Key

4.3 AP Practice Problems (p. 290-291)

1. A spherical balloon is inflated at the rate of 50 m^3/min. Find the rate at which the radius of the balloon is increasing when the diameter is 20 m.

(A) $\frac{1}{2\pi}$ m/min (B) $\frac{5}{8\pi}$ m/min

$\boxed{\text{(C)}}$ $\frac{1}{8\pi}$ m/min (D) $\frac{5}{4\pi}$ m/min

$\frac{dV}{dt} = 50\ m^3/min$

$\frac{dr}{dt} = \_\_\_?$

$d = 20$

$\hookrightarrow r = 10$

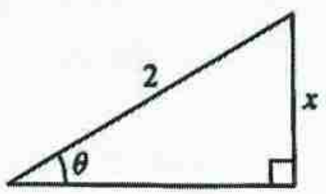
$V = \frac{4}{3}\pi r^3$ | $\frac{dV}{dt} = 3 \cdot \frac{4}{3}\pi r^2 \left(\frac{dr}{dt}\right)$ | $\frac{dV}{dt} = 4\pi r^2 \left(\frac{dr}{dt}\right)$

$50 = 4\pi(10)^2\left(\frac{dr}{dt}\right)$

$\frac{50}{400\pi} = \frac{dr}{dt}$

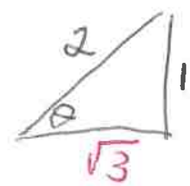
$\boxed{\frac{dr}{dt} = \frac{1}{8\pi}\ m/min}$

2. In the right triangle below, θ is changing at the rate of 2 radians per second. At what rate is x changing at the instant when $x = 1$ cm?



(A) 2 cm/s $\boxed{\text{(B)}}$ $2\sqrt{3}$ cm/s (C) $\sqrt{3}$ cm/s (D) $4\sqrt{3}$ cm/s

$\frac{d\theta}{dt} = 2\ rad/sec$ $\frac{dx}{dt} = \_\_\_?$



$\rightarrow \cos\theta = \frac{\sqrt{3}}{2}$

*create trig equation involving the given information:

$\sin\theta = \frac{x}{2}$

$\sin\theta = \frac{1}{2}x$

$\cos\theta\left(\frac{d\theta}{dt}\right) = \frac{1}{2}\left(\frac{dx}{dt}\right)$

$\left(\frac{\sqrt{3}}{2}\right)(2) = \frac{1}{2}\left(\frac{dx}{dt}\right)$

$\frac{\sqrt{3}}{\cancel{2}} \cdot \cancel{2} \cdot \frac{2}{1} = \frac{dx}{dt}$

$\boxed{\frac{dx}{dt} = 2\sqrt{3}\ cm/sec}$

3. The radius of a circle is decreasing at a constant rate of 2 in./min. What is the rate of change in the area of the circle when its area is 25π in.^{2}?

$\boxed{\text{(A)}}$ -20π in.^{2}/min (B) -25π in.^{2}/min

(C) 20π in.^{2}/min (D) $20\pi^2$ in.^{2}/min

$A = \pi r^2$

$25\pi = \pi r^2$

$25 = r^2$

$r = 5$

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r\left(\frac{dr}{dt}\right)$

$\frac{dr}{dt} = -2\ in/min$

$\frac{dA}{dt} = \_\_\_?$ $A = 25\pi$

$\frac{dA}{dt} = 2\pi(5)(-2)$

$\frac{dA}{dt} = -20\pi$

$\boxed{\frac{dA}{dt} = -20\pi\ in^2/min}$

4. The radius r of a sphere is increasing at a rate of 2 cm/s. At the instant when $r = 12$ cm, what is the rate of change in the surface area S of the sphere? (The surface area S of a sphere with radius r is $S = 4\pi r^2$.)

(A) $96\pi\ \text{cm}^2/\text{s}$ (B) $1152\pi\ \text{cm}^2/\text{s}$

(C) $576\pi\ \text{cm}^2/\text{s}$ $\boxed{\text{(D)}}$ $192\pi\ \text{cm}^2/\text{s}$

$S = 4\pi r^2$

$\frac{dS}{dt} = 2\cdot 4\pi r\left(\frac{dr}{dt}\right)$

$\frac{dS}{dt} = 8\pi r\left(\frac{dr}{dt}\right)$

$\frac{dr}{dt} = 2\text{cm/sec}$

$r = 12$

$\frac{dS}{dt} = \_\_\_?$

$\frac{dS}{dt} = 8\pi(12)(2)$

$\boxed{\frac{dS}{dt} = 192\pi\ \text{cm}^2/\text{sec}}$

5. The sides of the rectangle shown below are increasing so that the rate of change of y with respect to time t is three times the rate of change of x with respect to t. If $\frac{dc}{dt} = 1$, what is the rate of change of x when $x = 6$ and $y = 8$?

