2017

8.5 Partial Fractions p.549 #1-34

1) $\dfrac{4}{x^2-8x} = \dfrac{A}{x} + \dfrac{B}{x-8} = \dfrac{-1}{2x} + \dfrac{1}{2(x-8)}$

$x(x-8)$; $x=0$, $A = -1/2$; $x=8$, $B = 1/2$

4) $\dfrac{2x-1}{x(x^2+1)^2} = \dfrac{A}{x} + \dfrac{Bx+C}{x^2+1} + \dfrac{Dx+E}{(x^2+1)^2}$

7) $\displaystyle\int \frac{5}{x^2+3x-4}\,dx = \frac{A}{x+4} + \frac{B}{x-1} \rightarrow \int \frac{-1}{x+4} + \frac{1}{x-1}\,dx$

$(x+4)(x-1)$; $x=-4$, $A=-1$; $x=1$, $B=1$

$-\ln|x+4| + \ln|x-1| + C = \boxed{\ln\left|\dfrac{x-1}{x+4}\right| + C}$

10) $\displaystyle\int \frac{x^3-x+3}{x^2+x-2}\,dx = \int x-1\,dx + \int \frac{2x+1}{x^2+x-2}\,dx = \frac{A}{x+2} + \frac{B}{x-1}$

$(x+2)(x-1)$; $x=-2$, $A=1$; $x=1$, $B=1$

$$\begin{array}{r} x - 1 + \frac{2x+1}{x^2+x-2} \\ x^2+x-2 \overline{)\, x^3 - x + 3} \\ -(x^3 + x^2 - 2x) \\ \hline -x^2 + x + 3 \\ -(-x^2 - x + 2) \\ \hline 2x + 1 \end{array}$$

$\displaystyle\int x - 1 + \frac{1}{x+2} + \frac{1}{x-1}\,dx$

$\boxed{\dfrac{x^2}{2} - x + \ln|x+2| + \ln|x-1| + C}$

or

$\boxed{\dfrac{x^2}{2} - x + \ln|x^2+x-2| + C}$

13) $\displaystyle\int \frac{4x^2+2x-1}{x^3+x^2}\,dx = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$

$x^2(x+1)$

$4x^2+2x-1 = Ax(x+1) + B(x+1) + Cx^2$

when $x=0 \rightarrow -1 = B(0+1)$ $\quad \underline{\underline{B=-1}}$

when $x=-1 \rightarrow 1 = 0+0+C(-1)^2$ $\quad \underline{\underline{C=+1}}$

when $x=1 \rightarrow 5 = A(2)+(-1)(2)+1(1) \rightarrow 5 = 2A-1,\ 2A=6,\ \underline{\underline{A=3}}$

$\displaystyle\int \frac{3}{x} + \frac{-1}{x^2} + \frac{1}{x+1}\,dx = \int \frac{3}{x} - 1x^{-2} + \frac{1}{x+1}\,dx = 3\ln|x| + x^{-1} + \ln|x+1| + C$

$= \boxed{\frac{1}{x} + \ln|x^3(x+1)| + C}$

16) $\displaystyle\int \frac{8x}{x^3+x^2-x-1} = \int \frac{8x}{(x-1)(x+1)^2} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$

$x^2(x+1)-1(x+1)$ $\quad x=1 \quad x=-1 \quad x=-1$

$(x^2-1)(x+1)$

$(x-1)(x+1)(x+1)$

$8x = A(x+1)^2 + B(x+1)(x-1) + C(x-1)$

$x=1 \rightarrow 8 = A(4) + B(0) + C(0)$ $\quad 4A=8 \rightarrow \underline{A=2}$

$x=-1 \rightarrow -8 = 0 + B(0) + C(-2)$ $\quad 2C=8 \quad \underline{C=4}$

$x=0 \rightarrow 0 = 2 + B(-1) + 4(-1) \rightarrow -B=2$ $\quad \underline{B=-2}$

