2017

6.3 BC p.421 #47-60

Logistic Equation

$$y = \frac{L}{1+Ce^{\cdots}}$$

Logistic differential equation

$$\frac{dP}{dt} = KP\left(1-\frac{P}{L}\right)$$

47) $y = \dfrac{12}{1+3e^{-x}}$ $y(0)=6$, $\boxed{D}$

48) $y = \dfrac{12}{1+3e^{-x}}$ $y(0)=3$ $\boxed{A}$

49) $y = \dfrac{12}{1+\frac{1}{2}e^{-x}}$ $y(0)=8$ $\boxed{B}$

50) $y = \dfrac{12}{1+e^{-2x}}$ $y(0)=6$ $\boxed{C}$

Find a) value of K b) carrying capacity (L)
c) initial population d) time when population reaches 50% of carrying capacity
e) write logistic differential equation with solution of P(t)

51) $P(t) = \dfrac{2100}{1+29e^{-0.75t}}$

a) $K = +0.75$

b) $L = 2100$

c) $P(0) = \dfrac{2100}{1+29} = 70$

d) $\dfrac{2100}{2} = 1050 = \dfrac{2100}{1+29e^{-0.75t}}$

$1+29e^{-0.75t} = 2$

$e^{-0.75t} = \dfrac{1}{29}$

$\ln e^{-0.75t} = \ln(1/29)$

$-0.75t = \ln(1/29)$

$t = \dfrac{-\ln(1/29)}{0.75} \approx \boxed{4.489 \text{ yrs}}$

e) $\dfrac{dP}{dt} = KP\left(1-\dfrac{P}{L}\right)$

$\dfrac{dP}{dt} = 0.75P\left(1-\dfrac{P}{2100}\right)$

$P(0) = 70$

52) $P(t) = \dfrac{5000}{1+39e^{-0.2t}}$

$y = \dfrac{L}{1+Ce^{-kt}}$

$\dfrac{dP}{dt} = kP\left(1-\dfrac{P}{L}\right)$

a) $k = 0.2$

b) $L = 5000$

c) $P(0) = \dfrac{5000}{1+39} = \dfrac{5000}{40} = 125$

$\boxed{\dfrac{dP}{dt} = 0.2P\left(1-\dfrac{P}{5000}\right)}$

d) $\dfrac{5000}{2} = \dfrac{5000}{1+39e^{-0.2t}}$

$2500 = \dfrac{5000}{1+39e^{-0.2t}}$

$1+39e^{-0.2t} = 2$

$e^{-0.2t} = \dfrac{1}{39}$

$\ln e^{-0.2t} = \ln(1/39)$

$-0.2t = \ln(1/39)$

$t = \dfrac{-\ln(1/39)}{0.2}$

$\boxed{t \approx 18.3178}$

53) $\dfrac{dP}{dt} = 3P\left(1-\dfrac{P}{100}\right)$

a) $k=3$

b) $L=100$

d) population growth rate is greatest at $\frac{L}{2} \rightarrow P = \frac{100}{2} = 50$

