2017

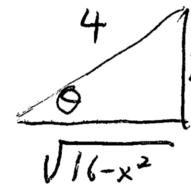
8.4 Trig Substitution p. 539 #5-45

use $x = 4\sin\theta$

5) $\displaystyle\int \frac{1}{(16-x^2)^{3/2}}\,dx$

$\dfrac{dx}{d\theta} = 4\cos\theta$

$dx = 4\cos\theta\, d\theta$



$\sin\theta = \dfrac{x}{4}$

$\sin\theta = \dfrac{x}{4}$

$\displaystyle\int \frac{1}{\left(\sqrt{16-x^2}\right)^3}\,dx$

$\cos\theta = \dfrac{\sqrt{16-x^2}}{4}$

$4\cos\theta = \sqrt{16-x^2}$

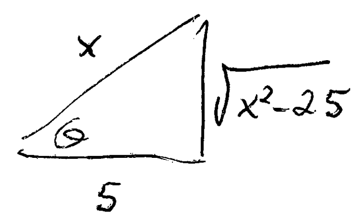
$$\int \frac{1}{(4\cos\theta)^3}\cdot 4\cos\theta\, d\theta = \int \frac{1}{16\cos^2\theta}\,d\theta = \frac{1}{16}\int \sec^2\theta\, d\theta$$

$$= \frac{1}{16}\tan\theta + c = \boxed{\frac{1}{16}\left(\frac{x}{\sqrt{16-x^2}}\right) + c}$$

9) use $x = 5\sec\theta$

$\sec\theta = \dfrac{x}{5}$

$\displaystyle\int \frac{1}{\sqrt{x^2-25}}\,dx$



$\dfrac{dx}{d\theta} = 5\sec\theta\tan\theta$

$dx = 5\sec\theta\tan\theta\, d\theta$

$\tan\theta = \dfrac{\sqrt{x^2-25}}{5}$

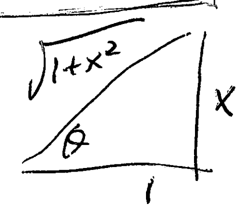
$5\tan\theta = \sqrt{x^2-25}$

$\displaystyle\int \frac{1}{\cancel{5\tan\theta}}\cdot \cancel{5}\sec\theta\,\cancel{\tan}\theta\, d\theta$

$$\int \sec\theta\, d\theta = \ln\left|\sec\theta + \tan\theta\right| + C = \boxed{\ln\left|\frac{x}{5} + \frac{\sqrt{x^2-25}}{5}\right| + C}$$

13) use $x = \tan\theta$

$\displaystyle\int x\sqrt{1+x^2}\,dx$



$\dfrac{dx}{d\theta} = \sec^2\theta$

$dx = \sec^2\theta\, d\theta$

$\sec\theta = \dfrac{\sqrt{1+x^2}}{1}$

$\displaystyle\int \tan\theta\cdot\sec\theta\cdot\sec^2\theta\, d\theta$

$\displaystyle\int \sec^2\theta\cdot \sec\theta\tan\theta\, d\theta$

$u = \sec\theta$

$\dfrac{du}{d\theta} = \sec\theta\tan\theta$

$\dfrac{\sec^3\theta}{3} + C$

$= \boxed{\dfrac{\left(\sqrt{1+x^2}\right)^3}{3} + C}$

17) Use Theorem 8.2 (p. 537)

$$* \int \sqrt{u^2+a^2}\,du = \frac{1}{2}\left(u\sqrt{u^2+a^2} + a^2 \ln\left|u+\sqrt{u^2+a^2}\right|\right) + C$$

$$\int \sqrt{9+16x^2}\,dx$$

$u = 4x$

$a = 3$

$\frac{du}{dx} = 4$

$dx = \frac{du}{4}$

$$= \int \sqrt{a^2+u^2} \cdot \frac{du}{4} = \frac{1}{4}\int \sqrt{a^2+u^2}\,du = \frac{1}{2}\left(4x\sqrt{9+16x^2} + 9\ln\left|4x+\sqrt{9+16x^2}\right|\right) + C$$

$$\boxed{= \frac{1}{8}\left[4x\sqrt{9+16x^2} + 9\ln\left|4x+\sqrt{9+16x^2}\right|\right] + C}$$

