8.3 Trig Integrals p. 530 # 1-11, 19-35, 41-61, 71, 75

1) $\int \cos^5 x \sin x\,dx$ $\quad \int (\cos x)^5 \sin x\,dx$ $\quad u = \cos x$ $\quad dx = \frac{du}{-\sin x}$

$\frac{du}{dx} = -\sin x$

$\int u^5 \cdot \cancel{\sin x} \cdot \frac{du}{-\cancel{\sin x}} = \frac{-u^6}{6} + C$ $\quad = \boxed{-\frac{1}{6}(\cos x)^6 + C}$

5) $\int \sin^3 x \cos^2 x\,dx = \int [\sin x]^2 \cdot \cos^2 x \cdot \sin x\,dx$

$u = \cos x$

$\int (1 - \cos^2 x)\cos^2 x \cdot \sin x\,dx = \int \cos^2 x - \cos^4 x \cdot \sin x\,dx$ $\quad \frac{du}{dx} = -\sin x$

$dx = \frac{du}{-\sin x}$

$\int u^2 - u^4 \cdot \cancel{\sin x} \cdot \frac{du}{-\cancel{\sin x}} = -\int u^2 - u^4\,du = -\frac{u^3}{3} + \frac{u^5}{5} + C$

$= \boxed{-\frac{1}{3}\cos^3 x + \frac{1}{5}\cos^5 x + C}$

9) $\int \cos^2 3x\,dx$ $\quad$ * Recall half-angle : $\cos^2 u = \frac{1}{2}(1 + \cos 2u)$

$= \int \frac{1}{2}(1 + \cos 2(3x))\,dx = \int \frac{1}{2} + \frac{1}{2}\cos 6x\,dx$ $\quad u = 6x \quad \frac{du}{dx} = 6 \quad dx = \frac{du}{6}$

$\int \frac{1}{2}\cos u \cdot \frac{du}{6}$

$= \boxed{\frac{1}{2}x + \frac{1}{12}\sin 6x + C}$

* Recall $\int \sec u\,du = \ln|\sec u + \tan u| + C$

19) $\int \sec 4x\,dx$

$u = 4x$ | $dx = \frac{du}{4}$ | $\int \sec u \cdot \frac{du}{4} = \frac{1}{4}\ln|\sec u + \tan u| + C$

$\frac{du}{dx} = 4$

$= \boxed{\frac{1}{4}\ln|\sec 4x + \tan 4x| + C}$

23) $\int \tan^5\left(\frac{x}{2}\right)dx = \int \tan^3\left(\frac{x}{2}\right) \cdot \tan^2\left(\frac{x}{2}\right)dx = \cancel{\int \left[\tan\left(\frac{x}{2}\right)\right]^3 \cdot \tan^2\left(\frac{x}{2}\right)dx}$

$\cancel{u = \tan\left(\frac{x}{2}\right)}$

$\cancel{\frac{du}{dx} = \sec^2\left(\frac{x}{2}\right)\cdot\frac{1}{2}}$

$\cancel{dx = \frac{2du}{\sec^2(x/2)}}$

$\int \tan^3\left(\frac{x}{2}\right) \cdot \left[\sec^2\left(\frac{x}{2}\right) - 1\right] dx$

$-\int \tan^3\left(\frac{x}{2}\right) + \int \tan^3\left(\frac{x}{2}\right)\sec^2\left(\frac{x}{2}\right) dx$

$-\int \tan^2\left(\frac{x}{2}\right) \cdot \tan\left(\frac{x}{2}\right) dx$

$-\int \left[\sec^2\left(\frac{x}{2}\right) - 1\right] \cdot \tan\left(\frac{x}{2}\right) dx$

$-\int \sec^2\left(\frac{x}{2}\right)\tan\left(\frac{x}{2}\right) + \int \tan\left(\frac{x}{2}\right)dx$

$u = \tan(x/2)$

$\frac{du}{dx} = \sec^2(x/2)\cdot\frac{1}{2}$

$dx = \frac{2du}{\sec^2(x/2)}$

$-2\ln|\cos x/2|$

$-\int u \cdot \cancel{\sec^2(x/2)} \cdot \frac{2du}{\cancel{\sec^2(x/2)}}$

$-\frac{2u^2}{2}$

$u = \tan(x/2)$

$\frac{du}{dx} = \sec^2(x/2)\cdot\frac{1}{2}$

$dx = \frac{2du}{\sec^2(x/2)}$

$\int u^3 \cdot \cancel{\sec^2(x/2)} \cdot \frac{2du}{\cancel{\sec^2(x/2)}}$

$\frac{2u^4}{4}$

$\boxed{-\tan^2\left(\frac{x}{2}\right) - 2\ln\left|\cos\left(\frac{x}{2}\right)\right| + \frac{1}{2}\tan^4\left(\frac{x}{2}\right) + C}$

27) $\int \sec^6(4x)\tan(4x)\,dx = \int \sec^5(4x)\cdot\sec(4x)\tan(4x)\,dx$

$u = \sec(4x)$

$\frac{du}{dx} = \sec 4x \tan 4x \cdot 4$

$dx = \frac{du}{4\sec 4x \tan 4x}$

$\int u^5 \cdot \sec 4x\cancel{\tan 4x}\cdot\frac{du}{4\sec 4x\cancel{\tan 4x}} = \frac{1}{4}\int u^5\,du$

