2017

8.1 Basic Integration Rules p. 512 #1-72

1) a) $\dfrac{2x}{\sqrt{x^2+1}}$ c) $\dfrac{x}{2\sqrt{x^2+1}}$

$\boxed{\text{b) } \dfrac{x}{\sqrt{x^2+1}}}$ d) $\dfrac{2x}{x^2+1}$

2) $\boxed{\text{a) } \dfrac{x}{x^2+1}}$ c) $\dfrac{1}{1+x^2}$

b) $\dfrac{2(1-3x^2)}{(x^2+1)^3}$ d) $\dfrac{2x}{x^2+1}$

4) a) (product rule) $4x^2\cos(x^2+1)+\sin(x^2+1)$

b) $-x\cos(x^2+1)$

$\boxed{\text{c) } x\cos(x^2+1)}$

d) ~~prodc~~

6) $\displaystyle\int \frac{2t+1}{t^2+t-4}\,dt$ $u=t^2+t-4$ use Rule $\displaystyle\int \frac{1}{u}\,du$

8) $\displaystyle\int \frac{2}{(2t-1)^2+4}\,dt$ $u=2t-1$, $a=1$ use $\displaystyle\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\left(\frac{u}{a}\right)+C$

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10) $\int \frac{-2x}{\sqrt{x^2-4}}\,dx$ $\quad u = x^2-4$ $\quad$ use power Rule $\int u^n\,du$

12) $\int \sec 5x \tan 5x\,dx$ $\quad u = 5x$

14) $\int \frac{1}{x\sqrt{x^2-4}}\,dx$ $\quad u = x$, $a = 2$ $\quad$ use $\int \frac{du}{|u|\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\left(\frac{|u|}{a}\right) + C$

16) $\int \frac{5}{(t+6)^3}\,dt$ $\quad u = t+6$ $\quad \int \frac{5}{u^3}\,du = \int 5u^{-3}\,du$

$= \frac{5u^{-2}}{-2} + c = \boxed{\frac{-5}{2(t+6)^2} + c}$

18) $\int t^3\sqrt{t^4+1}\,dt$ $\quad u = t^4+1$ $\quad = \boxed{\frac{1}{6}(t^4+1)^{3/2} + c}$

20) $\int \left[4x - \frac{2}{(2x+3)^2}\right]dx = \int 4x\,dx - 2\int \frac{1}{(2x+3)^2}\,dx$ $\quad u = 2x+3$

$= \boxed{2x^2 + \frac{1}{2x+3} + c}$

22) $\displaystyle\int \frac{x+1}{\sqrt{3x^2+6x}}\,dx$ $u = 3x^2+6x$ $\frac{du}{dx} = 6x+6 = 6(x+1)$ $\boxed{\frac{1}{3}\sqrt{3x^2+6x} + C}$

24) $\displaystyle\int \frac{3x}{x+4}\,dx$

$$\begin{array}{r} 3 - \frac{12}{x+4} \\ x+4 \overline{)\,3x} \\ \ominus\, 3x \overset{\ominus}{+} 12 \\ \hline -12 \end{array}$$

$\displaystyle\int 3 - \frac{12}{x+4}\,dx$

$\boxed{3x - 12\ln|x+4| + C}$

26) $\displaystyle\int \frac{1}{2x+5} - \frac{1}{2x-5}\,dx = \frac{1}{2}\ln|2x+5| - \frac{1}{2}\ln|2x-5| + C$

$= \boxed{\frac{1}{2}\ln\left|\frac{2x+5}{2x-5}\right| + C}$

28) $\displaystyle\int x\left(3+\frac{2}{x}\right)^2 = \int 9x + 12 + \frac{4}{x}\,dx = \boxed{\frac{9}{2}x^2 + 12x + 4\ln|x| + C}$ (Expand)

30) $\displaystyle\int \csc \pi x \cot \pi x\,dx$ $u = \pi x$ $= \boxed{-\frac{1}{\pi}\csc \pi x + C}$

32) $\displaystyle\int \csc^2 x \cdot e^{\cot x}\,dx$ $u = \cot x$ $\frac{du}{dx} = -\csc^2 x$ $\boxed{-e^{\cot x} + C}$

34) $\displaystyle\int \frac{2}{7e^x+4}\,dx \cdot \frac{e^{-x}}{e^{-x}} = \int \frac{2e^{-x}}{7+4e^{-x}}\,dx$ $\boxed{-\frac{1}{2}\ln|7+4e^{-x}| + C}$