21) $$\int \frac{1}{\sqrt{16-x^2}}\,dx$$

$$* \int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\left(\frac{u}{a}\right) + C$$

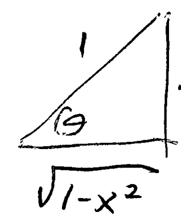
$u = x$

$a = 4$

$du = dx$

$$= \boxed{\arcsin\left(\frac{x}{4}\right) + C}$$

25) $$\int \frac{\sqrt{1-x^2}}{x^4}\,dx$$



$\cos\theta = \frac{\sqrt{1-x^2}}{1}$

$x = \sin\theta$

$\frac{dx}{d\theta} = \cos\theta \quad dx = \cos\theta\,d\theta$

$$\int \frac{\cos\theta}{\sin^4\theta} \cdot \cos\theta\,d\theta$$

$$\int \frac{\cos^2\theta}{\sin^4\theta}\,d\theta$$

$$\int \frac{\cos^2\theta}{\sin^2\theta \cdot \sin^2\theta}\,d\theta$$

$$\int \cot^2\theta \csc^2\theta\,d\theta$$

$$\int (\cot\theta)^2 \cdot \csc^2\theta\,d\theta$$

$u = \cot\theta$

$\frac{du}{d\theta} = -\csc^2\theta$

$$-\frac{u^3}{3}$$

$$\boxed{-\frac{1}{3}\cot^3\theta + C}$$

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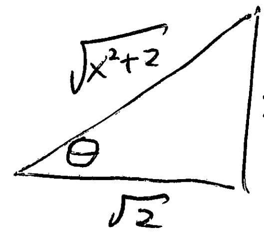
29) $\int \frac{-3x}{(x^2+3)^{3/2}}dx$ $\quad u=x^2+3 \quad dx=\frac{du}{2x}$

$\frac{du}{dx}=2\cancel{x}$

$$\int \frac{-3\cancel{x}}{u^{3/2}}\cdot\frac{du}{2\cancel{x}} = -\frac{3}{2}\int u^{-3/2}\,du = \frac{-3}{2}\cdot\frac{u^{-1/2}}{-1/2}+c$$

$$3(x^2+3)^{-1/2}+C = \boxed{\frac{3}{\sqrt{x^2+3}}+C}$$

33) $\int \frac{1}{4+4x^2+x^4}dx = \int \frac{1}{(x^2+2)^2}dx$



$\tan\theta=\frac{x}{\sqrt{2}}$

$x=\sqrt{2}\tan\theta$

$\frac{dx}{d\theta}=\sqrt{2}\sec^2\theta$

$dx=\sqrt{2}\sec^2\theta\,d\theta$

$\cos\theta=\frac{\sqrt{2}}{\sqrt{x^2+2}}$

$\sqrt{x^2+2}=\frac{\sqrt{2}}{\cos\theta}$

$\sqrt{x^2+2}=\sqrt{2}\sec\theta$

$$\int \frac{1}{(\sqrt{x^2+2})^4}dx = \int \frac{1}{(\sqrt{2}\sec\theta)^4}\sqrt{2}\sec^2\theta\,d\theta$$

$$\int \frac{\sqrt{2}\sec^2\theta}{4\sec^4\theta}d\theta = \frac{\sqrt{2}}{4}\int \cos^2\theta\,d\theta$$

$*\cos 2\theta = 2\cos^2\theta-1$

$\cos 2\theta+1 = 2\cos^2\theta$

$\frac{1}{2}(\cos 2\theta+1)=\cos^2\theta$

$$\frac{\sqrt{2}}{4}\int \frac{1}{2}(\cos 2\theta+1)\,d\theta = \frac{\sqrt{2}}{8}\int \cos 2\theta + 1\,d\theta$$

$$\frac{\sqrt{2}}{8}\cdot\frac{1}{2}\sin 2\theta + \frac{\sqrt{2}}{8}\theta + c = \frac{\sqrt{2}}{16}\left[2\sin\theta\cos\theta + \theta\right]+C$$

