BC Calculus Ch. 6.3 p. 422 #59

59) L = carrying capacity $= 200$

(time, Population)

(t, P)

$(0, 25)$

$(2, 39)$

$$P = \frac{L}{1 + Ce^{-kt}}$$

plug in, solve for C.

$$25 = \frac{200}{1 + Ce^{-k(0)}}$$

$$25 = \frac{200}{1 + C} \rightarrow 25(1 + C) = 200$$

$$1 + C = 8 \qquad \underline{\underline{C = 7}}$$

$$P = \frac{200}{1 + 7e^{-k(t)}}$$

plug in to solve for k

$$39 = \frac{200}{1 + 7e^{-k(2)}}$$

$$39(1 + 7e^{-2k}) = 200$$

$$1 + 7e^{-2k} = \frac{200}{39}$$

$$7e^{-2k} = \frac{200}{39} - 1$$

$$e^{-2k} = \frac{23}{39}$$

$$\ln e^{-2k} = \ln\left(\frac{23}{39}\right)$$

$$-2k = \ln(23/39)$$

$$k = -\frac{1}{2}\ln(23/39)$$

$$\underline{\underline{k \approx 0.2640}}$$

a) $$\boxed{P = \frac{200}{1 + 7e^{-0.2640t}}}$$

b) Find population after 5 years $(t = 5)$

$$P = \frac{200}{1 + 7e^{-0.2640(5)}} \approx \boxed{70 \text{ panthers}}$$

c) Find t when $P = 100$

$$100 = \frac{200}{1 + 7e^{-0.2640t}}$$

$$1 + 7e^{-0.2640t} = \frac{200}{100}$$

$$7e^{-0.2640t} = 1$$

$$e^{-0.2640t} = \frac{1}{7}$$

$$\ln e^{-0.2640t} = \ln(1/7)$$

$$-0.2640t = \ln(1/7)$$

$$\boxed{t \approx 7.37 \text{ yrs.}}$$

d) $\frac{dP}{dt} = kP\left(1 - \frac{P}{2}\right)$ $k = 0.264$, $L = 200$

differential equation $\frac{dP}{dt} = 0.264P\left(1 - \frac{P}{200}\right)$

Eulers' Method: step size $h = 1$ $\Delta x = 1$

t	P	$m = \frac{dP}{dt}$	$\Delta P = m\Delta x$	$P_{new} = P + \Delta P$
0	25	5.775	$\Delta P = 5.775(1)$	$P = 25 + 5.775 = 30.775$
1	30.775	6.874	$\Delta P = 6.874$	$P = 30.775 + 6.874$
2	37.649	8.068	8.068	$P = 45.717$
3	45.717	9.310	9.310	$P = 55.027$
4	55.027	10.53	10.53	$P = 55.027 + 10.53$
5	65.557			

$\boxed{P(5) \approx 65.6 \text{ panthers}}$

e) P is increasing most rapidly at $\frac{P}{2} = \frac{200}{2} = 100$, so $t \approx 7.37$ yrs. (found in part c)