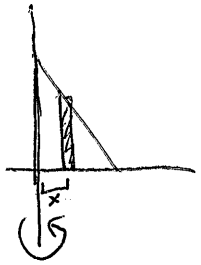
7.3 Shell Method p. 462 #1-22 D251
#23-36 D251

$V = 2\pi \int [\text{shell radius}] \cdot [\text{shell height}]\, dx$

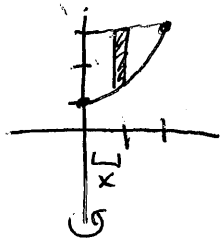
2) $y = 1 - x$



$V = 2\pi \int_0^1 x(1-x)\,dx = 2\pi \int_0^1 x - x^2\,dx$

$2\pi \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = \boxed{\frac{\pi}{3}}$

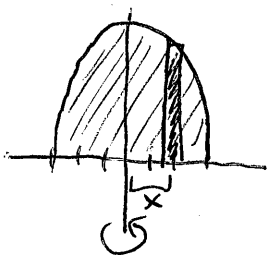
4) $y = \frac{1}{2}x^2 + 1$



$V = 2\pi \int_0^2 x\left(\frac{1}{2}x^2 + 1\right) dx$

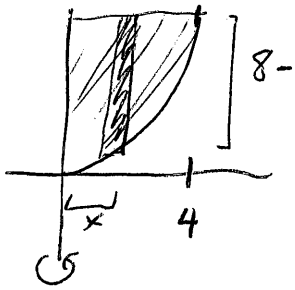
$V = 2\pi \left[x^2 - \frac{x^4}{8}\right]_0^2 = 2\pi(4-2) = \boxed{4\pi}$

8) $y = 9 - x^2$, $y = 0$



$V = 2\pi \int_0^3 x(9-x^2)\,dx = \boxed{\frac{81\pi}{2}}$

10) $y = x^{3/2}$ $y = 8$, $x = 0$

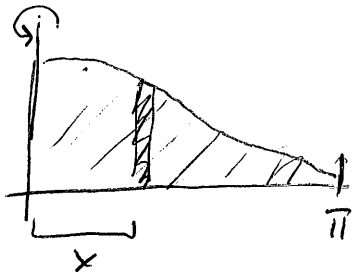


$V = 2\pi \int_0^4 x(8 - x^{3/2})\,dx$

$V = 2\pi \left[4x^2 - \frac{2}{7}x^{7/2}\right]_0^4$

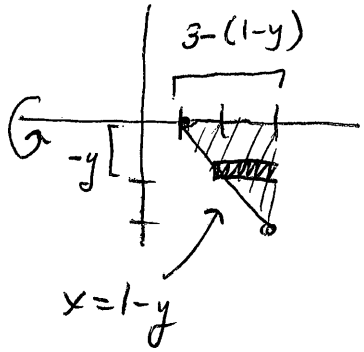
$= \boxed{\frac{384\pi}{7}}$

14) $y = \begin{cases} \frac{\sin x}{x}, & x > 0 \\ 1, & x = 0 \end{cases}$ $y=0, x=0, x=\pi$



$$V = 2\pi \int_0^{\pi} x\left[\frac{\sin x}{x}\right] dx = -2\pi \cos x \Big]_0^{\pi} = \boxed{4\pi}$$

16) $y = 1 - x$

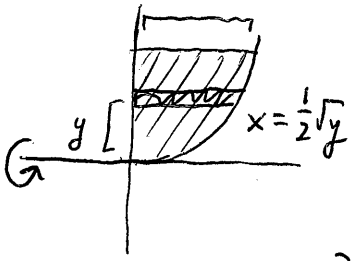


$$V = 2\pi \int_{-2}^{0} (-y)(2+y)\,dy$$

$$= 2\pi \left[-y^2 - \frac{y^3}{3}\right]_{-2}^{0} = 2\pi\left(\frac{4}{3}\right)$$

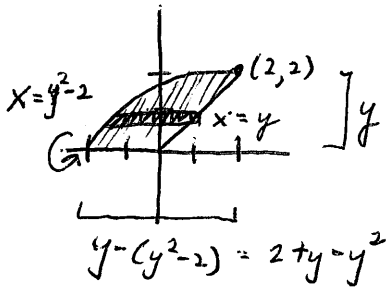
$$= \boxed{8\pi/3}$$

20) $y = 4x^2$, $x = 0$, $y = 4$



$$V = 2\pi \int_0^4 y\left(\tfrac{1}{2}\sqrt{y}\right) dy = \pi \int_0^4 y^{3/2}\,dy = \pi \cdot \frac{2}{5} y^{5/2} \Big]_0^4 = \boxed{\frac{64}{5}\pi}$$

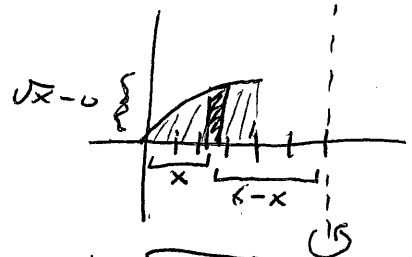
22) $y = \sqrt{x+2}$ $y = x$ $y = 0$



$$V = 2\pi \int_0^2 y(2+y-y^2)\,dy$$

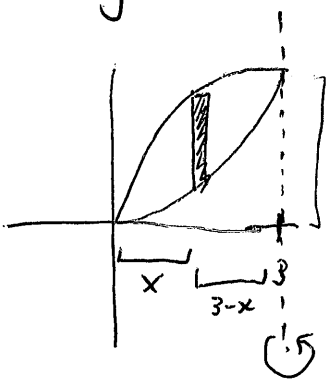
$$= 2\pi\left(4 + \tfrac{8}{3} - 4\right) = \boxed{\frac{16\pi}{3}}$$

