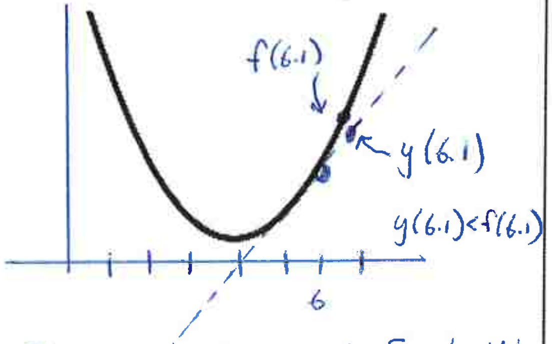
AP Calculus – 4.2 Notes Linear Approximation and rates of change other than motion

Key

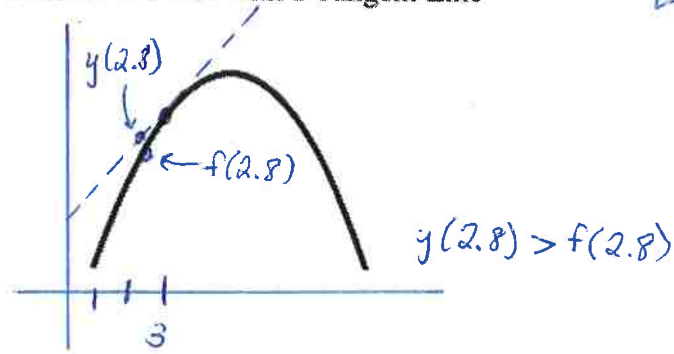
The tangent line of the function $f(x)$ at $x = a$ can give you an approximate value of $f(x)$ for points close to $x = a$.

Concave UP with a Tangent Line



Tangent Line Underestimates Function Value

Concave DOWN with a Tangent Line



Tangent Line Overestimates Function Value

1. f is concave up on its domain and $f(4) = 5$ and $f'(4) = 3$.
   a. What is the estimate for $f(3.8)$ using the local linear approximation for f at $x = 4$?

*Create Tangent line using the given information:

point: $(4,5)$ slope: $m = 3$ | $y - 5 = 3(x-4)$ | $y = 3(x-4)+5$ | $y(3.8) = 3(3.8-4)+5 = \boxed{4.4}$

   b. Is it an underestimate or overestimate? Explain.

Since $f(x)$ is concave up, the linear approximation will produce an underapproximation

2. The function $f(x) = 5x - 2x^3 - 2$ is concave down at $x = 1$?.
   a. Find the tangent line of f at $x = 1$.

$f(1) = 5(1) - 2(1)^3 - 2 = 1$ | $f'(1) = 5 - 6(1)^2 = -1$ | point: $(1,1)$ slope: $m = -1$ | $\boxed{y - 1 = -1(x-1)}$

$f'(x) = 5 - 6x^2$

   b. What is the estimate for $f(1.1)$ using the local linear approximation for f at $x = 1$?

$y = -1(x-1)+1$ | $y(1.1) = -1(1.1-1)+1 = \boxed{0.9}$

   c. Is it an underestimate or overestimate? Explain.

Overestimation since $f(x)$ is concave down

3. Consider the differential equation $\frac{dy}{dx} = e^y(2x^2 - 5x)$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(2) = 0$.

*differential equation is just a fancy way of saying a derivative equation

   a. Write an equation for the line tangent to the graph of f at the point (2,0).

$\left.\frac{dy}{dx}\right|_{(2,0)} = e^0[2(2)^2 - 5(2)] = -2$ | point: $(2,0)$ slope: $m = -2$ | $\boxed{y - 0 = -2(x-2)}$

   b. Use the tangent line to approximate $f(2.2)$.

$\boxed{y(2.2) = -2(2.2-2) = -0.4}$

Rates of Change other than Motion:

Increasing or Decreasing?

To know if something is increasing or decreasing, check the sign of it's derivative

Height is *increasing* if $h'(t) > 0$

Velocity is *decreasing* if $V'(t) < 0$

Recall: Derivative on a calculator.

Find $f'(3)$ if $f(x) = 5^{\sin x}$

$f'(3) = -1.9996$

Is the function already a rate of change?

- If $f(x)$ is the bunny population after x years, than what is $f'(x)$?

  The rate of change of the bunny population per year at year "x"

- If $f(x)$ is the rate at which a bunny population increases (bunnies per year), than what is $f'(x)$?

  The rate at which the bunny population is increasing or decreasing (bunny/yr^2)

Rate of Change from a Table

t (years)	0	10	20	30
$P(t)$ (people)	100	120	150	200

Estimate $P'(15)$

$\frac{P(20)-P(10)}{20-10} \rightarrow \frac{150-120}{20-10} = \boxed{3 \text{ people/yr}}$

Estimate $P'(20)$

$\frac{p(30)-p(20)}{30-20}$ or $\frac{p(20)-p(10)}{20-10}$

Practice Problems:

1. A store is having a 12-hour sale. The total number of shoppers who have entered the store t hours after the sale begins is modeled by the function E defined by $E(t) = 0.3t^4 - 14t^3 + 110t^2$ for $0 \le t \le 12$. At what rate are shoppers entering the store 5 hours after the start of the sale?

   $E'(5) = 200$ shoppers per hour.

2. The function $t = f(P)$ models the time, in days, for a small pond to evaporate as a function of the size P of the pond, measured in liters. What are the units for $f''(P)$?

   $\boxed{\text{days/Liters}^2}$