivative of $y = \underbrace{\sin x}_{f}\,\underbrace{\tan x}_{g}$ ← *product Rule

$y' = \underbrace{\cos x}_{f'} \cdot \underbrace{\tan x}_{g} + \underbrace{\sin x}_{f} \cdot \underbrace{\sec^2 x}_{g'}$

$\boxed{y' = \cos x \tan x + \sin x \sec^2 x}$ → $\boxed{y' = \sin x + \sin x \sec^2 x}$

2. Find $f'\left(\frac{\pi}{6}\right)$ if $f(x) = \frac{x}{\sec x}$

$f'(x) = \frac{(1)(\sec x) - x \cdot \sec x \tan x}{\sec^2 x}$

$f'(x) = \frac{\sec x(1 - x\tan x)}{\sec^2 x}$

$f'(x) = \frac{1 - x\tan x}{\sec x}$

$f'\left(\frac{\pi}{6}\right) = \frac{1 - \frac{\pi}{6}\tan \pi/6}{\sec(\pi/6)}$

$f'(\pi/6) = \frac{1 - \frac{\pi}{6}\left(\frac{1}{\sqrt{3}}\right)}{\frac{2}{\sqrt{3}}}$ $\quad \frac{\frac{6\sqrt{3} - \pi}{6\sqrt{3}}}{\frac{2}{\sqrt{3}}}$

$\frac{6\sqrt{3} - \pi}{6\cancel{\sqrt{3}}} \cdot \frac{\cancel{\sqrt{3}}}{2} \rightarrow \frac{6\sqrt{3}}{12} - \frac{\pi}{12}$

$\boxed{f'\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} - \frac{\pi}{12}}$

Find the derivative of each function

3. $h(x) = 2x\tan(x)$ ← *product Rule

$h'(x) = \underbrace{2}_{f'} \cdot \underbrace{\tan x}_{g} + \underbrace{2x}_{f} \cdot \underbrace{\sec^2 x}_{g'}$

$\boxed{h'(x) = 2\tan x + 2x\sec^2 x}$

4. $f(x) = \frac{1}{2\cos x} = \frac{1}{2}\sec x$

$\boxed{f'(x) = \frac{1}{2}\sec x \tan x}$

Find the derivative at the given x-value. Show your work!

5. $f(x) = 2\sec x$ at $x = \frac{\pi}{4}$.

$f'(x) = 2\sec x \tan x$

$f'(\pi/4) = 2\sec(\pi/4)\tan(\pi/4)$

$f'(\pi/4) = 2\left(\frac{2}{\sqrt{2}}\right)(1) = \frac{4}{\sqrt{2}} \to \frac{4\sqrt{2}}{2} \to 2\sqrt{2}$

$\boxed{f'(\pi/4) = 2\sqrt{2}}$

* product Rule

6. $f(x) = \underbrace{x}_{f}\underbrace{\cot x}_{g}$ at $x = \frac{\pi}{6}$.

$f'(x) = \underbrace{1}_{f'} \cdot \underbrace{\cot x}_{g} + \underbrace{x}_{f} \cdot \underbrace{-\csc^2 x}_{g'}$

$f'(\pi/6) = \cot(\pi/6) - \left(\frac{\pi}{6}\right)\left[\csc(\pi/6)\right]^2$

$= \sqrt{3} - \frac{\pi}{6}[2]^2$

$\boxed{f'\left(\frac{\pi}{6}\right) = \sqrt{3} - \frac{2\pi}{3}}$

Find the equations of both the normal line and the tangent line.

7. $y = \sec x$ at $x = \pi$

$y' = \sec x \tan x$

$y'(\pi) = \sec\pi \tan\pi$

$y'(\pi) = (-1)(0) = 0$

slope $m_1 = 0$

slope $m_2 =$ undefined

point: $y(\pi) = \sec(\pi)$

$y(\pi) = -1$

point: $(\pi, -1)$

$y - y_1 = m(x - x_1)$

Tangent: $y + 1 = 0(x - \pi) \to \boxed{y = -1}$

Normal: $x = \pi$ (vertical line through the point)

8. $y = \tan x$ at $x = \frac{\pi}{3}$

$y' = \sec^2 x$

$y'(\pi/3) = \left[\sec\frac{\pi}{3}\right]^2$

$y'(\pi/3) = (2)^2 = 4$

slope: $m_1 = 4$

$m_2 = -1/4$

$y\left(\frac{\pi}{3}\right) = \tan(\pi/3)$

$y(\pi/3) = \sqrt{3}$

point: $\left(\frac{\pi}{3}, \sqrt{3}\right)$

Tangent: $y - \sqrt{3} = 4(x - \pi/3)$

Normal: $y - \sqrt{3} = \frac{-1}{4}(x - \pi/3)$

Find the equation of the tangent line at the given x-value.

15. $f(x) = 3\cos x + x$ at $x = \frac{\pi}{2}$

$f\left(\frac{\pi}{2}\right) = 3\cos(\pi/2) + \frac{\pi}{2} = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

$f'(x) = -3\sin x + 1$

$f'(\pi/2) = -3\sin(\pi/2) + 1 = -3(1) + 1 = -2$

point: $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$

slope: $m = -2$

$\boxed{y - \frac{\pi}{2} = -2\left(x - \frac{\pi}{2}\right)}$

16. $f(x) = 4e^x - 3\sin x + x^2$ at $x = 0$

$f(0) = 4e^0 - 3\sin 0 + 0^2 = 4(1) = 4$

$f'(x) = 4e^x - 3\cos x + 2x$

$f'(0) = 4e^0 - 3\cos 0 + 2(0) = 4 - 3 = 1$

point: $(0, 4)$

slope: $m = 1$

$\boxed{y - 4 = 1(x - 0)}$