Key

Ch. 4.3 Related Rates Exercise Problems (Day 1)

Pg. 286-291 #9, 10, 22, 23, 35, 38

9. Volume of a Cube If each edge of a cube is increasing at the constant rate of 3 cm/s, how fast is the volume of the cube increasing when the length x of an edge is 10 cm?

$V = x^3$

$\frac{dV}{dt} = 3x^2\left(\frac{dx}{dt}\right)$

$\frac{dx}{dt} = 3 \text{ cm/sec}$

$x = 10\text{cm}$

$\frac{dV}{dt} = \_\_\_ ?$

$\frac{dV}{dt} = 3(10)^2(3)$

$\boxed{\frac{dV}{dt} = 900 \text{ cm}^3/\text{sec}}$

10. Volume of a Sphere If the radius of a sphere is increasing at 1 cm/s, find the rate of change of its volume when the radius is 6 cm.

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 3 \cdot \frac{4}{3}\pi r^2\left(\frac{dr}{dt}\right)$

$\frac{dV}{dt} = 4\pi r^2\left(\frac{dr}{dt}\right)$

$\frac{dr}{dt} = 1 \text{cm/sec}$

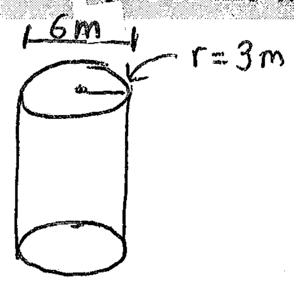
$r = 6\text{cm}$

$\frac{dV}{dt} = \_\_\_ ?$

$\frac{dV}{dt} = 4\pi(6)^2(1)$

$\boxed{\frac{dV}{dt} = 144\pi \approx 452.389 \text{ cm}^3/\text{sec}}$

22. Filling a Tank Water is flowing into a vertical cylindrical tank of diameter 6 m at the rate of 5 m³/min. Find the rate at which the depth of the water is rising.



$\frac{dV}{dt} = 5 \text{ m}^3/\text{min}$

$\frac{dh}{dt} = \_\_\_ ?$

$V = \pi r^2 h$

$V = \pi(3)^2 h$

$V = 9\pi h$

$\frac{dV}{dt} = 9\pi\left(\frac{dh}{dt}\right)$

$5 = 9\pi\left(\frac{dh}{dt}\right)$

$\frac{5}{9\pi} = \frac{dh}{dt}$

$\boxed{\frac{dh}{dt} = \frac{5}{9\pi} \text{ m/min} \approx 0.177 \text{ m/min}}$

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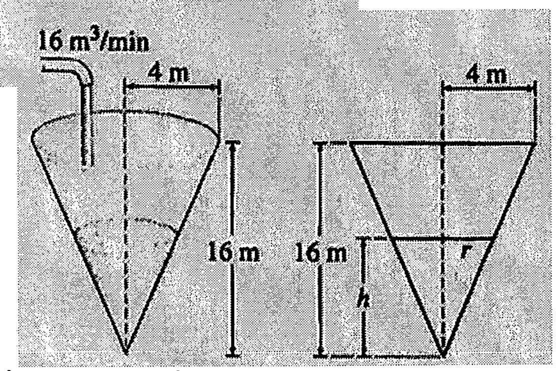
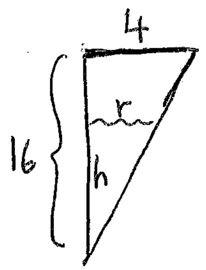
23. Fill Rate A container in the form of a right circular cone (vertex down) has radius 4 m and height 16 m. See the figure. If water is poured into the container at the constant rate of 16 m^3/min, how fast is the water level rising when the water is 8 m deep? *Hint:* The volume V of a cone of radius r and height h is $V = \frac{1}{3}\pi r^2 h$.





$\frac{r}{4} = \frac{h}{16}$

$16r = 4h$

$r = \frac{4}{16}h = \frac{1}{4}h = \frac{h}{4}$

$V = \frac{\pi}{3}r^2 h$

$V = \frac{\pi}{3}\left(\frac{h}{4}\right)^2 h$

$V = \frac{\pi}{3}\cdot\frac{h^2}{16}\cdot h$

$V = \frac{\pi}{48}h^3$

$\frac{dV}{dt} = 3\cdot\frac{\pi}{48}h^2\left(\frac{dh}{dt}\right)$

$\frac{dV}{dt} = \frac{3\pi}{48}h^2\left(\frac{dh}{dt}\right)$

$\frac{dV}{dt} = \frac{\pi}{16}h^2\left(\frac{dh}{dt}\right)$

$16 = \frac{\pi}{16}(8)^2\left(\frac{dh}{dt}\right)$

Given: $\frac{dV}{dt} = 16 m^3/min$

$\frac{dh}{dt} = \_\_?$ $h = 8$

$16 = \frac{\pi}{16}\cdot 64\left(\frac{dh}{dt}\right)$

$16\cdot\frac{16}{\pi\cdot 64} = \frac{dh}{dt}$

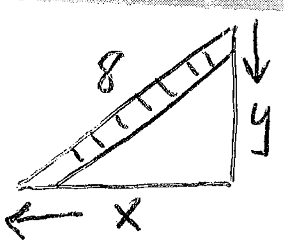
$\boxed{\frac{dh}{dt} = \frac{4}{\pi}\ m/min}$

35. Falling Ladder An 8-m ladder is leaning against a vertical wall. If a person pulls the base of the ladder away from the wall at the rate of 0.5 m/s, how fast is the top of the ladder moving down the wall when the base of the ladder is

(a) 3 m from the wall?

