AP Calculus – 4.1 Notes Interpreting Derivatives & Particle Motion (Position-Velocity-Acceleration)

Key

Recall:

Slope between two points: $\frac{\Delta y}{\Delta x}$ or $\frac{\Delta \text{dependent}}{\Delta \text{independent}}$

Units for the Derivative:

The derivative of $f(x)$ is $\frac{\text{unit for } f}{\text{unit for } x}$

If $f'(x) > 0$, then $f(x)$ is increasing   If $f'(x) < 0$, then $f(x)$ is decreasing

1. Mr. Sullivan wants Mr. Brust to finish creating his packets in Algebra 2. The number of packets Mr. Brust has completed is modeled by $p(w)$, where w is measured in weeks.
   a. Interpret $p(10) = 1$ in the context of the problem.

      After 10 weeks, Mr. Brust has completed 1 packet

   b. Interpret $p'(39) = 4$ in the context of the problem.

      at the 39th week, he is making packets at a rate of 4 packets per week

2. The rate at which Mr. Kelly is buying baseball cards per year is modeled by $b(t)$, where t is measured in years.
   a. Interpret $b(3) = 150$ in the context of the problem.

      On the 3rd year, Mr. Kelly is buying cards at a rate of 150 cards per year.

   b. Interpret $b'(4) = 10$ in the context of the problem.

      On the 4th year, the rate at which Mr. Kelly is buying baseball cards is increasing at a rate of 10 cards per year per year

Practice problems:

For each problem, a differentiable function is given along with a definition of the variables. Interpret the values in the context of the problem.

1. The percentage grade a student receives on a test, is modeled by $G(t)$ where t is the number of hours spent studying for the test. Interpret $G'(1) = 3$.

   At 1 hour of studying, the grade will improve at a rate of 3% per hour.

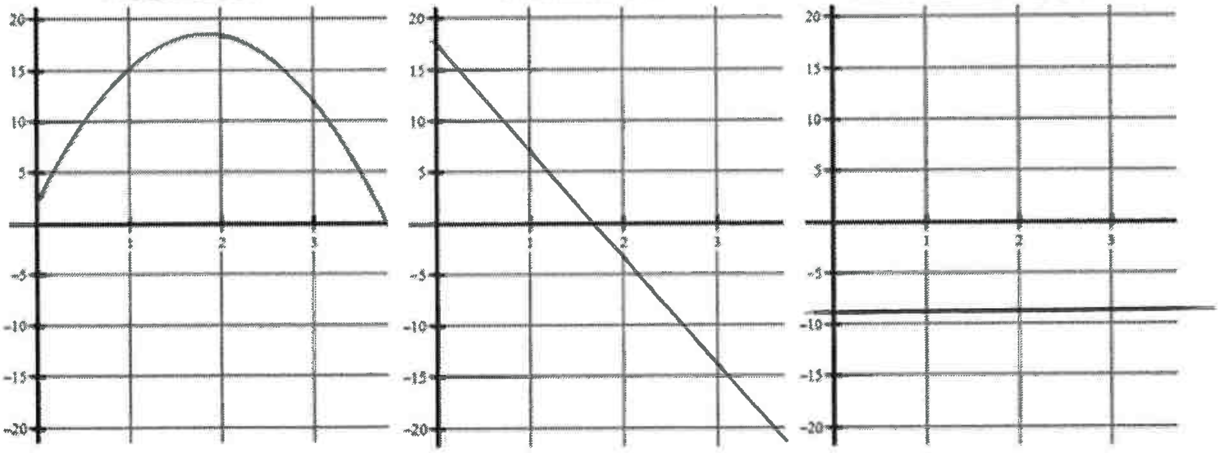
2. Mr. Bean rides his motor scooter to work some days. His distance from home can be modeled by $d(t)$ meters where t is measured in minutes. Interpret $d'(15) = 650$.

   At 15 mins, the distance from home is increasing at a rate of 650 meters per minute.

Particle Motion – Position – Velocity – Acceleration (PVA)

Mr. Brust is playing catch with his best friend, himself. He throws a tennis ball straight up into the air. The height of the ball is modeled by $s(t) = -4.9t^2 + 18t + 2$ where t is time in seconds and s is the height of the ball from the ground in meters.