24) $y = \sqrt{x}$, $y = 0$, $x = 4$ (about $x = 6$)



$$V = 2\pi \int_0^4 (6-x)(\sqrt{x})\,dx = 2\pi \cdot 4x^{3/2} - \frac{2}{5}x^{5/2} \Big]_0^4 = \boxed{\frac{192\pi}{5}}$$

26) $y = \frac{1}{3}x^3$ $y = 6x - x^2$ about line $x = 3$

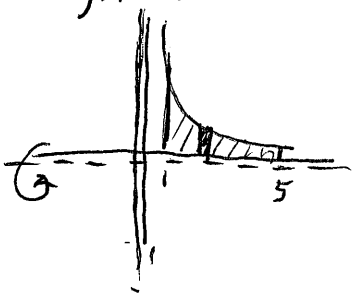


$$V = 2\pi \int_0^3 (3-x)\left[6x - x^2 - \frac{1}{3}x^3\right] dx$$

$$= 2\pi \left[\frac{x^5}{15} - 3x^3 + 9x^2\right]_0^3 = \boxed{\frac{162\pi}{5}}$$

30) $y = \frac{10}{x^2}$, $y = 0$, $x = 1$, $x = 5$

a) AOR: x-axis



Disc Method:

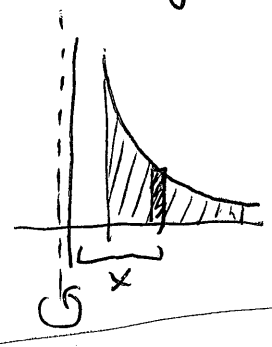
$$V = \pi \int_1^5 R(x)^2 dx$$

$$= \pi \int_1^5 \left(\frac{10}{x^2}\right)^2 dx$$

$$100\pi \int x^{-4}$$

$$100\pi \cdot \frac{x^{-3}}{-3}\Big]_1^5 = \boxed{\frac{496\pi}{15}}$$

b) AOR: y-axis

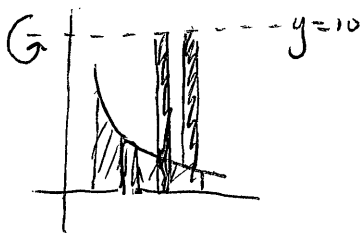


Shell Method:

$$V = 2\pi \int_1^5 x \cdot \left(\frac{10}{x^2}\right) dx = 20\pi \int_1^5 \frac{1}{x} dx = 20\pi \ln|x|\Big|_1^5$$

$$= \boxed{20\pi \ln 5}$$

c) AOR: $y = 10$



Washer Method:

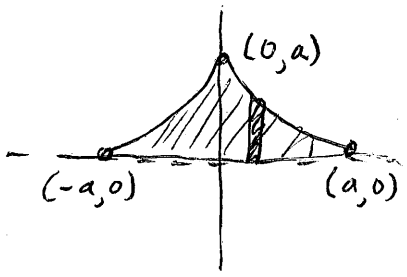
$R(x) = 10 - 0 = 10$

$r(x) = 10 - \frac{10}{x^2}$

$$V = \pi \int_1^5 10^2 - \left[10 - \frac{10}{x^2}\right]^2 dx$$

$$= \pi \left[\frac{100}{3x^3} - \frac{200}{x}\right]_1^5 = \boxed{\frac{1904}{15}\pi}$$

32) $x^{2/3} + y^{2/3} = a^{2/3}$ $\quad a > 0$



a) x-axis (AOR) $\quad y^{2/3} = a^{2/3} - x^{2/3}$

$$y = \left(a^{2/3} - x^{2/3}\right)^{3/2}$$

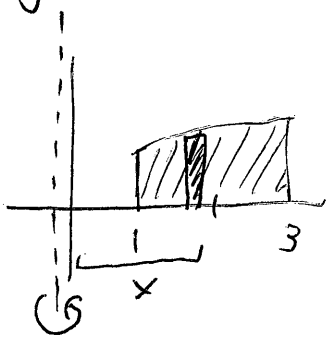
$$V = \pi \int_{-a}^{a} \left(\left[a^{2/3} - x^{2/3}\right]^{3/2}\right)^2 dx$$

$$V = \pi \int_{-a}^{a} \left[a^{2/3} - x^{2/3}\right]^3 dx$$

$$2\pi\left(a^3 - \frac{9}{5}a^3 + \frac{9}{7}a^3 - \frac{1}{3}a^3\right) = \boxed{\frac{32\pi a^3}{105}}$$

b) AOR: y-axis : same due to symmetry $\left(\frac{32\pi a^3}{105}\right)$

36) $y = \dfrac{2}{1+e^{1/x}}$ $\quad y = 0,\ x = 1,\ x = 3$



AOR: (y-axis)

Shell Method:

$$V = 2\pi \int_{1}^{3} x\left[\frac{2}{1+e^{1/x}}\right] dx \approx \boxed{19.0162}$$