Ch. 9.2 p. 601 #3-54 O1S2

Find sequence of Partial Sums S_1, S_2, S_3, S_4, S_5

3) $3 - \frac{9}{2} + \frac{27}{4} - \frac{81}{8} + \frac{243}{16} - \dots$

$S_1 = 3$

$S_2 = 3 - 9/2 = -1.5$

$S_3 = 3 - 9/2 + 27/4 = 5.25$

$S_4 = 3 - 9/2 + 27/4 - \frac{81}{8} = -4.875$

$S_5 = 3 - 9/2 + 27/4 - \frac{81}{8} + \frac{243}{16} = 10.3125$

6) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n!} = 1 - 1/2 + \frac{1}{6} - \frac{1}{24} + \frac{1}{120}$

$S_1 = 1$

$S_2 = 1 - 1/2 = 0.5$

$S_3 = 1 - 1/2 + 1/6 = 0.6667$

$S_4 = 1 - 1/2 + 1/6 - \frac{1}{24} \approx 0.6250$

$S_5 = 1 - 1/2 + 1/6 - \frac{1}{24} + \frac{1}{120} \approx 0.6333$

<u>Recall n^{th}-Term Test for Divergence:</u>

*If $\lim_{n\to\infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ diverges

<u>Careful</u>: The converse of this test is <u>not</u> true.
If $\lim_{n\to\infty} a_n = 0$, this n^{th} term test is inconclusive.

12) $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2+1}}$ $\quad \lim_{n\to\infty} \frac{n}{\sqrt{n^2+1}} = 1 \neq 0$, Diverges by n^{th}-term test.

<u>Recall Geometric Series Test: (GST)</u>

a) Geometric series with ratio $|r| \geq 1$ will diverge and converge if $0 < |r| < 1$

b) For converging geometric series, Sum $= \frac{a_1}{1-r}$ $\quad \boxed{\sum_{n=0}^{\infty} ar^n = \frac{a_1}{1-r}}$

18) $\sum_{n=0}^{\infty} (-0.6)^n$ $\quad |r| = |-0.6| < 1$, converges by GST

← Partial Fraction Decomposition

$$A=2 \qquad B=-2$$

21) $$\sum_{n=1}^{\infty} \frac{6}{n(n+3)} = \frac{A}{\underset{n=0}{n}} + \frac{B}{\underset{n=-3}{n+3}} = 2\sum_{n=1}^{\infty} \frac{1}{n} - \frac{1}{n+3} =$$

$$= 2\left[\left(1-\tfrac{1}{4}\right)+\left(\tfrac{1}{2}-\tfrac{1}{5}\right)+\left(\tfrac{1}{3}-\tfrac{1}{6}\right)+\left(\tfrac{1}{4}-\tfrac{1}{7}\right)+\ldots\right] = 2\left[1+\frac{1}{2}+\frac{1}{3}\right] = \boxed{\frac{11}{3}}$$

24) $$\sum_{n=1}^{\infty} 10\left(-\tfrac{1}{4}\right)^{n-1} = \sum_{n=1}^{\infty} 10\left(-\tfrac{1}{4}\right)^{n-1} \qquad a_1 = 10 \quad r = -\tfrac{1}{4} \qquad \text{Sum} = \frac{a_1}{1-r} = \frac{10}{1-\left(-\tfrac{1}{4}\right)}$$

$$\text{Sum} = \frac{10}{5/4} = \frac{40}{5} = \boxed{8}$$

$$A=2 \qquad B=-2$$

27) $$\sum \frac{4}{n(n+2)} = \frac{A}{\underset{n=0}{n}} + \frac{B}{\underset{n=-2}{n+2}} = 2\sum_{n=1}^{\infty} \frac{1}{n} - \frac{1}{n+2}$$

$$= 2\left[\left(\frac{1}{1}-\cancel{\frac{1}{3}}\right)+\left(\frac{1}{2}-\cancel{\frac{1}{4}}\right)+\left(\cancel{\frac{1}{3}}-\cancel{\frac{1}{5}}\right)+\ldots\right] = 2\left[1+\tfrac{1}{2}\right] = 2\left(\tfrac{3}{2}\right) = \boxed{3}$$

30) $$\underbrace{9-3}_{-1/3}+1-\tfrac{1}{3} \qquad r=-\tfrac{1}{3} \qquad \text{Sum} = \frac{a_1}{1-r} = \frac{9}{1-\left(-\tfrac{1}{3}\right)} = \frac{9}{4/3} = \boxed{\frac{27}{4}}$$

33) $$\sum_{n=1}^{\infty} (\sin 1)^n$$

since $|\sin 1| < 1$, series converge by GST (Geometric Series Test)

$$\text{Sum} = \frac{a_1}{1-r} = \frac{\sin 1}{1-\sin 1} \approx \boxed{5.3080}$$

Using Geometric Series:

36) $0.\overline{36} = \sum \frac{36}{100}\left(\frac{1}{100}\right)^n$ $\quad \text{Sum} = \frac{1}{1-r} = \frac{\frac{36}{100}}{1-\frac{1}{100}} = \frac{\frac{36}{100}}{\frac{99}{100}} = \frac{36}{99}$

$0.363636 = \underbrace{0.36 + 0.0036}_{\frac{1}{100}} + 0.000036 + \ldots$

$\frac{36}{99} = \boxed{\frac{4}{11}}$

39) $0.0\overline{75} = \underbrace{0.075 + 0.00075}_{\frac{1}{100}} + 0.0000075$

$\sum \frac{75}{1000}\left(\frac{1}{100}\right)^n$ $\quad \text{Sum} = \frac{\frac{75}{1000}}{1-\frac{1}{100}} = \frac{3/40}{\frac{99}{100}} = \frac{3}{40} \cdot \frac{100}{99} = \boxed{\frac{5}{66}}$

Determine Convergence/divergence of series:

42) $\sum_{n=0}^{\infty} \frac{3^n}{1000} = \lim_{n\to\infty} \frac{3^n}{1000} = \infty \neq 0$, diverges by n^{th} term test.

45) $\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+2}\right)$

$S_n = \left(1 - \cancel{\tfrac{1}{3}}\right) + \left(\tfrac{1}{2} - \cancel{\tfrac{1}{4}}\right) + \left(\cancel{\tfrac{1}{3}} - \cancel{\tfrac{1}{5}}\right) + \ldots = \frac{3}{2}$, converges

51) $\sum_{n=1}^{\infty} \left(1 + \frac{k}{n}\right)^n$

54) $\sum_{n=1}^{n} \ln\left(\frac{n+1}{n}\right)$

$= \ln\left(\frac{2}{1}\right) + \ln\left(\frac{3}{2}\right) + \ldots$

$= \ln 2 - \ln 1 + \ln 3 - \ln 2 + \ln 4 - \ln 3 + \ldots \ln(n+1) - \cancel{\ln}(n)$

$= \ln(n+1) - \ln(1) = \ln(n+1) \neq 0$ Diverges