$\frac{dy}{dt} = 3\left(\frac{dx}{dt}\right)$

$x = 6$

$y = 8$

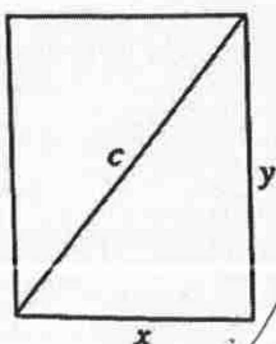
$c = 10$

$\frac{dc}{dt} = 1$

$\frac{dx}{dt} = \_\_\_?$

(A) 3 $\boxed{\text{(B)}}$ $\frac{1}{3}$

(C) 1 (D) $\frac{1}{6}$



$x^2 + y^2 = c^2$

$2x\left(\frac{dx}{dt}\right) + 2y\left(\frac{dy}{dt}\right) = 2c\left(\frac{dc}{dt}\right)$

$2(6)\left(\frac{dx}{dt}\right) + 2(8)\cdot 3\left(\frac{dx}{dt}\right) = 2(10)(1)$

$12\left(\frac{dx}{dt}\right) + 48\left(\frac{dx}{dt}\right) = 20$

$60\left(\frac{dx}{dt}\right) = 20$

$\boxed{\frac{dx}{dt} = \frac{20}{60} = \frac{1}{3}}$

6. The area of a circle is increasing at a rate of $48\pi\ \text{ft}^2/\text{h}$. How fast is the radius of the circle increasing when its area is $36\pi\ \text{ft}^2$?

$\boxed{\text{(A)}}$ 4 ft/h (B) 6 ft/h (C) $4\sqrt{3}$ ft/h (D) $\frac{4}{\pi}$ ft/h

$\boxed{\frac{dr}{dt} = 4\ \text{ft/h}}$

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r\left(\frac{dr}{dt}\right)$

$\frac{dA}{dt} = 48\pi\ \text{ft}^2/\text{h}$

$\frac{dr}{dt} = \_\_\_?$

$A = 36\pi$

$A = \pi r^2$

$36\pi = \pi r^2$

$36 = r^2$

$\underline{6 = r}$

$\frac{dA}{dt} = 2\pi r\left(\frac{dr}{dt}\right)$

$48\pi = 2\pi(6)\left(\frac{dr}{dt}\right)$

$\frac{48\pi}{12\pi} = \frac{dr}{dt}$

7. The radius r and height h of a right circular cone are both increasing at a constant rate of 2 cm/h. At what rate in centimeters cubed per hour is the volume V of the cone increasing when $r = 6$ cm and $h = 15$ cm? (The volume V of a right circular cone of height h and radius r is $V = \frac{1}{3}\pi r^2 h$.)

$\frac{dr}{dt} = 2\,cm/h$

$\frac{dh}{dt} = 2\,cm/h$

$r = 6\,cm$

$h = 15\,cm$

(A) 24π cm^3/h
(B) 96π cm^3/h
$\boxed{(C)}$ 144π cm^3/h
(D) 180π cm^3/h

$$V = \overbrace{\frac{\pi}{3}r^2}^{f}\overbrace{h}^{g}$$

$$\frac{dV}{dt} = \overbrace{2\cdot\frac{\pi}{3}r\left(\frac{dr}{dt}\right)}^{f'}\cdot\overbrace{h}^{g} + \overbrace{\frac{\pi}{3}r^2}^{f}\cdot\overbrace{\left(\frac{dh}{dt}\right)}^{g'}$$

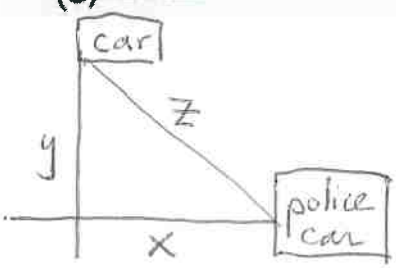
$$\frac{dV}{dt} = \frac{2\pi}{3}(6)(2)(15) + \frac{\pi}{3}(6)^2(2)$$

$$\frac{dV}{dt} = 120\pi + 24\pi$$

$$\boxed{\frac{dV}{dt} = 144\pi\ cm^3/h}$$

8. Two roads cross at right angles. A police officer sits in a car 65 m east of the crossing and observes a car speeding northbound at 84 m/s. At what speed (in meters per second) is the car distancing itself from the police officer 5 seconds after it passes the crossing?