$\displaystyle\int \frac{2}{x-1} + \frac{-2}{x+1} + \frac{4}{(x+1)^2}\,dx$ $\quad u=(x+1) \quad \frac{du}{dx}=1 \quad \displaystyle\int 4u^{-2}\,du = 4\left(\frac{u^{-1}}{-1}\right)$

$2\ln|x-1| - 2\ln|x+1| - 4(x+1)^{-1} + C$

$\boxed{2\ln\left|\frac{x-1}{x+1}\right| - \frac{4}{x+1} + C}$

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19) $\int \frac{x^2}{x^4-2x^2-8}\,dx = \frac{A}{x-2} + \frac{B}{x+2} + \frac{Cx+D}{x^2+2}$ $\Big]\,(x-2)(x+2)(x^2+2)$

$(x^2-4)(x^2+2)$

$(x-2)(x+2)(x^2+2)$

$x^2 = A(x+2)(x^2+2) + B(x-2)(x^2+2) + (Cx+D)(x-2)(x+2)$

$x=-2 \rightarrow 4 = A(0) + B(-4)(6) + (-2C+D)(0)$

$4 = -24B \qquad \boxed{B = -\frac{1}{6}}$

$x=2 \rightarrow 4 = A(4)(6) + 0 + 0 \qquad \boxed{A = \frac{1}{6}}$

$x=0 \rightarrow 0 = \frac{1}{6}(2)(2) + -\frac{1}{6}(-2)(2) + (C(0)+D)(-2)(2)$

$0 = \frac{4}{6} + \frac{4}{6} + -4D, \quad -4D = \frac{-8}{6} \qquad \boxed{D = \frac{1}{3}}$

$x=1 \rightarrow 1 = \frac{1}{6}(3)(3) + -\frac{1}{6}(-1)(3) + (C + \frac{1}{3})(-1)(3)$

$1 = \frac{9}{6} + \frac{3}{6} - 3C - 1, \quad 1 = 2 - 3C - 1 \qquad \boxed{C = 0}$

$\int \frac{1}{6(x-2)} + \frac{-1}{6(x+2)} + \frac{0x + \frac{1}{3}}{x^2+2}\,dx \rightarrow \frac{1}{3}\int \frac{dx}{x^2+2} \leftarrow \frac{1}{3}\int \frac{dx}{(x)^2+(\sqrt{2})^2}$

$$\boxed{\frac{1}{6}\ln|x-2| - \frac{1}{6}\ln|x+2| + \frac{1}{3}\cdot\frac{1}{\sqrt{2}}\arctan\left(\frac{x}{\sqrt{2}}\right) + C}$$

22) $\int \frac{x^2+6x+4}{x^4+8x^2+16}\,dx = \frac{Ax+B}{(x^2+4)} + \frac{Cx+D}{(x^2+4)^2}$

$(x^2+4)^2$

match 'like' coefficients

$x^2+6x+4 = (Ax+B)(x^2+4) + Cx + D$

$x^2+6x+4 = Ax^3 + Bx^2 + 4Ax + 4B + Cx + D$

$\underline{0x^3} + \underline{1x^2} + \underline{6x} + \underline{4} = \underline{Ax^3} + \underline{Bx^2} + \underline{(4A+C)}x + \underline{4B+D}$

$\underline{A=0}$

$\underline{B=1}$

$4A+C=6 \rightarrow \underline{C=6}$

$4B+D=4 \quad 4+D=4$

$\underline{D=0}$

$\int \frac{0x+1}{x^2+4}\,dx + \int \frac{6x+0}{(x^2+4)^2}$

$\int \frac{dx}{(x)^2+(2)^2} \qquad u=x^2+4, \quad \frac{du}{dx}=2x$

$\frac{1}{2}\arctan\left(\frac{x}{2}\right) + \int 3u^{-2}\,du = 3\frac{u^{-1}}{-1}$

$$\boxed{\frac{1}{2}\arctan\left(\frac{x}{2}\right) - \frac{3}{x^2+4} + C}$$

25) $\displaystyle\int_1^2 \frac{x+1}{x(x^2+1)}dx = \frac{A}{x} + \frac{Bx+C}{x^2+1}$

