2017

8.8 Improper Integrals p.575 #19-44 D2S2

19) $\int_1^\infty \frac{3}{\sqrt[3]{x}}\,dx$

$\lim_{b\to\infty}\int_1^b 3x^{-1/3}\,dx$

$\lim_{b\to\infty} \frac{3x^{2/3}}{2/3}\Big]_1^b$

$\frac{9}{2}x^{2/3}\Big]_1^b =$

$\lim_{b\to\infty} \frac{9}{2}b^{3/2} - \frac{9}{2}(1)^{3/2} = \boxed{\infty}$

20) $\int_1^\infty \frac{4}{\sqrt[4]{x}}\,dx$

$\lim_{b\to\infty}\int_1^b 4x^{-1/4}\,dx$

$\lim_{b\to\infty} \frac{4x^{3/4}}{3/4}$

$\frac{16}{3}x^{3/4}\Big]_1^b$

$\lim_{b\to\infty} \frac{16}{3}b^{3/4} - \frac{16}{3}(1)^{3/4} = \boxed{\infty}$

23) $\int_0^\infty x^2 e^{-x}\,dx$

*IBP, Tab Method

	u	dv
+	x^2	e^{-x}
−	$2x$	$-e^{-x}$
+	2	e^{-x}
−	0	$-e^{-x}$

$\lim_{b\to\infty} -x^2e^{-x} - 2xe^{-x} - 2e^{-x}\Big]_0^b$

$\lim_{b\to\infty} \frac{-x^2-2x-2}{e^x}\Big]_0^b = \lim_{b\to\infty} \frac{-b^2-2b-2}{e^b} - \frac{(-2)}{e^0} = 0 + 2 = \boxed{2}$

by L'Hopital's or comparative growth rate $= 0$

24) $\int_0^\infty e^{-x}\cos x\,dx$

IBP (LIPET) $u = e^{-x}$ $dv = \cos x$

$du = -e^{-x}dx$ $v = \sin x$

$\int u\,dv = uv - \int v\,du$

$\lim_{b\to\infty} e^{-x}\sin x - \int \sin x(-e^{-x})\,dx = e^{-x}\sin x - e^{-x}\cos x - \int \cos x e^{-x}\,dx$

$2\int e^{-x}\cos x\,dx = e^{-x}(-\cos x + \sin x)$

$\int e^{-x}\cos x\,dx = \frac{1}{2}e^{-x}(-\cos x + \sin x)\Big]_0^b$

$= \frac{1}{2e^b}(-\cos b + \sin b) - \frac{1}{2e^0}(-\cos 0 + \sin 0)$

$0 - \frac{1}{2}(-1)$

$= \boxed{1/2}$

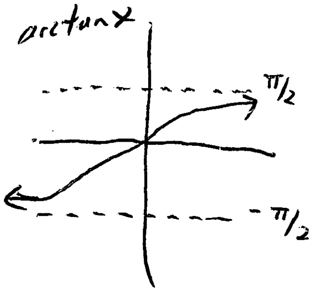
27) $\int_{-\infty}^{\infty} \frac{4}{16+x^2}dx = \int_{-\infty}^{0} \frac{4}{16+x^2}dx + \int_{0}^{\infty} \frac{4}{16+x^2}dx$ $\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\left(\frac{u}{a}\right)$

$a=4$
$u=x$

$\lim_{b\to-\infty} 4\cdot\frac{1}{4}\arctan\left(\frac{x}{4}\right)\Big]_b^0$ | $\lim_{b\to\infty} 4\cdot\frac{1}{4}\arctan\left(\frac{x}{4}\right)\Big]_0^b$

$0 - \arctan\left(\frac{b}{4}\right) = -\left(-\frac{\pi}{2}\right)$ | $\lim_{b\to\infty} \arctan\left(\frac{b}{4}\right) - \arctan 0 = \frac{\pi}{2}$

arctan x



$\frac{\pi}{2} + \frac{\pi}{2} = \frac{2\pi}{2} = \boxed{\pi}$

28) $\int_0^{\infty} \frac{x^3}{(x^2+1)^2}dx = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{(x^2+1)^2}$

Partial Fraction Decomp.

$x^3 = (Ax+B)(x^2+1) + Cx + D$
$x^3 = Ax^3 + Bx^2 + Ax + B + Cx + D$
$x^3 = Ax^3 + Bx^2 + (A+C)x + B + D$

$A=1$
$B=0$
$A+C=0 \to 1+C=0,\ C=-1$
$B+D=0 \quad 0+D=0,\ D=0$

$\int_0^{\infty} \frac{1x+0}{x^2+1}dx + \int_0^{\infty} \frac{-1x+0}{(x^2+1)^2}$ $u=x^2+1 \quad dx=\frac{du}{2x}$ $\frac{du}{dx}=2x$ $\int \frac{-x}{u^2}\cdot\frac{du}{2x} = \frac{-1}{2}\int u^{-2} = \frac{-1}{2}\left(\frac{u^{-1}}{-1}\right)$

