Key

4.2 AP Practice Problems (p.281)

1. Let f be a function for which $f(2) = 6$ and $f'(2) = -3$. If the tangent line to the graph of f at 2 is used to approximate a zero of f, then the approximation is

(A) 0 **(B) 4** (C) 6 (D) 12

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2. For small, positive values of h, $\sqrt[3]{8+h}$ is best approximated by

(A) $4-\frac{h}{12}$ (B) $4+\frac{h}{12}$ (C) $2-\frac{h}{12}$ **(D)** $2+\frac{h}{12}$

$y = \sqrt[3]{x}$

$y' = \frac{1}{3}x^{-2/3}$

$y'(8) = \frac{1}{3}\left(\frac{1}{8}\right)^{2/3} = \frac{1}{12}$

point: $y(8) = \sqrt[3]{8} = 2$

slope: $y'(8) = \frac{1}{12}$

$y - 2 = \frac{1}{12}(x-8)$

$y = \frac{1}{12}(x-8)+2$

$y(8+h) = \frac{1}{12}[8+h-8]+2 = \boxed{\frac{h}{12}+2}$

3. A linear approximation to $f(x) = x\sin\left(\frac{\pi x}{2}\right) + x^2$ at $x = 3$ is

(A) $y = 5x+6$ **(B)** $y = 5x-9$

(C) $y = 7x-9$ (D) $y = 7x+9$

$f'(x) = 1\cdot\sin\left(\frac{\pi}{2}x\right) + x\cdot\cos\left(\frac{\pi}{2}x\right)\cdot\frac{\pi}{2} + 2x$ (f′g + fg′)

$f'(3) = \sin\left(\frac{3\pi}{2}\right) + 3\left(\cos\frac{3\pi}{2}\right)\cdot\frac{\pi}{2} + 2(3)$

$f'(3) = -1 + 6 = 5$

$f(3) = 3\sin\left(\frac{3\pi}{2}\right) + 3^2 = 6$

point: (3, 6)

slope: $m = 5$

$y - 6 = 5(x-3)$

$y = 5x - 15 + 6$

$\boxed{y = 5x-9}$

4. Using the tangent line to the graph of $f(x) = xe^x + 2$ at 0, the approximate value of $f(-0.3)$ is

(A) 2.3 (B) 1.3 **(C) 1.7** (D) -2.3

$f'(x) = 1\cdot e^x + x\cdot e^x + 0$ (f′g + fg′)

$f'(0) = e^0 + 0e^0 = 1$

$f(0) = 0+2 = 2$

point: (0, 2)

slope: $m = 1$

$y - 2 = 1(x-0)$

$y = x + 2$

$y(-0.3) = -0.3+2 = \boxed{1.7}$

5. If $f'(x) = 2xe^{x^2-1} - 3\pi\sin(\pi x)$ and $f(1) = 4$, approximate $f(1.03)$ using a linear approximation.

(A) 4.06 (B) 5.06 (C) 4 (D) 3.94

$f'(1) = 2(1)e^0 - 3\pi\sin\pi$

$f'(1) = 2$

point: (1, 4)

slope: $m = 2$

$y - 4 = 2(x-1)$

$y = 2(x-1)+4$

$y(1.03) = 2(1.03-1)+4$

$\boxed{y(1.03) = 4.06}$

6. The tangent line to the graph of $f(x) = x^3 + 1$ at $x = 1$ is used to approximate $f(x)$ near 1. Which number below is the greatest value of x that results in an error less than or equal to 0.5?

(A) 1.30 (B) 1.35 (C) 1.40 (D) 1.45

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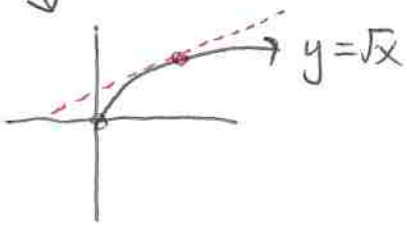
7. A linear approximation L is used to approximate $f(x) = \sqrt{x}$, at c, $c > 0$. The approximation

(A) always underestimates the true value of f at c.
(B) always overestimates the true value of f at c.
(C) sometimes overestimates the true value of f at c.
(D) does not provide enough information to determine whether the true value of f at c is over- or underestimated.



8. Suppose $y = f(x)$ is a differentiable function. The table below gives values of f and f' for select numbers x in the domain of f. Use a linear approximation to approximate $f(3.1)$.

x	-3	0	1	3	5
$f(x)$	4	4	-1	-2	3
$f'(x)$	1	-1	-2	3	4

(A) 4.1 (B) -2.3 (C) 0.1 (D) -1.7

$f(3) = -2$
$f'(3) = 3$

point: $(3, -2)$
slope: $m = 3$

$y + 2 = 3(x - 3)$
$y = 3(x - 3) - 2$
$y(3.1) = 3(3.1 - 3) - 2$

$y(3.1) = -1.7$