2017

8.7 Indeterminate Forms, L'Hopital's Rule p.564 #5-58

45) $\lim_{x\to\infty}\left(x\sin\frac{1}{x}\right) \rightarrow \lim_{x\to\infty}\frac{\sin\frac{1}{x}}{\frac{1}{x}} = \frac{0}{0} \xrightarrow{L'H} \frac{\cos(\frac{1}{x})\cdot -\cancel{1x^{-2}}}{-\cancel{1x^{-2}}}$ $\quad \sin(x^{-1})$

$\lim_{x\to\infty}\cos(\frac{1}{x}) = \boxed{1}$

46) $\lim_{x\to\infty} x\tan(\frac{1}{x}) = \lim_{x\to\infty}\frac{\tan(1/x)}{(1/x)} = \lim_{x\to\infty}\frac{\sec^2(1/x)\cdot -1\cancel{x^{-2}}}{-1\cancel{x^{-2}}} = \boxed{1}$

49) $\lim_{x\to\infty} x^{1/x}$

$y = \lim_{x\to\infty} x^{1/x}$

$\ln y = \lim_{x\to\infty}\ln x^{1/x}$

$\ln y = \lim_{x\to\infty}\frac{1}{x}\cdot\ln x$

$\ln y = \lim_{x\to\infty}\frac{\ln x}{x} = \frac{\infty}{\infty} \rightarrow L'H \quad \frac{\frac{1}{x}}{1}$

$\ln y = \lim_{x\to\infty}\frac{1}{x}$

$\ln y = 0$

$e^0 = y = 1$

$\boxed{\lim_{x\to\infty} x^{1/x} = 1}$

50) $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$

$y = \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$

$\ln y = \lim_{x\to\infty}\ln\left(1+\frac{1}{x}\right)^x$

$\ln y = \lim_{x\to\infty} x\cdot\ln\left(1+\frac{1}{x}\right)$

$\ln y = \lim_{x\to\infty}\frac{\ln(1+\frac{1}{x})}{\frac{1}{x}} = \frac{0}{0} \rightarrow L'H$

$\ln y = \lim_{x\to\infty}\frac{\frac{-\cancel{x^{-2}}}{1+\frac{1}{x}}}{-\cancel{x^{-2}}}$

$\ln y = \lim_{x\to\infty}\frac{1}{1+\frac{1}{x}} = 1$

$\ln y = 1$

$e^1 = y$

$\boxed{\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e}$

53) $\lim_{x\to 0^+}\left[3(x)^{x/2}\right]$

$\ln y = \lim_{x\to 0^+}\left(\ln(3(x)^{x/2})\right)$

$\ln y = \lim_{x\to 0^+}\ln 3 + \ln x^{x/2}$

$\ln y = \lim_{x\to 0^+}\ln 3 + \frac{x}{2}\ln x$

$= \lim_{x\to 0^+}\ln 3 + \frac{\ln x}{(\frac{2}{x})} = \frac{\infty}{\infty}\ (L'H)$

$\ln y = \lim_{x\to 0^+}\ln 3 + \frac{\frac{1}{x}}{-2x^{-2}}$

$= \lim_{x\to 0^+}\ln 3 - \frac{x^2}{2} \rightarrow 0$

$= \ln 3$

$\ln y = \ln 3 \quad y = 3$

$\boxed{\lim_{x\to 0^+}\left[3(x)^{x/2}\right] = 3}$

54) $\lim_{x\to 4^+} [3(x-4)]^{x-4}$

$\ln y = \lim_{x\to 4^+} \ln [3(x-4)]^{x-4}$

$\ln y = \lim_{x\to 4^+} (x-4)\cdot \ln 3(x-4)$

$\ln y = \lim_{x\to 4^+} \frac{\ln 3(x-4)}{1/x-4} = \frac{\ln(3)(x-4)}{(x-4)^{-1}} \; \frac{0}{0}$

$\ln y = \lim_{x\to 4^+} \frac{\frac{3}{3(x-4)}}{-1(x-4)^{-2}} = \frac{-3(x-4)^2}{3(x-4)} = -(x-4)$

$\ln y = \lim_{x\to 4^+} -(x-4) = 0$

$\ln y = 0 \to e^0 = 1 = y$

$\boxed{\lim_{x\to 4^+} [3(x-4)]^{x-4} = 1}$

57) $\lim_{x\to 2^+} \left(\frac{8}{x^2-4} - \frac{x}{x-2}\right)$

$\lim_{x\to 2^+} \frac{8-x(x+2)}{x^2-4}$

$\lim_{x\to 2^+} \frac{8-x^2-2x}{x^2-4}$

$\lim_{x\to 2^+} \frac{-(x^2+2x-8)}{(x+2)(x-2)}$

$\lim_{x\to 2^+} \frac{-(x+4)\cancel{(x-2)}}{(x+2)\cancel{(x-2)}} = -\frac{6}{4} = \boxed{\frac{-3}{2}}$

58) $\lim_{x\to 2^+} \left(\frac{1}{x^2-4} - \frac{\sqrt{x-1}}{x^2-4}\right)$

$\lim_{x\to 2^+} \frac{1-\sqrt{x-1}}{x^2-4} = \frac{0}{0} \to L'H$

$\lim_{x\to 2^+} \frac{0-\frac{1}{2}(x-1)^{-1/2}(1)}{2x}$

$\lim_{x\to 2^+} \frac{-1}{4x\sqrt{x-1}} = \frac{-1}{4(2)(1)} = \boxed{-\frac{1}{8}}$