$x+1 = A(x^2+1) + (Bx+c)(x)$

$x=0 \rightarrow 1 = A(1) + (\cancel{Bx+c})(0)$ $\boxed{A=1}$

$x=1 \rightarrow 2 = (1)(2) + (B(1)+c)(1)$ $2 = 2+B+C$, $B+C=0$

$x=-1 \rightarrow 0 = (1)(2) + (B(-1)+c)(-1)$ $0 = 2+B-C$, $B-C=-2$

$2B=-2$

$B=-1$

$C=1$

$\displaystyle\int_1^2 \frac{1}{x} + \frac{-x+1}{x^2+1}dx$

$\displaystyle= \int_1^2 \frac{1}{x} - \frac{x}{x^2+1} + \frac{1}{x^2+1} = \ln x - \frac{1}{2}\ln|x^2+1| + \frac{1}{1}\arctan(x)\Big]_1^2$

$u=x^2+1$

$\cancel{du/dx=2x}$

$\frac{1}{2}\ln(8/5) - \frac{\pi}{4} + \arctan 2 \approx \boxed{0.557}$

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28) $\displaystyle\int \frac{5\cos x}{\sin^2 x + 3\sin x - 4}dx$ $\cancel{(\sin x+4)(\sin x-1)}$ $= \boxed{\begin{matrix}u=\sin x\\ \frac{du}{dx}=\cos x\end{matrix}} = \displaystyle\int \frac{5}{u^2+3u-4}du = \frac{A}{u+4} + \frac{B}{u-1}$

$(u+4)(u-1)$

$A=-1$ $B=1$

$u=-4$ $u=1$

$\displaystyle\int \frac{-1}{u+4}du + \int \frac{1}{u-1} = -\ln|u+4| + \ln|u-1| = \ln\left|\frac{u-1}{u+4}\right|$

$= \boxed{\ln\left|\frac{\sin x - 1}{\sin x + 4}\right| + C}$

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31) $\int \frac{e^x}{(e^x-1)(e^x+4)}\,dx$ $\quad u=e^x,\ \frac{du}{dx}=e^x$ $\quad = \int \frac{du}{(u-1)(u+4)} = \frac{A}{u-1} + \frac{B}{u+4}$

$u=1 \qquad u=-4$

$A=\frac{1}{5} \qquad B=\frac{-1}{5}$

$$\int \frac{1}{5(u-1)} + -\frac{1}{5(u+4)}\,du = \frac{1}{5}\ln|u-1| - \frac{1}{5}\ln|u+4| + C$$

$$= \frac{1}{5}\ln\left|\frac{u-1}{u+4}\right| + C = \boxed{\frac{1}{5}\ln\left|\frac{e^x-1}{e^x+4}\right| + C}$$

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34) $\int \frac{1}{\sqrt{x}-\sqrt[3]{x}}\,dx = \int \frac{1}{x^{1/2}-x^{1/3}}\,dx$ $\quad u=x^{1/6} \rightarrow u^6=x$

$\frac{du}{dx}=\frac{1}{6}x^{-5/6} \qquad dx=6u^5\,du$

$u^2=x^{1/3}$

$u^3=x^{1/2}$

$$\longrightarrow \int \frac{6u^5\,du}{u^3-u^2} = 6\int \frac{u^3}{u-1}\,du$$

$$u-1\overline{)u^3} = u^2+u+1+\frac{1}{u-1}$$

$-u^3 + u^2$

u^2

$-u^2 + u$

$+u$

$-u + 1$

1

$$6\int u^2+u+1+\frac{1}{u-1}\,du$$

$$6\left[\frac{u^3}{3}+\frac{u^2}{2}+u+\ln|u-1|\right]+C = \boxed{2\sqrt{x}+3x^{1/3}+6x^{1/6}+6\ln|x^{1/6}-1|+C}$$