$u = 7+4e^{-x}$

$\frac{du}{dx} = -4e^{-x}$

36) $\int \tan x \cdot \ln(\cos x)\,dx$

$u = \ln(\cos x)$

$\frac{du}{dx} = \frac{-\sin x}{\cos x} = -\tan x$

$\boxed{-\frac{[\ln(\cos x)]^2}{2} + C}$

38) $\int \frac{1}{\cos\theta - 1}\,d\theta$

$\frac{1}{\cos\theta-1} \cdot \frac{\cos\theta+1}{\cos\theta+1} = \frac{\cos\theta+1}{\cos^2\theta-1} = \frac{\cos\theta+1}{-\sin^2\theta}$

$= \frac{\cos\theta}{-\sin^2\theta} - \frac{1}{\sin^2\theta} = \int -\cot\theta\csc\theta - \csc^2\theta\,d\theta = \csc\theta + \cot\theta + C$

$= \boxed{\frac{1+\cos\theta}{\sin\theta} + C}$

40) $\int \frac{1}{25+4x^2}\,dx$

$u = 2x$

$a = 5$

$\int \frac{du}{u^2+a^2} = \frac{1}{a}\arctan\left(\frac{u}{a}\right) + C$

$= \boxed{\frac{1}{10}\arctan\left(\frac{2x}{5}\right) + C}$

42) $\int \frac{e^{1/t}}{t^2}\,dt$

$u = 1/t$

$\frac{du}{dt} = \frac{-1}{t^2}$

$\boxed{-e^{1/t} + C}$

* complete the square

44) $\int \frac{1}{(x-1)\sqrt{4x^2-8x+3}}\,dx = \int \frac{1}{(x-1)\sqrt{4(x^2-2x+\underline{1})+3-\underline{4}}} = \int \frac{1}{(x-1)\sqrt{4(x-1)^2-1}}\,dx$

$\int \frac{1}{(x-1)\sqrt{(2(x-1))^2-(1)^2}}\,dx$

$u = 2(x-1)$ $a = 1$

$\frac{du}{dx} = 2$, $dx = \frac{du}{2}$

$\int \frac{du}{|u|\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\left(\frac{|u|}{a}\right) + C$

$\int \frac{1}{2(x-1)\sqrt{[2(x-1)]^2-[1]^2}}\,dx$

$\int \frac{du}{|u|\sqrt{u^2-a^2}}$

$= \frac{1}{1}\operatorname{arcsec}\left(\frac{|2(x-1)|}{1}\right) + C$

$= \boxed{\operatorname{arcsec}|2(x-1)| + C}$

46) $\int \frac{1}{x^2-4x+9}\,dx$ * complete the square $= \int \frac{1}{x^2-4x+\underline{4}+9-\underline{4}}\,dx$

$\int \frac{1}{(x-2)^2+5}\,dx = \int \frac{1}{(x-2)^2+(\sqrt{5})^2}\,dx$ | $\int \frac{du}{u^2+a^2} = \frac{1}{a}\arctan\left(\frac{u}{a}\right)+C$

$\boxed{= \frac{1}{\sqrt{5}}\arctan\left(\frac{x-2}{\sqrt{5}}\right)+C}$

48) $\frac{dy}{dx} = \frac{1}{\sqrt{4x-x^2}}$ $(2, \frac{1}{2})$ $\int \frac{1}{\sqrt{-1(x^2-4x+\underline{4})+\underline{4}}}\,dx = \int \frac{1}{\sqrt{-(x-2)^2+(2)^2}}\,dx$

$= \int \frac{1}{\sqrt{2^2-(x-2)^2}}\,dx$ | * $\int \frac{1}{\sqrt{a^2-u^2}} = \arcsin\left(\frac{u}{a}\right)+C$ | $\frac{1}{2} = \arcsin(0)+C$

$\frac{1}{2} = C$

$u=(x-2)$ $a=2$ | $y = \arcsin\left(\frac{x-2}{2}\right)+C$ | $\boxed{y = \arcsin\left(\frac{x-2}{2}\right)+\frac{1}{2}}$

50) $\frac{dy}{dx} = 5-y$ $y(0)=1$ | $-\ln|5-y| = x+C$ | $|5-1| = Ce^0$ $4=C$

$\int \frac{dy}{5-y} = \int dx$ | $e^{\ln|5-y|} = e^{-x+C}$ | $|5-y| = 4e^{-x}$ | $\boxed{y = -4e^{-x}+5}$