$\ln|x^2+1|\Big]_0^b$ | $+\frac{1}{2(x^2+1)}\Big]_0^b$

$\lim_{b\to\infty} \ln|b^2+1| - 0$ | $\frac{1}{2(b^2+1)} - \frac{1}{2}$

$\infty + 0 - \frac{1}{2} = \boxed{\infty \text{ (diverges)}}$

31) $\int_0^\infty \cos(\pi x)\,dx$

$u = \pi x$ | $dx = \frac{du}{\pi}$

$\frac{du}{dx} = \pi$

$\int \cos u \cdot \frac{du}{\pi}$

$\frac{1}{\pi}\int \cos u\,du$

$\frac{1}{\pi}\sin(\pi x)\Big]_0^b$

$\lim_{b\to\infty} \frac{1}{\pi}\sin(\pi b) - \frac{1}{\pi}\sin(0) = \boxed{\infty\text{, diverges}}$

32) $\int_0^\infty \sin(x/2)\,dx = \lim_{b\to\infty} -2\cos(x/2)\Big|_0^b = -2\cos(b/2) + 2\cos(0) = \boxed{\infty \text{ diverges}}$

$u = x/2$ | $dx = 2du$

$\frac{du}{dx} = \frac{1}{2}$

$\int \sin u \cdot 2du$

35) $\int_0^2 \frac{1}{\sqrt[3]{x-1}}\,dx = \int_0^{1^-} \frac{1}{(x-1)^{1/3}}\,dx + \int_{1^+}^2 \frac{1}{(x-1)^{1/3}}\,dx$

* Interior Discontinuity at $x=1$

$\int (x-1)^{-1/3}dx \to \lim_{b\to 1^-} \frac{(x-1)^{2/3}}{2/3}\Big]_0^b$

$\lim_{b\to 1^-} \frac{3}{2}(b-1)^{2/3} - \frac{3}{2}(0-1)^{2/3} = 0 - \frac{3}{2}$

$= -\frac{3}{2}$

$\lim_{b\to 1^+} \frac{3}{2}(x-1)^{2/3}\Big]_b^2$

$\frac{3}{2}(1)^{2/3} - \frac{3}{2}(0)^{2/3} = \frac{3}{2}$

$-\frac{3}{2} + \frac{3}{2} = \boxed{0}$

36) $\int_0^8 \frac{3}{\sqrt{8-x}}\,dx$

$= \int 3(8-x)^{-1/2}dx$

$u = 8-x$

$\frac{du}{dx} = -1$

$3\int u^{-1/2}(-du)$

$-3\frac{u^{1/2}}{1/2}$

$-6(8-x)^{1/2}\Big]_0^b$

$\lim_{b\to 8^-} -6(8-b)^{1/2} - (-6(8-0)^{1/2})$

$= 0 + 6\sqrt{8} = 6\cdot 2\sqrt{2}$

$= \boxed{12\sqrt{2}}$

39) $\int_0^{\pi/2} \tan\theta \, d\theta = \lim_{b \to \pi/2^-} -\ln|\cos\theta| \Big]_0^b = -\ln(\cos b) - (-\ln \cos 0)$

$= -\infty + 0 = \boxed{-\infty \text{ diverges}}$

40) $\int_0^{\pi/2} \sec\theta \, d\theta = \lim_{b \to \pi/2^-} \ln|\sec\theta + \tan\theta| \Big]_0^b = \boxed{\infty \text{ diverges}}$

43) $\int_3^5 \frac{1}{\sqrt{x^2-9}} \, dx$

$u = x$

$a = 3$

Use 8.6 Integration Table
Formula # 98

$\int \frac{du}{\sqrt{u^2-a^2}} = \ln\left(u + \sqrt{u^2-a^2}\right) + C$

$= \lim_{b \to 3^+} \ln\left(x + \sqrt{x^2-9}\right) \Big]_b^5$

$= \lim_{b \to 3^+} \ln\left(5 + \sqrt{16}\right) - \ln(3+0) = \ln(9) - \ln 3$

$= \ln(9/3) = \boxed{\ln 3}$

44) $\int_0^5 \frac{1}{25-x^2} \, dx$

$A = \frac{1}{10}$ $\quad B = \frac{1}{10}$

$\int_0^5 \frac{1}{(5-x)(5+x)} = \frac{A}{5-x} + \frac{B}{5+x}$

$x = 5 \qquad x = -5$

$\frac{1}{10} \int \frac{dx}{5-x} = -\frac{1}{10} \ln|5-x| + \frac{1}{10} \ln|5+x|$

$\frac{1}{10} \ln\left|\frac{5+x}{5-x}\right|$

$u = 5-x$

$\frac{du}{dx} = -1$

$\lim_{b \to 5^-} \frac{1}{10} \ln\left|\frac{5+x}{5-x}\right| \Big]_0^b$

$\frac{1}{10} \ln\left|\frac{5+b}{5-b}\right| - \frac{1}{10} \ln\left|\frac{5}{5}\right|$

$\infty - 0$

$= \boxed{\infty, \text{ diverges}}$