POSITION **VELOCITY** **ACCELERATION**



Position function: $s(t)$

Velocity function: $v(t) = s'(t)$

Acceleration function: $a(t) = v'(t) = s''(t)$

Velocity = Rate of Change of Position

$v(t) < 0$ means the particle is moving left (x-axis) or down (y-axis)

$v(t) > 0$ means the particle is moving right (x-axis)

$v(t) = 0$ means the particle is at rest

Average velocity = $\frac{\text{change in position}}{\text{change in time}} \rightarrow \frac{s(b)-s(a)}{b-a}$

(Avg Rate of change)

Speed = $|\text{velocity}|$

Speeding Up or Slowing Down?

If velocity and acceleration have the **same** sign, the particle is speeding up

If velocity and acceleration have **different** signs, the particle is slowing down

t	-5	1	2	4
$v(t)$	3	−2	1	−1
$a(t)$	−4	7	0.1	−1
Conclusion	slowing down	slowing down	speeding up	speeding up

Displacement: The net change in position

$\boxed{s(b) - s(a)}$

Rate of change of velocity

Is velocity increasing or decreasing at $t=1$? Since $a(1) > 0$, velocity is increasing at $t=1$

Particle Motion from an equation.

The position (x-coordinate) of a particle moving on the x-axis is modeled by the function

$$x(t) = t^3 - 4t^2 + 3 \text{ for } t \geq 0,$$

Where x is measured in cm and t is measured in minutes.

1. Find the displacement of the particle during the first 2 minutes.

$x(0) = 3$ | $x(2) - x(0) = -5 - 3 = \boxed{-8\text{cm}}$

$x(2) = -5$

This means that after 2 mins, particle is 8 cm to the left of where it started.

2. Find the average velocity of the particle during the first 2 minutes.

$x(0) = 3$ | Avg. ROC | $\dfrac{x(2)-x(0)}{2-0} \rightarrow \dfrac{-5-3}{2-0} = \dfrac{-8}{2} = \boxed{-4\text{ cm/min}}$

$x(2) = -5$

3. Find the velocity of the particle when $t = 4$.

*instantaneous velocity | $x'(t) = v(t) = 3t^2 - 8t$ | $v(4) = 3(4)^2 - 32 = \boxed{16\text{ cm/min}}$

4. Find the acceleration of the particle when $t = 4$.

$x''(t) = v'(t) = a(t) = 6t - 8$ $\quad a(4) = 6(4) - 8 = \boxed{16\text{ cm/min}^2}$

5. Is the particle speeding up or slowing down at $t = 4$? Justify.

particle is speeding up at $t=4$ since $v(4) > 0$ and $a(4) > 0$ (velocity and acceleration have same signs)

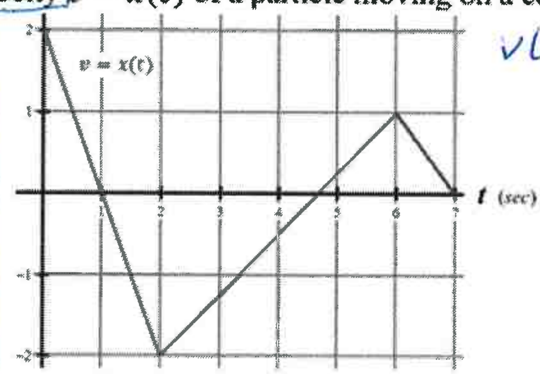
Particle Motion from a graph

The figure shows the velocity $v = x(t)$ of a particle moving on a coordinate line.

v(t) graph

positive velocity

negative velocity



6. When is the particle moving right? Justify.

particle moves right on interval $(0,1)$, $(4.8, 7)$ since $v(t) > 0$

7. When is the particle moving left? Justify.

particle moves left on interval $(1, 4.8)$ since $v(t) < 0$

8. When is the particle's acceleration Positive? Negative? Zero?

* $a(t) > 0$ when $v(t)$ have positive slope | particle's acceleration is positive on $(2, 6)$ since $v'(t) > 0$. particle's acceleration is negative on $(0,2)$ and $(6,7]$ since $v'(t) < 0$. Accelention is never zero since $v'(t)$ is never equal to zero.

9. When does the particle have the greatest speed?

* speed = |velocity| | particle has greatest speed at $t = 0$ (2 units/sec) and $t = 2$ (-2 units/sec)

10. When is the particle speeding up? Justify.

particle speeding up on $(5,6)$ since $v(t) > 0$ and $a(t) > 0$ (same signs)
also speeding up on $(1,2)$ since $v(t) < 0$ and $a(t) < 0$ (same signs)

11. When is the particle slowing down? Justify.

slowing down $(0,1)$, $(2, 4.8)$, and $(6,7]$ since $v(t)$ and $a(t)$ have opposite signs.