Ch. 9.1 p. 593 #1-62 D252

Describe pattern

arithmetic sequence

13) $\underbrace{2, 5}_{3}, 8, 11, \ldots$ $\boxed{14, 17}$

$a_n = a_1 + d(n-1)$

$a_n = 2 + 3(n-1)$

$a_n = 2 + 3n - 3$

$a_n = 3n - 1$

$a_5 = 3(5) - 1 = 14$

$a_6 = 3(6) - 1 = 17$

14) $\underbrace{8, 13}_{5}, 18, 23, 28, \_\_, \_\_$

$a_n = a_1 + d(n-1)$

$a_n = 8 + 5(n-1)$

$a_n = 8 + 5n - 5$

$a_n = 5n + 3$

$\boxed{a_6 = 5(6) + 3 = 33 \quad a_7 = 5(7) + 3 = 38}$

18) Simplify ratio of factorials: $\frac{n!}{(n+2)!} = \frac{\cancel{n!}}{(n+2)(n+1)\cancel{n!}} = \boxed{\frac{1}{(n+2)(n+1)}}$

22) $\lim_{n \to \infty} 6 + \frac{2}{n^2} = 6 + 0 = \boxed{6}$

26) $\lim_{n \to \infty} \frac{1}{n^{3/2}} = \boxed{0}$

Determine Convergence/Divergence of sequence [Does $\lim_{n \to \infty} a_n$ = Real number?]

30) $\lim_{n \to \infty} \left(8 + \frac{5}{n}\right) = 8 + 0 = \boxed{8}$ Converges

34) $\lim_{n \to \infty} \frac{\sqrt[3]{n}}{\sqrt[3]{n} + 1} = \boxed{1}$, converges

38) $\lim_{n\to\infty} \frac{(n-2)!}{n!} = \lim_{n\to\infty} \frac{\cancel{(n-2)!}}{n\cdot(n-1)\cdot\cancel{(n-2)!}} = \lim_{n\to\infty} \frac{1}{n(n-1)} = \boxed{0}$, converges

42) $a_n = -3^{-n} = \frac{-1}{3^n}$ $\lim_{n\to\infty} \frac{-1}{3^n} = \boxed{0}$, converges

Find expression for n^{th} term of sequence

46) $1, \frac{1}{2}, \frac{1}{6}, \frac{1}{24}, \frac{1}{120}, \ldots$ $\boxed{\frac{1}{n!}}$

50) $2, 24, 720, 40{,}320, 3{,}628{,}800,$ $\boxed{(2n)!}$

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Determine if sequence is monotonic (always increasing or always decreasing)

Determine if sequence is bounded.

54) $a_n = \frac{3n}{n+2}$ $f(x) = \frac{3x}{x+2}$ $f'(x) = \frac{3(x+2)-(3x)(1)}{(x+2)^2} = \frac{3x+6-3x}{(x+2)^2}$

$f'(x) = \frac{6}{(x+2)^2} > 0$ always increasing

$\lim_{n\to\infty} \frac{3n}{n+2} = 3$ (bounded)

58) $a_n = \left(\frac{3}{2}\right)^n$ $\lim_{n\to\infty} \left(\frac{3}{2}\right)^n = \infty$, monotonic, unbounded

Use theorem 9.5 to show sequence converges:

9.5 Theorem: If a sequence a_n is bounded and monotonic, then it converges.

62) $a_n = 5 - \frac{2}{n}$ Since $5 - \frac{2}{n} < 5 - \frac{2}{n+1}$

$a_n < a_{n+1}$, montonic sequence

$\lim_{n\to\infty} 5 - \frac{2}{n} = 5$ (bounded)