$|5-y| = Ce^{-x}$ | $5-y = 4e^{-x}$

$-y = 4e^{-x}-5$

52) $\frac{dy}{dx} = (4-e^{2x})^2$ | $\boxed{y = 16x - 4e^{2x} + \frac{1}{4}e^{4x} + C}$

$dy = (4-e^{2x})^2\,dx$

$\int dy = \int 16 - 8e^{2x} + e^{4x}\,dx$

*u-sub

54) $\frac{dr}{dt} = \frac{(1+e^t)^2}{e^{3t}} = \frac{1+2e^t+e^{2t}}{e^{3t}} = e^{-3t}+2e^{-2t}+e^{-t}$

$r = \int e^{-3t}+2e^{-2t}+e^{-t}\,dt = \boxed{-\frac{1}{3}e^{-3t}-e^{-2t}-e^{-t}+c}$

56) $y' = \frac{1}{x\sqrt{4x^2-9}}$ | $\int \frac{1}{x\sqrt{(2x)^2-(3)^2}}dx$ | $\int \frac{du}{|u|\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\left(\frac{|u|}{a}\right)+c$

$y = \int \frac{1}{x\sqrt{4x^2-9}}dx$ | $u=2x$, $\frac{du}{dx}=2$, $a=3$ | $= \boxed{\frac{1}{3}\operatorname{arcsec}\left(\frac{|2x|}{3}\right)+c}$

58) $\int_0^{\pi} \sin^2 t\cos t\,dt$ | $u=\sin t$, $\frac{du}{dt}=\cos(t)$ | $\frac{1}{3}\sin^3 t\Big]_0^{\pi} = \boxed{0}$

60) $\int_1^e \frac{1-\ln x}{x}dx$ | $u=1-\ln x$, $\frac{du}{dx}=\frac{-1}{x}$ | $-\frac{1}{2}(1-\ln x)^2\Big]_1^e = \boxed{1/2}$

62) $\int_1^3 \frac{2x^2+3x-2}{x}dx = \int 2x+3-\frac{2}{x}\,dx = x^2+3x-2\ln|x|\Big]_1^3 = \boxed{14-2\ln 3}$

64) $\int_0^7 \frac{1}{\sqrt{100-x^2}}dx$ | $a=10$, $u=x$ | $\arcsin\left(\frac{x}{10}\right)\Big]_0^7 = \boxed{\arcsin\left(\frac{7}{10}\right)}$

* $\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\left(\frac{u}{a}\right)+c$

66) $\text{Area} = \int_0^5 \frac{3x+2}{x^2+9}\,dx = \underset{\text{(u-sub)}}{\int \frac{3x}{x^2+9}\,dx} + \underset{\text{(Arctan)}}{\int \frac{2}{x^2+9}\,dx}$

$u = x^2+9$ $\qquad u = x$

$\frac{du}{dx} = 2x$ $\qquad a = 3$

$$= \frac{3}{2}\ln|x^2+9| + \frac{2}{3}\arctan\left(\frac{x}{3}\right)\Big]_0^5 = \frac{3}{2}\ln\left(\frac{34}{9}\right) + \frac{2}{3}\arctan\left(\frac{5}{3}\right)$$

$$\approx \boxed{2.6806}$$

68) $A = \int_0^{\pi/2} \sin 2x = -\frac{1}{2}\cos(2x)\Big]_0^{\pi/2} = \boxed{1}$

70) $\int \frac{x-2}{x^2+4x+13}\,dx = \underset{\text{(u-sub)}}{\int \frac{x}{x^2+4x+13}\,dx} - \int \frac{2}{x^2+4x+13}\,dx$

$$= \boxed{\frac{1}{2}\ln(x^2+4x+13) - \frac{4}{3}\arctan\left(\frac{x+2}{3}\right) + C}$$

72) $\int \left(\frac{e^x+e^{-x}}{2}\right)^3 dx \quad \frac{1}{8}\int e^{3x} + \underbrace{3e^{2x}e^{-x}}_{3e^x} + \underbrace{3e^{x}e^{-2x}}_{+3e^{-x}} + e^{-3x}\,dx$

$$= \boxed{\frac{1}{8}\cdot\left[\frac{1}{3}e^{3x} + 3e^x - 3e^{-x} - \frac{1}{3}e^{-3x}\right] + C}$$