(b) 4 m from the wall?

(c) 6 m from the wall?



$x^2 + y^2 = 8^2$

$\boxed{2x\left(\frac{dx}{dt}\right) + 2y\left(\frac{dy}{dt}\right) = 0}$

a) $x = 3$

$y = \sqrt{55}$

$dx/dt = 0.5$

$\frac{dy}{dt} = \_\_?$

$2(3)(0.5) + 2\sqrt{55}\left(\frac{dy}{dt}\right) = 0$

$\boxed{\frac{dy}{dt} = \frac{-1.5}{\sqrt{55}}\ m/s}$

b) $x = 4$ $y = 4\sqrt{3}$

$\frac{dx}{dt} = 0.5$ $\frac{dy}{dt} = \_\_?$

$2(4)(0.5) + 2(4\sqrt{3})\left(\frac{dy}{dt}\right) = 0$

$\boxed{\frac{dy}{dt} = \frac{-0.5}{\sqrt{3}}\ m/s}$

c) $x = 6$ $y = 2\sqrt{7}$

$dx/dt = 0.5$ $dy/dt = \_\_?$

$2(6)(0.5) + 2(2\sqrt{7})\left(\frac{dy}{dt}\right) = 0$

$\boxed{\frac{dy}{dt} = \frac{-3}{2\sqrt{7}}\ m/s}$

38. **Tracking a Rocket** When a rocket is launched, it is tracked by a tracking dish on the ground located a distance D from the point of launch. The dish points toward the rocket and adjusts its angle of elevation θ to the horizontal (ground level) as the rocket rises. Suppose a rocket rises vertically at a constant speed of $2.0\ \mathrm{m/s}$, with the tracking dish located 150 m from the launch point. Find the rate of change of the angle θ of elevation of the tracking dish with respect to time t (tracking rate) for each of the following:

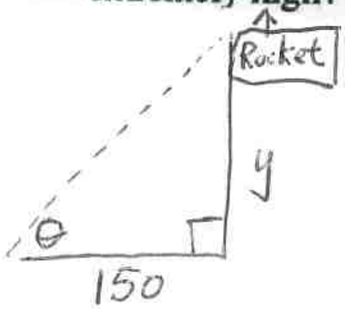
(a) Just after launch.

(b) When the rocket is 100 m above the ground.

(c) *When the rocket is 1.0 km above the ground.*

(d) *Use the results in (a)–(c) to describe the behavior of the tracking rate as the rocket climbs higher and higher. What limit does the tracking rate approach as the rocket gets extremely high?*

d) Based on the previous results, the tracking rate $\left(\frac{d\theta}{dt}\right)$ continues to decrease. As $\theta \to \frac{\pi}{2}$, $\sec\theta \to \infty$, so $\boxed{\frac{d\theta}{dt} \to 0}$

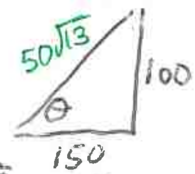


$\tan\theta = \frac{y}{150}$

$\tan\theta = \frac{1}{150}y$

$\boxed{\sec^2\theta\left(\frac{d\theta}{dt}\right) = \frac{1}{150}\left(\frac{dy}{dt}\right)}$

b) $y = 100$



$100^2 + 150^2 = c^2$

$c = \sqrt{32500}$

$c = 50\sqrt{13}$

$\sec\theta = \frac{50\sqrt{13}}{150} = \frac{\sqrt{13}}{3}$

$\sec^2\theta = \left(\frac{\sqrt{13}}{3}\right)^2 = \frac{13}{9}$

$\left(\frac{13}{9}\right)\left(\frac{d\theta}{dt}\right) = \frac{1}{150}(2)$

$\boxed{\frac{d\theta}{dt} = \frac{1}{75}\cdot\frac{9}{13} = \frac{9}{975}\ \mathrm{rad/sec}\ \text{or}\ \frac{3}{325}\ \mathrm{rad/sec}}$

a) Just after launch:

$y = 0$

$\tan\theta = 0$

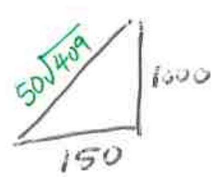
$\sec\theta = 1$

$\frac{dy}{dt} = 2\ \mathrm{m/sec}$

$(1)^2\left(\frac{d\theta}{dt}\right) = \frac{1}{150}(2)$

$\boxed{\frac{d\theta}{dt} = \frac{1}{75}\ \mathrm{rad/sec}}$

c) $y = 1000\ \mathrm{m}$



$1000^2 + 150^2 = c^2$

$c^2 = 1022500$

$c = 50\sqrt{409}$

$\left(\frac{50\sqrt{409}}{150}\right)^2\left(\frac{d\theta}{dt}\right) = \frac{1}{150}(2)$

$\left(\frac{409}{9}\right)\frac{d\theta}{dt} = \frac{1}{75}$

$\boxed{\frac{d\theta}{dt} = \frac{1}{75}\cdot\frac{9}{409} = \frac{9}{30675} \approx 0.000293\ \mathrm{rad/sec}}$