54) $\dfrac{dP}{dt} = 0.1P - 0.0004P^2$

$\dfrac{dP}{dt} = 0.1P(1-0.004P)$

$\dfrac{dP}{dt} = 0.1P\left(1-\dfrac{P}{250}\right)$

a) $k=0.1$

b) $L=250$

$P = \dfrac{L}{2} = \dfrac{250}{2} = \boxed{125}$

Solve Logistic Differential Equation

$$\frac{dy}{dt} = ky\left(1-\frac{y}{L}\right)$$

$$y = \frac{L}{1+Ce^{-kt}}$$

55) $\frac{dy}{dt} = y\left(1-\frac{y}{36}\right)$, $y(0)=4$

$k=1$, $L=36$

$y = \frac{L}{1+Ce^{-kt}} = \frac{36}{1+Ce^{-t}}$

$4 = \frac{36}{1+Ce^{-0}}$

$1+C=9$

$C=8$

$$\boxed{y = \frac{36}{1+8e^{-t}}}$$

56) $\frac{dy}{dt} = 2.8y\left(1-\frac{y}{10}\right)$ $y(0)=7$

$k=2.8$ $L=10$

$y = \frac{10}{1+Ce^{-2.8t}}$

$7 = \frac{10}{1+Ce^{0}}$

$(1+c)7=10$

$1+c=\frac{10}{7}$

$C=\frac{10}{7}-1=\frac{3}{7}$

$$\boxed{y = \frac{10}{1+\frac{3}{7}e^{-2.8t}}}$$

57) $\frac{dy}{dt} = \frac{4y}{5} - \frac{y^2}{150}$, $y(0)=8$

$\frac{dy}{dt} = \frac{4}{5}y\left[1-\frac{y}{120}\right]$

$k=4/5=0.8$ $L=120$

$y = \frac{120}{1+Ce^{-0.8t}}$

$8 = \frac{120}{1+Ce^{0}}$

$1+C=\frac{120}{8}$

$1+C=15$

$C=14$

$$\boxed{y = \frac{120}{1+14e^{-0.8t}}}$$

58) $\frac{dy}{dt} = \frac{3y}{20} - \frac{y^2}{1600}$, $y(0) = 15$

$\frac{dy}{dt} = \frac{3}{20}y\left[1 - \frac{y}{240}\right)$ $k = \frac{3}{20}$ $L = 240$

$y = \frac{240}{1+Ce^{-3/20t}}$ | $15 = \frac{240}{1+Ce^0}$ | $1 + c = \frac{240}{15} = 16$, $C = 15$ | $\boxed{y = \frac{240}{1+15e^{-3/20t}}}$

59) $P = \frac{L}{1+Ce^{-kt}}$ $L = 200$ $P(0) = 25$

$25 = \frac{200}{1+Ce^0}$ | $1 + C = 8$, $C = 7$

$P = \frac{200}{1+7e^{-kt}}$ $P(2) = 39$ plug in to find k

$39 = \frac{200}{1+7e^{-k(2)}}$

$1+7e^{-2k} = \frac{200}{39}$

$7e^{-2k} = \frac{161}{39}$

$e^{-2k} = \frac{23}{39}$

$\ln e^{-2k} = \ln(23/39)$

$-2k = \ln\left(\frac{23}{39}\right)$

$k = -\frac{1}{2}\ln\left(\frac{23}{39}\right) \approx 0.2640$

a) $P = \frac{200}{1+7e^{-0.264t}}$

b) at $t=5$, $P \approx 70$ panthers

c) $100 = \frac{200}{1+7e^{-0.264t}}$ $\boxed{t \approx 7.37 \text{ yrs.}}$

d) $\frac{dP}{dt} = kP\left(1 - \frac{P}{L}\right)$

$\frac{dP}{dt} = 0.264P\left(1 - \frac{P}{200}\right)$

e) P is increasing most rapidly when $P = \frac{L}{2} = \frac{200}{2} = \underline{100}$, $\boxed{t \approx 7.37 \text{ yrs.}}$

60) $y = \frac{L}{1+Ce^{-kt}}$ $L = 20$, $y(0) = 1$, $y(2) = 4$

$1 = \frac{20}{1+Ce^0}$

$1 + C = 20$

$C = 19$

$y = \frac{20}{1+19e^{-kt}}$

$4 = \frac{20}{1+19e^{-2k}}$

$1+19e^{-2k} = \frac{20}{4}$

$1+19e^{-2k} = 5$

$19e^{-2k} = 4$

$e^{-2k} = \frac{4}{19}$

$\ln e^{-2k} = \ln(4/19)$

$-2k = \ln(4/19)$

$k = -\frac{1}{2}\ln(4/19) \approx 0.7791$

$y = \frac{20}{1+19e^{-0.7791t}}$

$\boxed{y(5) \approx 14.43 \text{ grams}}$

c) $18 = \frac{20}{1+19e^{-0.7791t}}$

$\boxed{t \approx 6.60 \text{ hrs.}}$

d) $\frac{dy}{dt} = \frac{1}{2}\ln\left(\frac{19}{4}\right)y\left[1 - y/20\right)$

e) increasing most rapidly : $y = \frac{L}{2} = \boxed{\frac{20}{2} = 10,\ t \approx 3.78 \text{ hrs.}}$