$\tan\theta=\frac{x}{\sqrt{2}}$

$\theta=\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$

$$=\boxed{\frac{\sqrt{2}}{8}\left[\arctan\left(x/\sqrt{2}\right) + \frac{x}{\sqrt{x^2+2}}\cdot\frac{\sqrt{2}}{\sqrt{x^2+2}}\right]+C}$$

37) Complete the square $\displaystyle\int \frac{1}{\sqrt{4x-x^2}}\,dx$ $\quad \sqrt{-(x^2-4x+4)+4}$

$$\int \frac{1}{\sqrt{4-(x-2)^2}}\,dx$$

$*\displaystyle\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\left(\frac{u}{a}\right)+c$

$a=2$

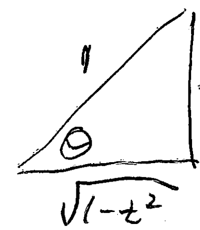
$u=x-2$

$\frac{du}{dx}=1$

$du=dx$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \boxed{\arcsin\left(\frac{x-2}{2}\right)+c}$$

41) $\displaystyle\int_0^{\sqrt{3}/2} \frac{t^2}{(1-t^2)^{3/2}}\,dt$ $\quad t=\sin\theta$



$\frac{dt}{d\theta}=\cos\theta \quad dt=\cos\theta\, d\theta$

$$\int \frac{\sin^2\theta}{\cos^3\theta}\cos\theta\, d\theta$$

$$\int \frac{\sin^2\theta}{\cos^2\theta}\,d\theta$$

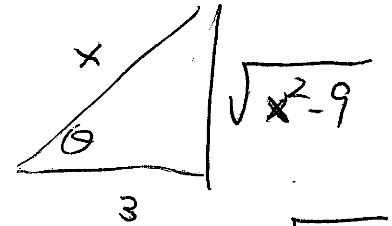
$\cos\theta = \sqrt{1-t^2}$

$$\int \frac{1-\cos^2\theta}{\cos^2\theta}\,d\theta = \int_0^{\sqrt{3}/2} \sec^2\theta - 1\, d\theta$$

a) $\tan\theta - \theta = \left.\frac{t}{\sqrt{1-t^2}} - \arcsin t\,\right]_0^{\sqrt{3}/2} = \frac{\sqrt{3}/2}{\sqrt{1/4}} - \arcsin\frac{\sqrt{3}}{2} \approx \boxed{0.685}$

b) $\tan\theta - \theta\,\Big]_6^{\pi/3} \approx \boxed{0.685}$

45) $\displaystyle\int_4^6 \frac{x^2}{\sqrt{x^2-9}}\,dx$ $\qquad \frac{x}{3} = \sec\theta \qquad x = 3\sec\theta$



$\frac{dx}{d\theta} = 3\sec\theta\tan\theta$

$dx = 3\sec\theta\tan\theta\,d\theta$

$\displaystyle\int \frac{(3\sec\theta)^2}{3\cancel{\tan\theta}} 3\sec\theta\cancel{\tan}\theta\,d\theta$

$\tan\theta = \frac{\sqrt{x^2-9}}{3}$

$\sqrt{x^2-9} = 3\tan\theta$

$= \displaystyle\int \frac{9\sec^3\theta}{1}\,d\theta = 9\int \sec^2\theta\,\sec\theta\,d\theta$

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$u = \sec\theta \qquad dv = \sec^2\theta$

$du = \sec\theta\tan\theta \qquad v = \tan\theta$

$uv - \int v\,du$

$\sec\theta\tan\theta - \int \sec\theta\tan^2\theta\,d\theta = \sec\theta\tan\theta - \int \sec\theta(\sec^2\theta - 1)\,d\theta$

$= 9\left[\frac{1}{2}\sec\theta\tan\theta + \ln|\sec\theta + \tan\theta\right]$

$= \frac{9}{2}\left[\frac{x}{3}\cdot\frac{\sqrt{x^2-9}}{3} + \ln\left|\frac{x}{3} + \frac{\sqrt{x^2-9}}{3}\right]\right]_4^6 = \boxed{12.644}$