4.4 AP Practice Problems (p. 301)

*L'Hopital's Rule

Key

1. $\lim_{x\to 0} \dfrac{e^{4x}-1}{\sin(2x)} = \dfrac{0}{0}$

(A) 0 **(B) 2** (C) 4 (D) does not exist

(L'H) $\lim_{x\to 0} \dfrac{e^{4x}(4)-0}{\cos(2x)\cdot 2} \to \dfrac{4e^0}{2\cos(0)} \to \dfrac{4(1)}{2(1)} \to \boxed{2}$

*$\sin 2x = 2\sin x\cos x$

2. $\lim_{x\to 0} \dfrac{1-\cos^2(3x)}{x^2} =$

(A) 18 **(B) 9** (C) 0 (D) 3

*$1-\cos^2\theta = \sin^2\theta$

$\lim_{x\to 0} \dfrac{\sin^2(3x)}{x^2} \to \dfrac{0}{0}$

$\lim_{x\to 0} \dfrac{[\sin(3x)]^2}{x^2}$

$\lim_{x\to 0} \dfrac{2[\sin(3x)]\cdot\cos(3x)\cdot 3}{2x} \to \dfrac{2\sin(0)\cos(0)\cdot 3}{0} \to \dfrac{0}{0}$

$\lim_{x\to 0} \dfrac{6\sin(3x)\cos(3x)}{2x} \to \dfrac{3\cdot\sin(2\cdot 3x)}{2x}$

$\lim_{x\to 0} \dfrac{3\cdot\cos(6x)\cdot 6}{2} \to \dfrac{18(1)}{2} = \boxed{9}$

3. Find $\lim_{x\to\infty} \dfrac{x^{-3/2}}{\sin\frac{1}{x}}$.

(A) $\frac{3}{2}$ (B) 1 **(C) 0** (D) ∞

$\lim_{x\to\infty} \dfrac{1}{x^{3/2}\sin\left(\frac{1}{x}\right)} = \boxed{0}$

4. $\lim_{x\to 1} \dfrac{\ln x^3}{x^2-1} =$

(A) 0 (B) 1 **(C) $\frac{3}{2}$** (D) 3

$\lim_{x\to 1} \dfrac{3\ln x}{x^2-1} \to \dfrac{0}{0} \to \lim_{x\to 1} \dfrac{\frac{3}{x}}{2x} \to \dfrac{\frac{3}{1}}{2} = \boxed{\dfrac{3}{2}}$

5. For any positive integer k, $\lim_{x\to\infty} \frac{\ln x}{x^k} =$

(A) 0 (B) 1 (C) $k+1$ (D) ∞

* comparative growth Rates $L < R < P < E$

$\lim_{x\to\infty} \frac{\text{logs}}{\text{polynomial}} \to 0$

6. $\lim_{\theta\to 0} \frac{1-\cos(2\theta)}{3\sin\theta} =$

(A) -2 (B) $\frac{2}{3}$ (C) 0 (D) $-\frac{1}{3}$

$\lim_{\theta\to 0} \frac{1-\cos(2\theta)}{3\sin\theta} \to \frac{0}{0} \to \lim_{\theta\to 0} \frac{-\sin(2\theta)\cdot 2}{3\cos\theta} \to \frac{0}{3} \to 0$

7. $\lim_{x\to \frac{\pi}{2}^-} \frac{\ln(\cos x)}{\tan x} =$

(A) $-\infty$ (B) 0 (C) 1 (D) ∞

$\lim_{x\to \frac{\pi}{2}^-} \frac{\ln(\cos x)}{\tan x} \to \frac{\infty}{\infty} \to \lim_{x\to \frac{\pi}{2}^-} \frac{\frac{-\sin x}{\cos x}}{\sec^2 x} \to \lim_{x\to \pi/2^-} \frac{-\sin x}{\cos x}\cdot\frac{1}{\cos^2 x} \to \frac{-\sin x}{\cos^3 x} \to \frac{1}{0.001} \to 0$

8. Which of the following are indeterminate forms at 0?

I $\frac{x}{\ln(x+1)} \to \frac{0}{0}$ II $\frac{e^x}{x^2-2x} \to$ III $\frac{x}{1-\cos(\pi x)} \to \frac{0}{0}$

(A) I only (B) I and III only

(C) II and III only (D) I, II, and III

II. $\lim_{x\to\infty} \frac{e^x}{x^2-2x} \to \infty$ ×