L'Hôpital's Rule (or Bernoulli's Rule)

If $\lim_{x\to a}\frac{f(x)}{g(x)}$ yields either of the indeterminate forms $\frac{0}{0}$ or $\pm\frac{\infty}{\infty}$, then $\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}$.

The rule works great, but it only works with the two forms $\frac{0}{0}$ or $\pm\frac{\infty}{\infty}$. There are other indeterminate forms including 0^0, 1^∞, $\infty-\infty$, $0\cdot\infty$, and ∞^0. We can still use the rule, but we have to first convert them to $\frac{0}{0}$ or $\pm\frac{\infty}{\infty}$.

1.If Indeterminate form is 0^0, 1^∞, or ∞^0 → rewrite as equation and use Log Differentiation

2. If Indeterminate form is $\infty-\infty$ → find common denominator, which will get the expression into a single quotient, ready to evaluate.

3. If Indeterminate form is $0\cdot\infty$ → rewrite as a quotient, bring ∞ or 0 down to denominator to create $\frac{0}{0}$ *or* $\pm\frac{\infty}{\infty}$

Example 1:

a) $\lim_{x\to\infty} e^{-x}\sqrt{x}=$

b) $\lim_{x\to 1^+}\left(\frac{1}{\ln x}-\frac{1}{x-1}\right)=$

Example 2:

a) $\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^x=$

b) $\lim_{x\to 0^+} x^x=$

c) $\lim_{x\to\infty} x^{1/x}=$