$= \frac{1}{4}\left(\frac{u^6}{6}\right) = \boxed{\frac{1}{24}\sec^6(4x) + C}$

31) $\int \frac{\tan^2 x}{\sec x}\,dx = \int \frac{\sec^2 x - 1}{\sec x}\,dx = \int \sec x - \cos x\,dx$

$= \boxed{\ln|\sec x + \tan x| - \sin x + C}$

35) Solve differential equation $y' = \tan^3 3x \sec 3x$

$\frac{dy}{dx} = \tan^3 3x \sec 3x$

$\int dy = \int \tan^3 3x \sec 3x$

$y = \int \tan^2 3x \cdot \tan 3x \sec 3x\,dx$

$= \int (\sec^2 3x - 1)\cdot \sec 3x \tan 3x\,dx$

$u = \sec 3x$

$\frac{du}{dx} = \sec 3x \tan 3x \cdot 3$

$dx = \frac{du}{3\sec 3x \tan 3x}$

$= \int u^2 - 1 \cdot \sec 3x\cancel{\tan 3x}\cdot\frac{du}{3\sec 3x\cancel{\tan 3x}}$

$= \frac{1}{3}\left[\frac{u^3}{3} - u\right] + C = \boxed{\frac{1}{9}\sec^3 3x - \frac{1}{3}\sec 3x + C}$

Use Product-to-Sum Identities

41) $\int \cos 2x \cos 6x \, dx$ $\qquad$ * $\cos mx \cos nx = \frac{1}{2}\left(\cos[m-n]x + \cos(m+n)x\right]$

$$\int \cos 6x \cos 2x \, dx = \frac{1}{2}\int \cos 4x + \cos 8x \, dx$$

$$\frac{1}{2}\cdot\frac{1}{4}\sin 4x + \frac{1}{2}\cdot\frac{1}{8}\sin 8x + C = \boxed{\frac{1}{8}\sin 4x + \frac{1}{16}\sin 8x + C}$$

45) $\int \sin\theta \sin 3\theta \, d\theta$ $\qquad$ * $\sin(mx)\sin(nx) = \frac{1}{2}\left[\cos(m-n)x - \cos(m+n)x\right]$

$$\int \sin 3\theta \sin\theta \, d\theta = \frac{1}{2}\int \cos 2\theta - \cos 4\theta \, d\theta$$

$$= \frac{1}{2}\cdot\frac{1}{2}\sin 2\theta - \frac{1}{2}\cdot\frac{1}{4}\sin 4\theta + C = \boxed{\frac{1}{4}\sin 2\theta - \frac{1}{8}\sin 4\theta + C}$$

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49) $\int \csc^4 3x \, dx = \int \csc^2 3x \cdot \csc^2(3x) \, dx$

$$\int \csc^2 3x \cdot \left[1 + \cot^2 3x\right] dx = \int \csc^2 3x + \int \csc^2 3x \cot^2 3x$$

$\to \int \left[\cot 3x\right]^2 \csc^2 3x \, dx$

$u = \cot 3x$

$\frac{du}{dx} = -\csc^2 x \cdot 3$

$-\frac{1}{3}\int u^2 \cdot du$

$$-\frac{1}{3}\cot 3x - \frac{1}{3}\frac{\cot^3(3x)}{3} + C$$

$$= \boxed{-\frac{1}{3}\cot 3x - \frac{1}{9}\cot^3 3x + C}$$

53) $\int \frac{1}{\sec x \tan x} dx = \int \frac{1}{\frac{1}{\cos x} \cdot \frac{\sin x}{\cos x}} dx = \int \frac{\cos^2 x}{\sin x} dx = \int \frac{1-\sin^2 x}{\sin x} dx$

$\int \frac{1}{\sin x} - \sin x \, dx = \int \csc x - \sin x dx = \boxed{-\ln|\csc x + \cot x| + \cos x + C}$

59) $\int_0^{\pi/4} 6\tan^3 x \, dx = \int 6 \tan^2 x \cdot \tan x = \int 6 \left[\sec^2 x - 1\right] \tan x \, dx$

$6\int \tan x \sec^2 x \, dx - 6\int \tan x \, dx = 6 \cdot \frac{\tan^2 x}{2} + 6 \ln|\cos x| \Big]_0^{\pi/4}$

$u = \tan x$
$\frac{du}{dx} = \sec^2 x$

$= 3(1) + 6\ln\left(\frac{\sqrt{2}}{2}\right) - 3(0) - 6\ln(1)$

$= 3 + 6\ln 2^{1/2} - 6\ln 2$

$3 + 3\ln 2 - 6\ln 2 = \boxed{3 - 3\ln 2}$

71) Find Area $y = \sin x \quad y = \sin^3 x \quad x = 0 \quad x = \pi/2$

75) Find volume of solid revolving about x-axis

$y = \tan x \quad y = 0 \quad x = -\pi/4 \quad x = \pi/4$