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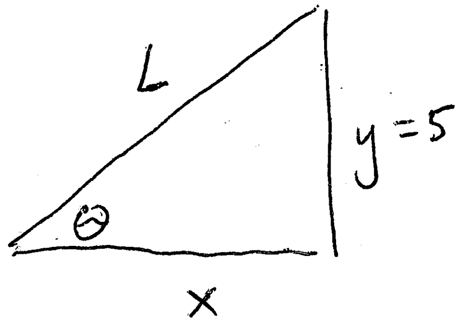
Ch. 2.6 Related Rates BC Select HW Problems

40)



$\tan\theta = \frac{y}{x}$, $y = 5$

$\frac{dx}{dt} = -600$ mph

$\tan\theta = \frac{y}{x} \quad \tan\theta = \frac{5}{x} \quad \tan\theta = 5x^{-1}$

$\sec^2\theta\left(\frac{d\theta}{dt}\right) = -5x^{-2}\left(\frac{dx}{dt}\right)$

$\frac{d\theta}{dt} = \frac{\cos^2\theta\cdot(-5)}{x^2}\left(\frac{dx}{dt}\right) = \cos^2\theta\cdot\left(\frac{-5}{x^2}\right)\left(\frac{dx}{dt}\right)$

$= \left(\frac{\cancel{x^2}}{L^2}\right)\left(\frac{-5}{\cancel{x^2}}\right)(-600) = \left(\frac{-5}{L^2}\right)(-600) = \sin^2\theta\cdot 600$

$\frac{d\theta}{dt} = 600\sin^2\theta$

a) $\theta = 30°$

$\frac{d\theta}{dt} = 120\left[\sin \pi/6\right]^2 = 120\left(\frac{1}{2}\right)^2 = \frac{120}{4} = \boxed{30 \text{ rad/hr} = \frac{1}{2} \text{ rad/min}}$

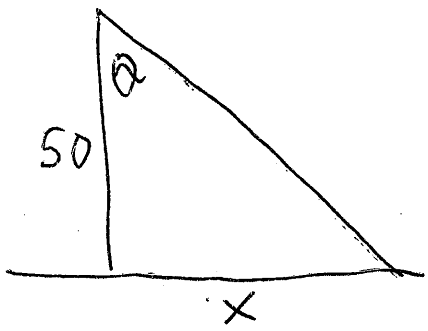
b) $\theta = 60°$

$\frac{d\theta}{dt} = 120\left[\sin \pi/3\right]^2 = 120\left(\frac{\sqrt{3}}{2}\right)^2 = 120\left(\frac{3}{4}\right) = \boxed{90 \text{ rad/hr} = \frac{3}{2} \text{ rad/min}}$

c) $\theta = 75°$

$\frac{d\theta}{dt} = 120\left[\sin 75\right]^2 = \boxed{111.96 \text{ rad/hr} = 1.87 \text{ rad/min}}$

44)



$\tan\theta = \frac{x}{50}$ $\qquad \frac{dx}{dt} = 2$ ft/s

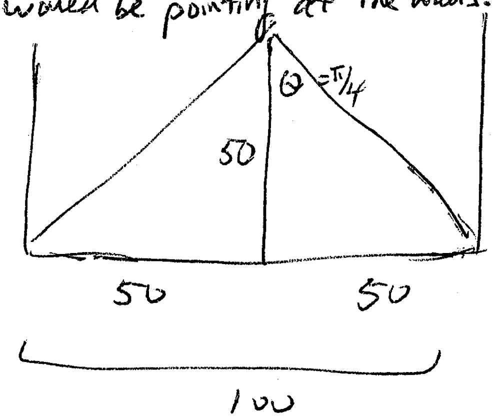
$\sec^2\theta\left(\frac{d\theta}{dt}\right) = \frac{1}{50}\left(\frac{dx}{dt}\right)$

$$\frac{d\theta}{dt} = \cos^2\theta \cdot \frac{1}{50} \cdot \frac{dx}{dt}$$

$$\frac{d\theta}{dt} = \cos^2\theta \cdot \frac{1}{50} \cdot (2)$$

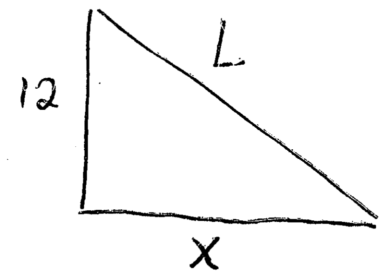
$$\frac{d\theta}{dt} = \frac{1}{25}\cos^2\theta \quad \text{when} \quad -\pi/4 \le \theta \le \pi/4$$

*If the angle extends beyond $\theta = \pi/4$ or $\theta = -\pi/4$, the security camera would be pointing at the walls.



Ch. 2.6 Related Rates BC HW Problem

#47



$$x^2 + 12^2 = L^2$$

$$\cancel{2}x\left(\frac{dx}{dt}\right) + 0 = \cancel{2}L\left(\frac{dL}{dt}\right)$$

$$x\left(\frac{dx}{dt}\right) = L\left(\frac{dL}{dt}\right)$$

Find 2nd Derivative (product rule)

$$\underbrace{1\left(\frac{dx}{dt}\right)}_{f'}\underbrace{\left(\frac{dx}{dt}\right)}_{g} + \underbrace{x}_{f}\underbrace{\left(\frac{d^2x}{dt^2}\right)}_{g'} = \underbrace{(1)\left(\frac{dL}{dt}\right)}_{f'} \cdot \underbrace{\frac{dL}{dt}}_{g} + \underbrace{L}_{f}\underbrace{\left(\frac{d^2L}{dt^2}\right)}_{g'}$$

$$\left(\frac{dx}{dt}\right)^2 + x\frac{d^2x}{dt^2} = \left(\frac{dL}{dt}\right)^2 + L\left(\frac{d^2L}{dt^2}\right) - \left(\frac{dx}{dt}\right)^2$$

$$\frac{d^2x}{dt^2} = \frac{1}{x}\left[\left(\frac{dL}{dt}\right)^2 + L\left(\frac{d^2L}{dt^2}\right) - \left(\frac{dx}{dt}\right)^2\right]$$

When $L = 13$, $x = 5$

$\frac{dx}{dt} = -10.4$

$\frac{dL}{dt} = -4$, $\frac{d^2L}{dt^2} = 0$

$$\frac{d^2x}{dt^2} = \frac{1}{5}\left[(-4)^2 + 13(0) - (-10.4)^2\right]$$

$$= \frac{1}{5}(-92.16) = \boxed{-18.432 \text{ ft/s}^2}$$