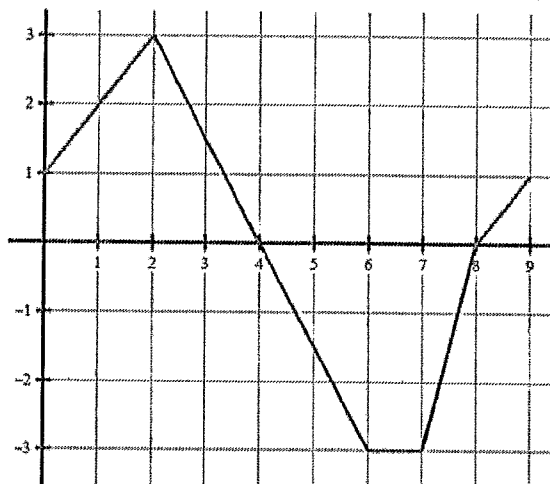


BC Calculus 4.1-4.4 Quiz Review (Calculators Allowed)

(Related Rates, Particle Motion, Linear Approximation, L'Hopital's Rule)

1. The figure shows the velocity  $v = \frac{ds}{dt} = f(t)$  of a body moving along a coordinate line in meters per second.

- When does the body reverse direction?
- When is the body moving at a constant speed?
- What is the body's maximum speed?
- At what time interval(s) is the body slowing down?



Find the following. Use L'Hospital's when possible.

2.  $\lim_{x \rightarrow 2} \frac{x-2}{x^2-7x+10}$

3.  $\lim_{x \rightarrow 0} \frac{3x^2}{e^x-1-x}$

4.  $\frac{d}{dx} \frac{3x-2}{5x+1}$

5. If the length  $l$  of a rectangle is decreasing at a rate of 2 inches per minute while its width  $w$  is increasing at a rate of 2 inches per minute, which of the following must be true about the area  $A$  of the rectangle?

- (A)  $A$  is always increasing.      (B)  $A$  is always decreasing.      (C)  $A$  is increasing only when  $l > w$ .  
 (D)  $A$  is increasing only when  $l < w$ .      (E)  $A$  remains constant.

6) A particle moves along the x-axis (in meters) so that at times  $t \geq 0$  seconds, its position is given by  $x(t) = t^3 - 3t^2 - 9t + 2$

a) Find the velocity and acceleration function

b) What is its velocity at  $t = 2$  seconds? (provide units of measure)

c) What is its acceleration at  $t = 4$  seconds? (provide units of measure)

d) At what times does the particle change directions? Justify

e) At  $t = 0$ , is the particle moving to the right or to the left? Justify.

f) Find the average velocity of particle in  $[1, 3]$

g) What is displacement of particle from  $t = 1$  to  $t = 4$ ? Show work.

h) What is the total distance of particle from  $t = 1$  to  $t = 4$ ? Show work.

i) Is velocity increasing or decreasing at  $t = 2$ ? Justify.

j) Is the speed increasing or decreasing at  $t = 4$ ? Justify

k) Find the average acceleration of particle on interval  $[1,2]$

7) The function  $f(x) = (1 - \sin x)^2$  is concave up at  $x = \frac{\pi}{6}$ ?

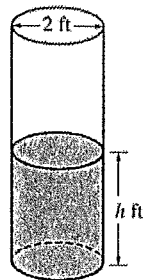
a. What is the estimate for  $f(0.5)$  using the local linear approximation for  $f$  at  $x = \frac{\pi}{6}$ ?

b. Is it an underestimate or overestimate? Explain.

8) A cylindrical barrel with a diameter of 2 feet contains collected rainwater, as shown in the figure above. The water drains out through a valve (not shown) at the bottom of the barrel. The rate of change of the height  $h$  of the water in the barrel with respect to time  $t$  is modeled by  $\frac{dh}{dt} = -\frac{1}{10}\sqrt{h}$ , where  $h$  is measured in feet and  $t$  is measured in seconds. (The volume  $V$  of a cylinder with radius  $r$  and height  $h$  is  $V = \pi r^2 h$ .)

(a) Find the rate of change of the volume of water in the barrel with respect to time when the height of the water is 4 feet. Indicate units of measure.

(b) When the height of the water is 3 feet, is the rate of change of the height of the water with respect to time increasing or decreasing? Explain your reasoning.



- 9) Person X and Person Y are walking on straight streets that meet at right angles. Y travels south and approaches the intersection at 2m/s. Person X travels east and moves away from the intersection at 1m/s.
- Find the rate at which the distance (Z) between Person X and Y is changing when Y is 10m from the intersection and X is 20 meters from the intersection.
  - At what rate is the angle  $\theta$  changing at the same moment?
- c) Find the rate of change of triangle at the same moment.

**Types of Related Rates Problems to Review:**

- 1) Pythagorean Theorem Problems
- 2) Area of (Right) Triangle  $A = \frac{1}{2}xy$
- 3) Sphere (Volume & Surface Area) – formulas will be provided
- 4) Trig Related Rates
- 5) Cone Problems (formula will be provided)
- 6) Cylinder Problems (formula will be provided)
- 7) Similar Right Triangle (shadow problem)

\*Formulas to memorize: Pythagorean theorem, Area and circumference of circle, area of square, area of triangle, SOH-CAH-TOA