(A) 166.024 m/s
$\boxed{(B)}$ 83.012 m/s
(C) 84 m/s
(D) 95.859 m/s



$x = 65$ $\quad \frac{dx}{dt} = \underline{0}$

$y = 5(84) = 420$ $\quad \frac{dy}{dt} = 84$

$z = \underline{425}$ $\quad \frac{dz}{dt} = \underline{\quad}$?

$$x^2 + y^2 = z^2$$

$$2x\left(\frac{dx}{dt}\right) + 2y\left(\frac{dy}{dt}\right) = 2z\left(\frac{dz}{dt}\right)$$

$$2\cancel{(65)(0)} + 2(420)(84) = 2(425)\left(\frac{dz}{dt}\right)$$

$$70560 = 850\left(\frac{dz}{dt}\right)$$

$$\boxed{\frac{dz}{dt} = \frac{70560}{850} \approx 83.012\ m/s}$$

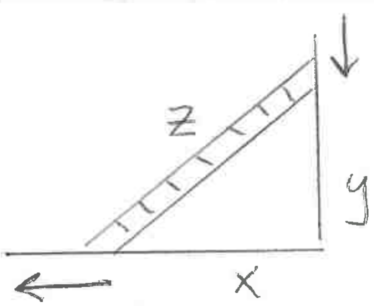
9. **A roofer's 13-meter ladder is placed against the wall of a building with its base on level ground. The top of the ladder slips down the wall as the bottom of the ladder slips away from the building at a constant rate of 5 m/s.**

a) -12 m/s
b) $-59.5\ m^2/sec$
c) -1 rad/sec

(a) At what rate is the top of the ladder moving when it is 5 meters from the ground?

(b) At what rate is the area of the triangle formed by the ladder, the wall, and the ground changing when the top of the ladder is 5 m from the ground?

(c) If θ is the angle formed by the ladder and the ground, what is the rate of change in θ when the top of the ladder is 5 m from the ground?



$x = 12 \qquad dx/dt = +5$ m/sec

$y = 5 \qquad dy/dt = \_\_\_?$

$z = 13 \qquad dz/dt = 0$

$$x^2 + y^2 = z^2$$

$$2x\left(\frac{dx}{dt}\right) + 2y\left(\frac{dy}{dt}\right) = 2z\left(\frac{dz}{dt}\right)$$

$$2(12)(5) + 2(5)\left(\frac{dy}{dt}\right) = 2(13)(0)$$

$$120 + 10\left(\frac{dy}{dt}\right) = 0$$

$$10\left(\frac{dy}{dt}\right) = -120$$

a) $\boxed{\frac{dy}{dt} = -12 \text{ m/s}}$

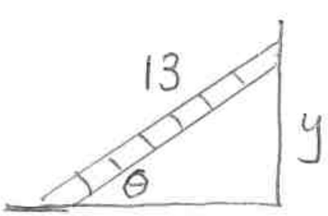
b) $\text{Area} = \underbrace{\tfrac{1}{2}}_{f}\underbrace{xy}_{g}$ *Area of triangle is $A = \frac{1}{2}bh$

$$\frac{dA}{dt} = \underbrace{\tfrac{1}{2}\left(\frac{dx}{dt}\right)}_{f'} \cdot \underbrace{y}_{g} + \underbrace{\tfrac{1}{2}x}_{f} \cdot \underbrace{\left(\frac{dy}{dt}\right)}_{g'}$$

$$\frac{dA}{dt} = \frac{1}{2}(5)(5) + \frac{1}{2}(12)(-12)$$

$$\boxed{\frac{dA}{dt} = \frac{25}{2} - \frac{144}{2} \rightarrow \frac{-119}{2} \text{ or } -59.5\ m^2/sec}$$

c)



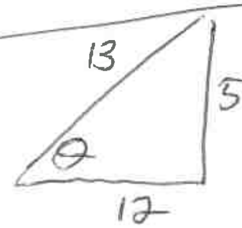
$$\sin\theta = \frac{y}{13}$$

$$\sin\theta = \frac{1}{13}y$$

$$\boxed{\cos\theta}\left(\frac{d\theta}{dt}\right) = \frac{1}{13}\left(\frac{dy}{dt}\right)$$

$$\boxed{\left(\frac{12}{13}\right)}\left(\frac{d\theta}{dt}\right) = \frac{1}{13}(-12)$$

$$\frac{d\theta}{dt} = \frac{13}{12} \cdot \frac{-12}{13} = -1$$



$\rightarrow \boxed{\cos\theta = \frac{12}{13}}$

$$\boxed{\frac{d\theta}{dt} = -1 \text{ rad/sec}}$$