Ch. 4.1 Exercise Problems Interpeting a Derivative

p. 270 #5-15 odds

5) $f(x) = x^3 - 4x^2 - 2$ at $c = 3$

a) $f'(x) = 3x^2 - 8x$

b) $f'(3) = 3(3)^2 - 8(3) \rightarrow f'(3) = 3$

c) find equation of tangent line:

$f(3) = 3^3 - 4(3)^2 - 2 = -11$

point: $f(3) = -11$

slope: $m = 3$

$\boxed{y + 11 = 3(x - 3)}$

7) $f(x) = \dfrac{e^x}{x^2+2}$ at $c = 0$

a) $f'(x) = \dfrac{\overbrace{e^x}^{f'}\,\overbrace{(x^2+2)}^{g} - \overbrace{e^x}^{f} \cdot \overbrace{2x}^{g'}}{\underbrace{(x^2+2)^2}_{g^2}} \rightarrow \dfrac{e^x(x^2+2-2x)}{(x^2+2)^2}$

b) $f'(0) = \dfrac{e^0(0+2-0)}{(0+2)^2} = \dfrac{2}{4} = \dfrac{1}{2}$

c) Tangent line equation:

$f(0) = \dfrac{e^0}{0^2+2} = \dfrac{1}{2}$

point: $(0, \tfrac{1}{2})$

slope: $m = \tfrac{1}{2}$

$\boxed{y - \tfrac{1}{2} = \tfrac{1}{2}(x - 0)}$

9) $s(t) = -16t^2 + 120t$ (position function)

$s'(t) = -32t + 120$ (velocity $v(t)$ equation)

$s''(t) = -32$ (acceleration $a(t)$ equation)

$\boxed{4.1}$

11) $s(t) = t\cos^2(t)$

$s(t) = \overbrace{t}^{f}\overbrace{(\cos(t))^2}^{g}$ *product Rule

*chain Rule out: $(\ )^2$ in: $\cos(t)$

$v(t) = s'(t) = \overbrace{1}^{f'} \cdot \overbrace{(\cos(t))^2}^{g} + \overbrace{t}^{f} \cdot \overbrace{2(\cos(t)) \cdot (-\sin(t))}^{g'}$

$v(t) = (\cos(t))^2 - t \cdot 2\sin t\cos t$ $\quad *2\sin\theta\cos\theta = \sin(2\theta)$

$v(t) = [\cos(t)]^2 - t \cdot \sin(2t)$

$a(t) = 2[\cos(t)] \cdot -\sin(t) - [\overbrace{1}^{f'}\ \overbrace{\sin(2t)}^{g} + \overbrace{t}^{f} \cdot \overbrace{\cos(2t) \cdot 2}^{g'}]$

$a(t) = -2\sin t\cos t - \sin(2t) - 2t\cos(2t)$

13) $P(t) = 30e^{0.2t}$

a) $P(0) = 30e^{0.2(0)} = 30$

b) $P(5) = 30e^{0.2(5)} \approx 81.548$

c) $P'(t) = 30e^{0.2t} \cdot 0.2 \rightarrow P'(t) = 6e^{0.2t}$

d) $P'(10) = 6e^{0.2(10)} = 6e^2 \approx 44.334$ flies/day at 10 days

(Rate of growth of fruit flies on day 10)

15) $S(t) = 4t - 1 \quad S'(t) = 4$

The rate at which the surface area is increasing is $4\,cm^2/sec$.