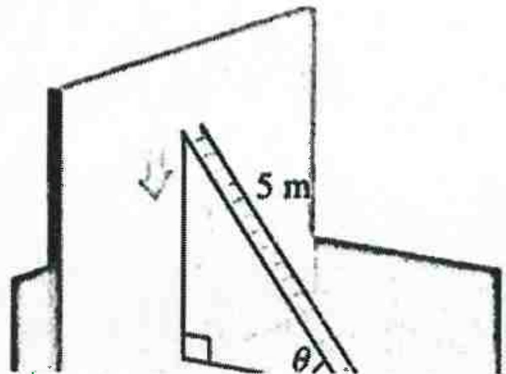
Key

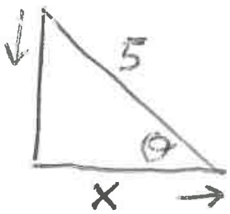
Ch. 4.3 Related Rates Exercise Problems (Day 2)

Pg. 286-291 #19, 39, 40, 53, 54

19. Change in Inclination A ladder 5 m long is leaning against a wall. If the lower end of the ladder slides away from the wall at the rate of 0.5 m/s, at what rate is the inclination θ of the ladder with respect to the ground changing when the lower end is 4 meter from the wall.



→ 0.5 m/sec



$x = 4$

$\frac{dx}{dt} = 0.5$

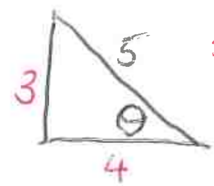
$\frac{d\theta}{dt} = \_\_\_ ?$

$\cos\theta = \frac{x}{5}$

$\cos\theta = \frac{1}{5}x$

$-\sin\theta\left(\frac{d\theta}{dt}\right) = \frac{1}{5}\left(\frac{dx}{dt}\right)$

$-\left(\frac{3}{5}\right)\left(\frac{d\theta}{dt}\right) = \frac{1}{5}(0.5)$

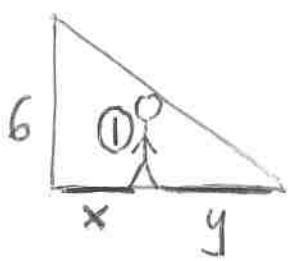


$\sin\theta = \frac{3}{5}$

$\frac{d\theta}{dt} = \frac{1}{5} \cdot \frac{1}{2} \cdot \frac{-5}{3}$

$\boxed{\frac{d\theta}{dt} = \frac{-1}{6} \text{ rad/sec}}$

39. Lengthening Shadow A child, 1 m tall, is walking directly under a street lamp that is 6 m above the ground. If the child walks away from the light at the rate of 20 m/min, how fast is the child's shadow lengthening?



$\frac{1}{6} = \frac{y}{x+y}$

$x + y = 6y$

$x = 5y$

$\frac{dx}{dt} = 5\left(\frac{dy}{dt}\right)$

$\frac{dx}{dt} = 20 \text{ m/min}$

$\frac{dy}{dt} = \_\_\_ ?$

RoC walking → $\frac{dx}{dt}$

RoC shadow → $\frac{dy}{dt}$

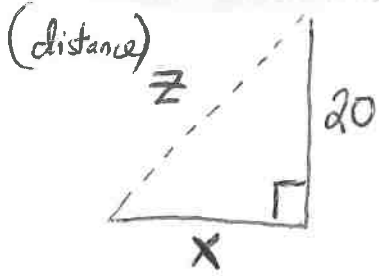
RoC tip of shadow → $\frac{dx}{dt} + \frac{dy}{dt}$

$20 = 5\left(\frac{dy}{dt}\right)$

$\frac{20}{5} = \frac{dy}{dt}$

$\boxed{\frac{dy}{dt} = 4 \text{ m/min}}$

40. Approaching a Pole A boy is walking toward the base of a pole 20 m high at the rate of 4 km/h. At what rate (in meters per second) is the distance from his feet to the top of the pole changing when he is 5 m from the pole?



$x^2+20^2=z^2$

$2x\left(\frac{dx}{dt}\right)+0=2z\left(\frac{dz}{dt}\right)$

$x=5\,m$

$\frac{dx}{dt}=\frac{-4\,Km}{h}\cdot\frac{1000m}{Km}\cdot\frac{1h}{3600sec}$

$\frac{dx}{dt}=\frac{-10}{9}\,m/sec$

$z^2=(5)^2+20^2 \rightarrow z=5\sqrt{17}$

$2(5)\left(\frac{-10}{9}\right)=2(5\sqrt{17})\frac{dz}{dt}$

$\boxed{\frac{dz}{dt}=\frac{-10}{9\sqrt{17}}\approx -0.269\ m/sec}$

53. Change in Volume The height h and width x of an open box with a square base are related to its volume by the formula $V=hx^2$. Discuss how the volume changes

(a) if h decreases with time, but x remains constant.

(b) if both h and x change with time.

a) $\boxed{\frac{dV}{dt}=\frac{dh}{dt}x^2}$

b) $V=\overbrace{h}^{f}\overbrace{x^2}^{g}$

$\frac{dV}{dt}=\overbrace{\frac{dh}{dt}}^{f'}\cdot\overbrace{x^2}^{g}+\overbrace{h}^{f}\cdot\overbrace{2x\left(\frac{dx}{dt}\right)}^{g'}$

$\boxed{\frac{dV}{dt}=x^2\left(\frac{dh}{dt}\right)+2hx\left(\frac{dx}{dt}\right)}$

54. Rate of Change Let $y=2e^{\cos x}$. If both x and y vary with time in such a way that y increases at a steady rate of 5 units per second, at what rate is x changing when $x=\frac{\pi}{2}$?

$\frac{dy}{dt}=5$

$x=\pi/2$

$\frac{dx}{dt}=\_\_?$

$y=2e^{\cos x}$

$\frac{dy}{dt}=2e^{\cos x}\cdot -\sin x\left(\frac{dx}{dt}\right)$

$5=2e^{\cos(\pi/2)}\cdot -\sin(\pi/2)\left(\frac{dx}{dt}\right)$

$5=2e^{0}(-1)\left(\frac{dx}{dt}\right)$

$\frac{5}{-2}=\frac{dx}{dt}$

$\boxed{\frac{dx}{dt}=\frac{-5}{2}\ units/sec.}$